Finite Automata

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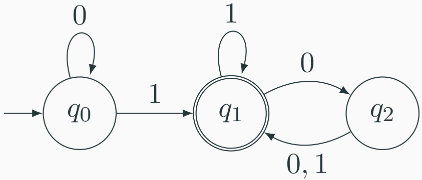
Finite Automaton: Example 1



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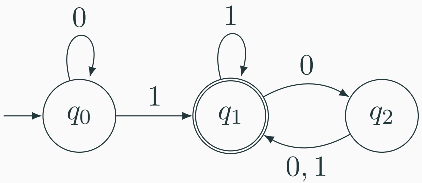
Finite Automaton: Example 1



Try running the automaton on the following strings.

1101, 1, 01, 11, 0101010101, 100, 0100,

110000, 0101000000, 0, 10, 101000

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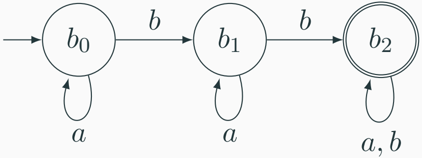
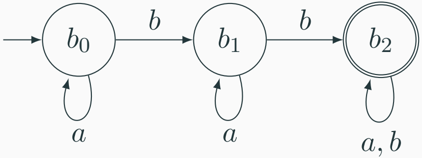
Finite Automaton: Example 2



Sipser chapter 1 4(a) – Part 2

Finite Automaton: Example 2

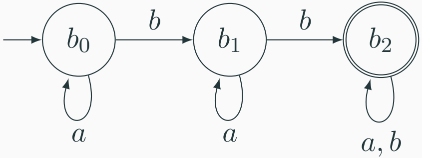


Try running the automaton on the following strings.

$aaaa$, $ababa$, $bababb$, $abaa$

Sipser chapter 1 4(a) – Part 2

Finite Automaton: Example 2



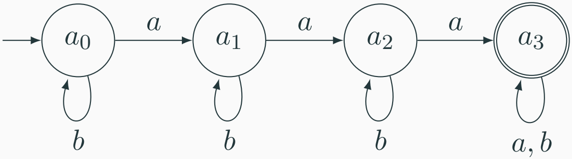
Try running the automaton on the following strings.

$aaaa$, $ababa$, $bababb$, $abaa$

Describe the strings that the automaton recognises.

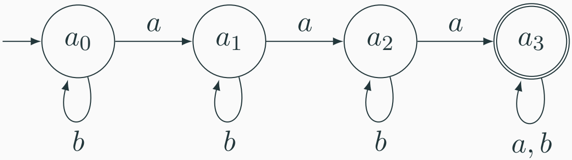
Sipser chapter 1 4(a) – Part 2

Finite Automaton: Example 3



Sipser chapter 1 4(a) – Part 1

Finite Automaton: Example 3



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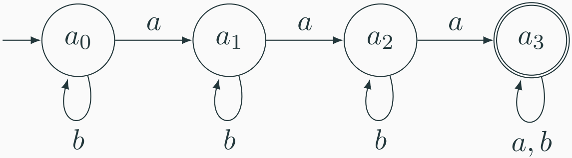
Finite Automaton: Example 3



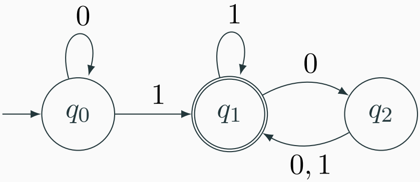
Try running the automaton on the following strings.

$aaaa$, $ababa$, $bababb$, $abaa$

Describe the strings that the automaton recognises.

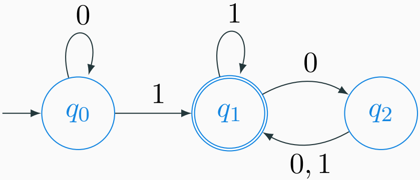
Sipser chapter 1 4(a) – Part 1

Finite Automaton: Concepts



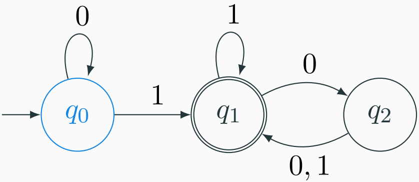
What are the essential concepts?

