Finite Automata

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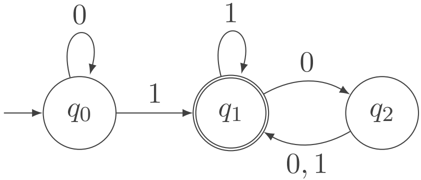
Finite Automaton: Example 1



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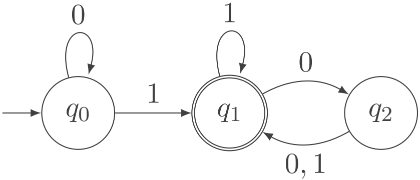
Finite Automaton: Example 1



Try running the automaton on the following strings.

1101, 1, 01, 11, 0101010101, 100, 0100,

110000, 0101000000, 0, 10, 101000

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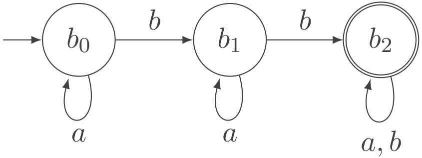
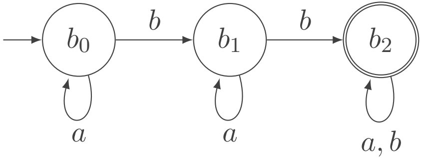
Finite Automaton: Example 2



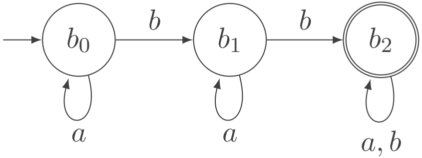
Finite Automaton: Example 2



Try running the automaton on the following strings.

$aaaa$, $ababa$, $bababb$, $abaa$

Finite Automaton: Example 2



Try running the automaton on the following strings.

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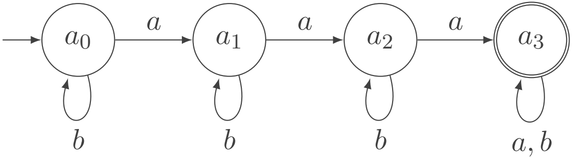
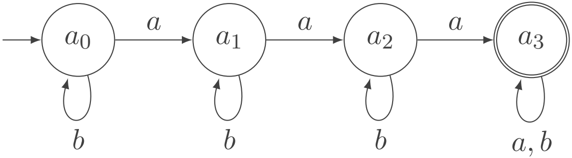
Describe the strings that the automaton recognises.

Finite Automaton: Example 3



Finite Automaton: Example 3



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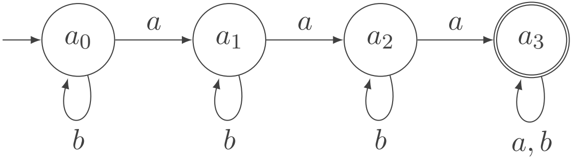
Finite Automaton: Example 3



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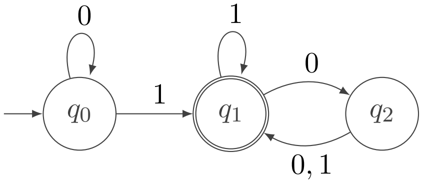
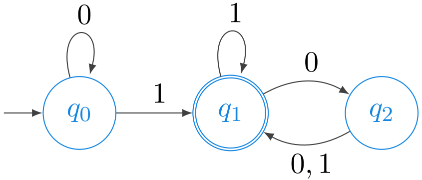
Finite Automaton: Concepts



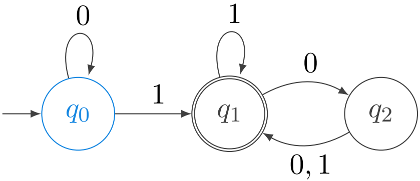
What are the essential concepts?

Finite Automaton: Concepts



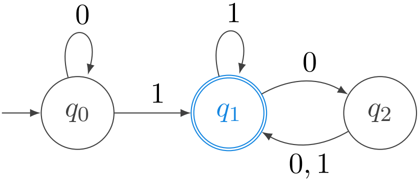
Set of states: $Q = \{q_0, q_1, q_2\}$

Finite Automaton: Concepts



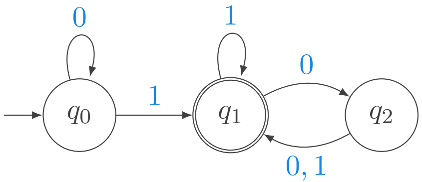
Initial state: $q_0 \in Q$

Finite Automaton: Concepts



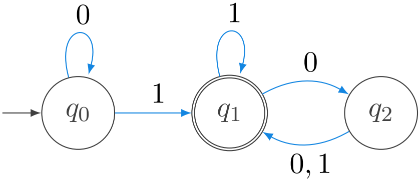
Set of final states: $F = \{q_1\} \subseteq Q$

Finite Automaton: Concepts



Alphabet: $\Sigma = \{0, 1\}$

Finite Automaton: Concepts



Transition function: $\delta = \{((q_0, 0), q_0), ((q_0, 1), q_1), ((q_1, 0), q_2), \ldots\}$

Deterministic Finite Automaton (DFA) definition

A DFA is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$ where

Q is a finite set of *states*,

Σ is a finite set called the *alphabet*,

δ is the *transition function* $(Q \times \Sigma \to Q)$,

q_0 is the *start state* $(\in Q)$, and

F is the set of *accept states* $(\subseteq Q)$.

