Algorithms with graphs

ian.mcloughlin@gmit.ie

Decision tree

Rooted tree where each vertex represents a decision.

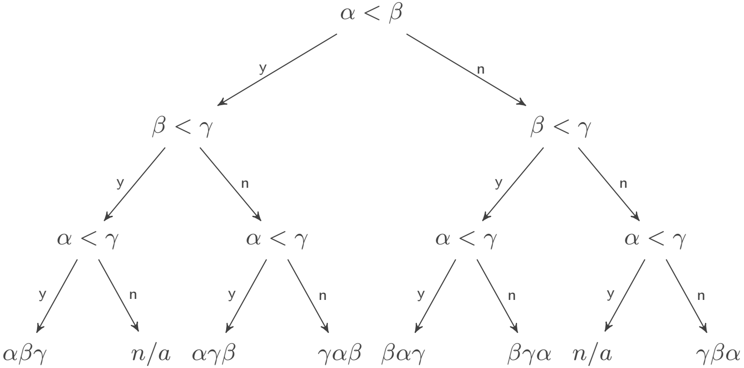
Results of a decision are represented by the edges to next level.

Decisions can connect to other decisions further down the tree.

Leaves represent final outcomes.

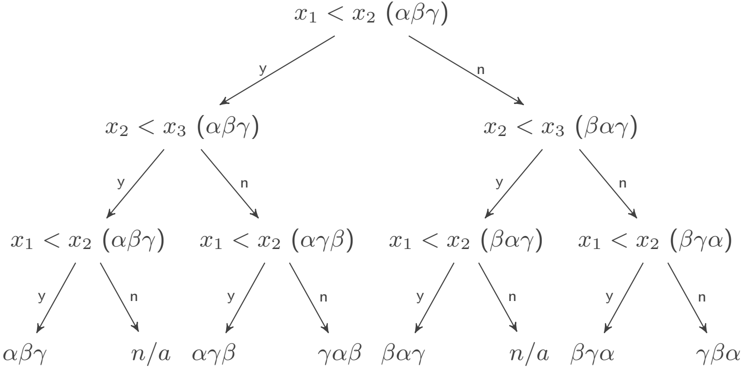
Sorting three items

Three items – (α, β, γ)



Decision tree for bubble sort with three items

Three items – (α, β, γ)



Heap sort

Comparison sorting algorithm.

Better worst case performance than quicksort: $O(n \log n)$ versus $O(n^2)$.

Same average case performance as quicksort: $O(n \log n)$.

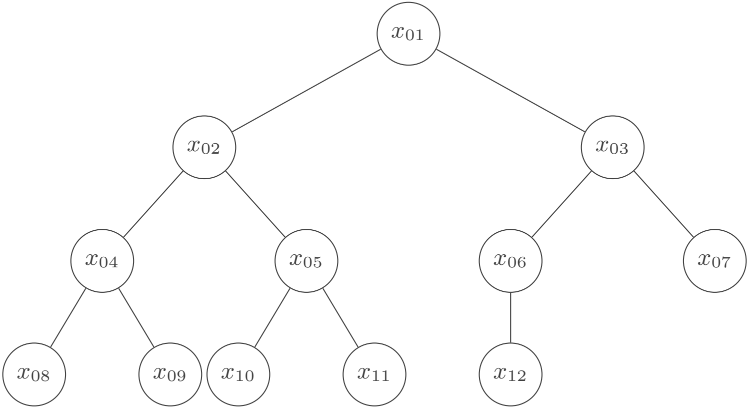
Better best case performance as quicksort: $O(n)$ versus $O(n \log n)$.

Claims that it's slower because it makes more swaps. Above is in terms of comparisons.

Heap is a binary tree where label of each parent is less than or equal to those of children.