Finite Automaton: Concepts



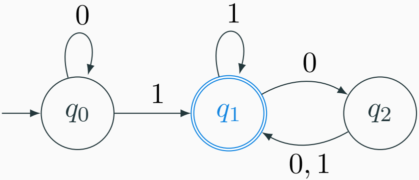
Set of states: $Q = \{q_0, q_1, q_2\}$

Finite Automaton: Concepts



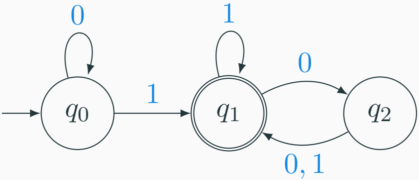
Initial state: $q_0 \in Q$

Finite Automaton: Concepts



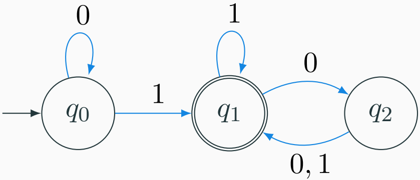
Set of final states: $F = \{q_1\} \subseteq Q$

Finite Automaton: Concepts



Alphabet: $\Sigma = \{0, 1\}$

Finite Automaton: Concepts



Transition function: $\delta = \{((q_0, 0), q_0), ((q_0, 1), q_1), ((q_1, 0), q_2), \ldots\}$

Deterministic Finite Automaton (DFA) definition

A DFA is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$ where

- Q is a finite set of *states*,
- Σ is a finite set called the *alphabet*,
- δ is the *transition function* $(Q \times \Sigma \rightarrow Q)$,
- q_0 is the *start state* $(\in Q)$, and
- F is the set of *accept states* $(\subseteq Q)$.

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Example 1 definition

$$
\begin{aligned}
Q &= \{q_0, q_1, q_2\} \\
\Sigma &= \{0, 1\} \\
\delta &= \{((q_0, 0), q_0), ((q_0, 1), q_1),\ ((q_1, 0), q_2), ((q_1, 1), q_1), \\
&\quad ((q_2, 0), q_1), ((q_2, 1), q_1)\} \\
q_0 &= q_0 \\
F &= \{q_1\}
\end{aligned}
$$

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Example 2 definition

$$
\begin{aligned}
Q &= \{b_0, b_1, b_2\} \\
\Sigma &= \{a, b\} \\
\delta &= \{((b_0, a), b_0), ((b_0, b), b_1),\ ((b_1, a), b_1), ((b_1, b), b_2), \\
&\quad ((b_2, a), b_2), ((b_2, b), b_2)\} \\
q_0 &= b_0 \\
F &= \{b_2\}
\end{aligned}
$$

Example 3 definition

$$
\begin{aligned}
Q &= \{a_0, a_1, a_2, a_3\} \\
\Sigma &= \{a, b\} \\
\delta &= \{((a_0, a), a_1), ((a_0, b), a_0),\ ((a_1, a), a_2), ((a_1, b), a_1), \\
&\quad ((a_2, a), a_3), ((a_2, b), a_2)\},\ ((a_3, a), a_3), ((a_3, b), a_3)\} \\
q_0 &= a_0 \\
F &= \{a_3\}
\end{aligned}
$$

Non-determinism

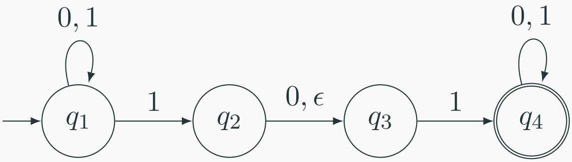
DFAs always have exactly one state to transition to when in any given state and reading any given symbol.

One arrow emerging from each state for each symbol. (Sometimes we use one arrow for two symbols for tidiness.)

Non-deterministic finite automata can have any number of arrows for each state and symbol.

Non-determinism simplifies automata theory, and it can be shown that NFAs and DFAs recognise the same set of languages.

NFA example



Try running the following strings on the automaton.

111101, 00001010, 1110, ϵ

Describe in words the strings that the automaton recognises.

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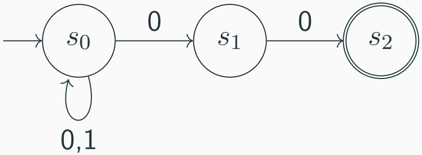
NFA example

Construct an NFA with alphabet $\{0, 1\}$ to recognise the language $\{w|w \text{ ends with } 00\}$. Try to do it with only three states.



Sipser Q 1. 7(a)

Non-deterministic Finite Automaton (NFA) definition

An NFA is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$ where

- Q is a finite set of *states*,
- Σ is a finite set called the *alphabet*,
- δ is the *transition function* ($Q \times \Sigma_\epsilon \to \mathcal{P}(Q)$),
- q_0 is the *start state* ($\in Q$), and
- F is the set of *accept states* ($\subseteq Q$).

By Σ_ϵ we mean $\Sigma \cup \{\epsilon\}$. e.g. When $\Sigma = \{0, 1\}$, $\Sigma_\epsilon = \{\epsilon, 0, 1\}$.

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Powerset example

Take any set, say $A = \{0, 1, 2\}$. Its powerset is the set of all its subsets, and is denoted $\mathcal{P}(A)$.

$$\mathcal{P}(A) = \Big\{ \, \{\} \, , \, \{0\} \, , \, \{1\} \, , \, \{2\} \, , \, \{0, 1\} \, , \, \{0, 2\} \, , \, \{1, 2\} \, , \, \{0, 1, 2\} \, \Big\}$$