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Example 1 definition

$$\begin{aligned}
Q =&\ \{q_0, q_1, q_2\} \\
\Sigma =&\ \{0, 1\} \\
\delta =&\ \{((q_0, 0), q_0), ((q_0, 1), q_1),\ ((q_1, 0), q_2), ((q_1, 1), q_1), \\
&\ ((q_2, 0), q_1), ((q_2, 1), q_1)\} \\
q_0 =&\ q_0 \\
F =&\ \{q_1\}
\end{aligned}$$

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Example 2 definition

$$
\begin{aligned}
Q &= \{b_0, b_1, b_2\} \\
\Sigma &= \{a, b\} \\
\delta &= \{((b_0, a), b_0), ((b_0, b), b_1),\ ((b_1, a), b_1), ((b_1, b), b_2), \\
&\quad ((b_2, a), b_2), ((b_2, b), b_2)\} \\
q_0 &= b_0 \\
F &= \{b_2\}
\end{aligned}
$$

Example 3 definition

$$
\begin{aligned}
Q &= \{a_0, a_1, a_2, a_3\} \\
\Sigma &= \{a, b\} \\
\delta &= \{((a_0, a), a_1), ((a_0, b), a_0),\ ((a_1, a), a_2), ((a_1, b), a_1), \\
&\quad ((a_2, a), a_3), ((a_2, b), a_2)\},\ ((a_3, a), a_3), ((a_3, b), a_3)\} \\
q_0 &= a_0 \\
F &= \{a_3\}
\end{aligned}
$$

Non-determinism

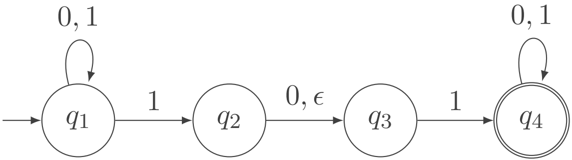
DFAs always have exactly one state to transition to when in any given state and reading any given symbol.

Non-deterministic finite automata can have any number of arrows for each state and symbol.

The empty string ϵ is also used to label arrows that are followed without reading a character from the input, while also remaining in the original state.

Non-determinism can simplify automata but it can be shown that NFAs and DFAs recognise the same set of languages.

NFA example



Try running the following strings on the automaton.

111101, 00001010, 1110, ϵ

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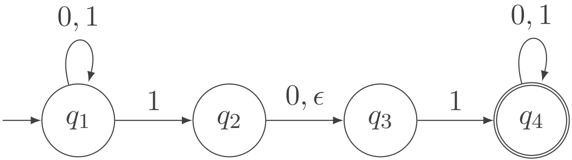
NFA example



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Describe in words the strings that the automaton recognises.

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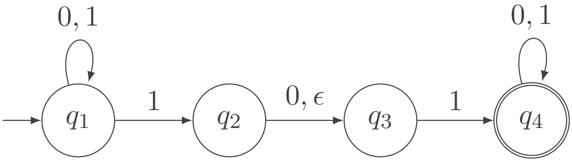
NFA example



Try running the following strings on the automaton.

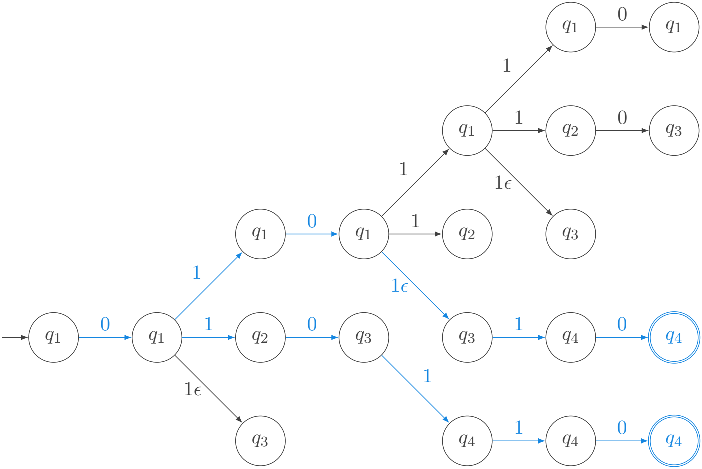
111101, 00001010, 1110, ϵ

Describe in words the strings that the automaton recognises.

