Maps

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Maps

Definition of map

A map from a set A to a set B is a subset M of $A \times B$ where each element of A appears as the first element of a tuple in M exactly once.

$$A = \{a, b, c\} \qquad B = \{x, y, z\}$$

Maps

- $\{(a, x), (b, x), (c, x)\}$
- $\{(a, x), (b, y), (c, z)\}$

Not maps

- $\{(a, x), (a, y), (b, x), (c, x)\}$
- $\{(a, x), (b, y)\}$

One-to-one map

$$A = \{1, 2, 3\} \quad B = \{a, b, c, d\} \quad M = \{(1, a), (2, b), (3, d)\}$$

- In a map, $M \subseteq A \times B$, two or more distinct elements of A can be mapped to the same element of B.
- A map where this does not happen is described as **one-to-one**.
- So, a map in which distinct elements of A go to distinct elements of B is one-to-one.

Onto map

$$A = \{1, 2, 3\} \quad B = \{a, b, c, d\} \quad M = \{(1, a), (2, a), (3, b)\}$$

- In a map, not all of the elements of B need to be involved in the map.
- A map in which they are all involved is described as **onto**.
- So, a map in which each element of B is paired with an element of A is onto.

Bijections

$$A = \{1, 2, 3\} \quad B = \{a, b, c\} \quad M = \{(1, a), (2, c), (3, b)\}$$

- Maps can be neither one-to-one nor onto, one or the other, or both.
- A **bijection** is map that is both one-to-one and onto.
- Both A and B must have the same size in a bijection.

Partial maps

$$A = \{1, 2, 3\} \quad B = \{a, b, c\} \quad P = \{(1, a), (3, c)\}$$

- A map M from a set A to a set B must involve every element of A.
- A partial map is like a map, but with that condition relaxed.
- A partial map from a set A to a set B is a subset of $A \times B$ where any element of A that appears as the first element in a tuple does so exactly once.
- The term *partial map* is a bit of a misnomer, as a partial map is not necessarily a map, but a map is a partial map.