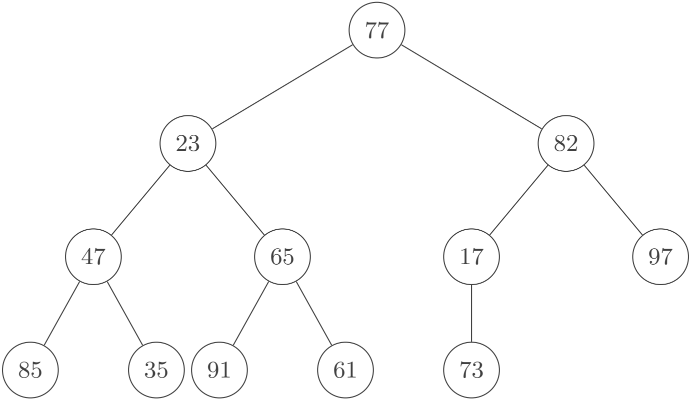
Heapsort initial tree

$$(x_{01}, x_{02}, x_{03}, x_{04}, x_{05}, x_{06}, x_{07}, x_{08}, x_{09}, x_{10}, x_{11}, x_{12})$$



Heapsort initial tree

$$(77, 23, 82, 47, 65, 17, 97, 85, 35, 91, 61, 73)$$



Tree to a heap

Start at the last parents and move backwards through the other parents as follows.

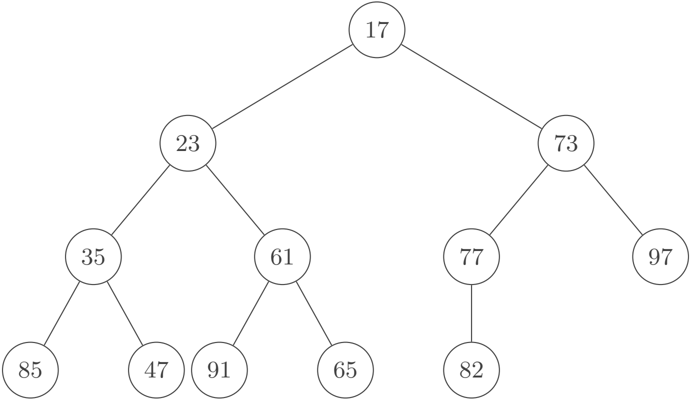
Suppose current parent is x_r and the trees at x_{2r} and x_{2r+1} are already heaps.

If x_{2r} or x_{2r+1} is smaller than x_r then swap x_r with the smaller child.

If necessary apply this procedure to the tree starting at the new child.

Heapsort - the heap

$$(17, 23, 73, 35, 61, 77, 97, 85, 47, 91, 65, 82)$$



$x_r < x_{2r}$ and $x_r < x_{2r+1}$

Transforming to a sorted list

Start with empty list.

Place the root of the heap at the end of the list.

Remove the last leaf and place it at the root.

Transform the tree to a heap again. This is relatively easy since the subtrees at x_2 and x_3 are already heaps.

Repeat until tree is empty.

$$(17, 23, 35, 47, 61, 65, 73, 77, 82, 85, 91, 97)$$

Searching trees and graphs

Searching is often visualised in graph form.

Usually on a spanning tree.

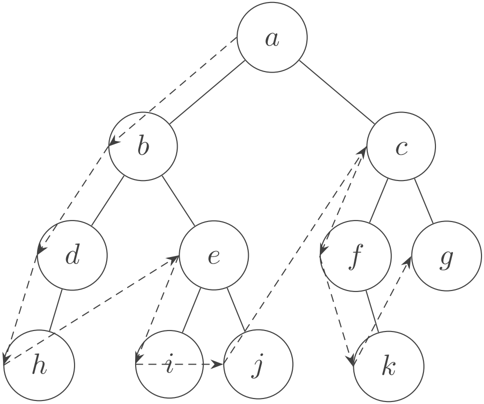
Main methods for searching through tree are depth-first and breadth-first.

Pick one node to start at (the root).

Depth-first means you go as far along each branch as possible before going to the next branch.

Breadth-first means you visit each vertex at level i before proceeding to level $i+1$.

Depth-first search



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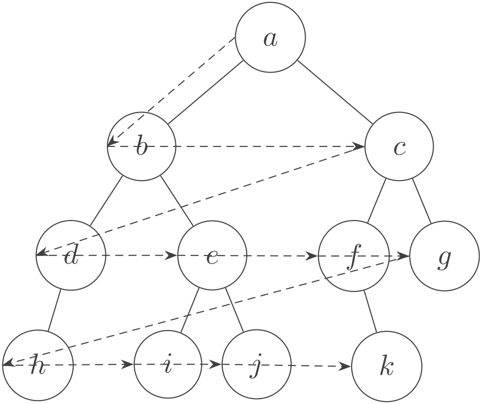
Breadth-first search



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