(Answer: all strings that contain either 11 or 101.)

NFA example

Example: 010110 on the previous NFA.



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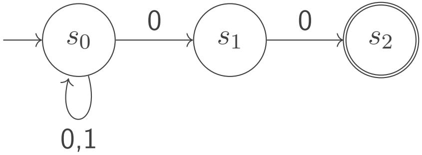
NFA example

Construct an NFA with alphabet $\{0, 1\}$ to recognise the language $\{w|w \text{ ends with } 00\}$. Try to do it with only three states.

Sipser Q 1. 7(a)

NFA example

Construct an NFA with alphabet $\{0, 1\}$ to recognise the language $\{w|w \text{ ends with } 00\}$. Try to do it with only three states.



Sipser Q 1. 7(a)

Non-deterministic Finite Automaton (NFA) definition

An NFA is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$ where

Q is a finite set of *states*,

Σ is a finite set called the *alphabet*,

δ is the *transition function* $(Q \times \Sigma_\epsilon \to \mathcal{P}(Q))$,

q_0 is the *start state* $(\in Q)$, and

F is the set of *accept states* $(\subseteq Q)$.

By Σ_ϵ we mean $\Sigma \cup \{\epsilon\}$. e.g. When $\Sigma = \{0, 1\}$, $\Sigma_\epsilon = \{\epsilon, 0, 1\}$.

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Powerset example

Take any set, say $A = \{0, 1, 2\}$. Its powerset is the set of all its subsets, and is denoted $\mathcal{P}(A)$.

$$\mathcal{P}(A) = \Big\{ \, \{\} \, , \, \{0\} \, , \, \{1\} \, , \, \{2\} \, , \, \{0,1\} \, , \, \{0,2\} \, , \, \{1,2\} \, , \, \{0,1,2\} \, \Big\}$$

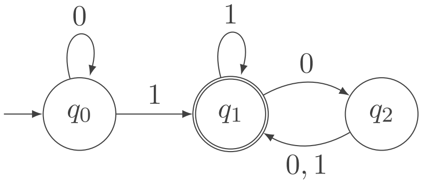
Strings of length p

Finite means that the number of states is finite.

Let p be the number of states in an automaton.

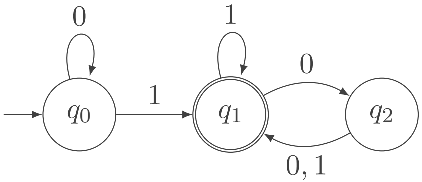
Every string of length at least p must visit one state twice.

Try the strings 000, 001, 010, etc on the following automaton.



Example of looping a string

Consider the string 110001 on the following automaton.



1	1	0	0	0	0	0	0	0	0	0	1
x		y		y		y		y		z	

Pumping lemma

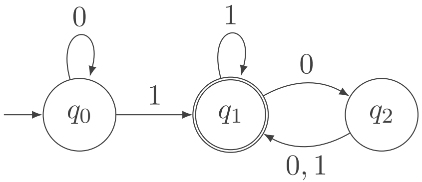
Theorem

Let A be a regular language. There is a positive integer p such that every string s of length at least p in A may be broken into three substrings $s = xyz$ where:

- xy^iz *is in A for all non-negative integers i.*
- *The length of y is greater than zero ($|y| > 0$).*
- *The length of xy is less than or equal to p ($|xy| \leq p$).*

Pumping lemma example

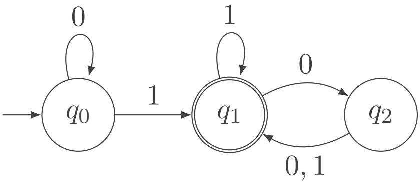
Consider the string 110001 on the following automaton.



Note that $|xy|$ must be less than p.

Pumping lemma example

Consider the string 110001 on the following automaton.



Note that $|xy|$ must be less than p.

1	1	1	1	1	1	1	1	0	0	0	1
x	y	y	y	y	y	y	y	z			

No automaton recognises $\{0^i 1^i\}$

Is there a finite automaton that recognises $\{0^i 1^i \mid i \in \mathbb{N}\}$?

If so, it has a finite number of states — let that number be p.

The string $0^p 1^p$ (p 0's followed by p 1's) must be accepted by the automaton.

By the pumping lemma, it can be broken into xyz where $xy^i z$ is also accepted for all $i \in \mathbb{N}$, y is of length greater than zero and xy is no longer than p.

So, $|xy| \leq p$ and $|y| > 0$, meaning y must be a string of 0's. However, then $xyyz$ contains more 0's than 1's — a contradiction.