Decision problems

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Decision problem

$$f : S \to T \text{ where } |T| = 2$$

A decision problem is a map to a set with two elements.

Example

$$f : \{0,1\}^* \to \{0,1\}$$

$$f(s) = 0 \Leftrightarrow |s| \equiv_2 0$$

Another example

$$f : \{0,1\}^* \to \{0,1\}$$

$$f(s) = 0 \Leftrightarrow wt(s) \equiv_2 0$$

Recap on Languages

Alphabet: finite set of symbols, denoted Σ.

String: tuple w over Σ.

Star: all strings over Σ, denoted Σ^*.

Language: subset L of Σ^*.

Length: of a string, denoted $|w|$.

Turing machines recap

For a given input a Turing machine does one of three things:

Accepts the input string by finishing in the accept state in a finite number of steps.

Rejects the input string by finishing in the reject/fail state in a finite number of steps.

Continues indefinitely in some sort of infinite loop.

Remember there are a finite number of states and tape symbols.

Deciders

$$f : \Sigma^* \to \{q_f, q_a\}$$

Decider: a Turing machine that always finishes in a finite number of steps.

Decides: decides the language it accepts.

Decidable: a language is called decidable if any Turing machine decides it.

Important question: are all languages decidable?

PRIMES

Is there a decider that decides if the input is a prime number?

Yes

We can make a Turing machine that counts up to a number, divides and checks the remainder is zero.

We call the language containing strings representing the prime numbers in binary form PRIMES.

$$\text{PRIMES} = \{x \mid x \in \mathbb{N} \text{ and } x \text{ is prime}\}$$

So, PRIMES is decidable.

An undecidable language

- Encode all Turing machines as strings over some finite alphabet.
- Consider the subset of Turing machines that are deciders.
- This set is undecidable.
- Closely related to the halting problem.