

# Subset sum

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# Subset-sum problem

## Problem

Given a set of integers  $S$ , is there a non-empty subset whose elements sum to zero?

## Example

Does  $\{1, 3, 7, -5, -13, 2, 9, -8\}$  have such a subset?

## Note

If somebody suggests a solution, it is very quick to check it. Being able to quickly verify a solution is a characteristic of NP problems.

# SUBSET-SUM

$$\{\langle S, t \rangle \mid S = \{x_1, x_2, \dots, x_k\} \mid \exists \{y_i\} \subseteq \{x_j\} \mid \sum_i y_i = t\}$$

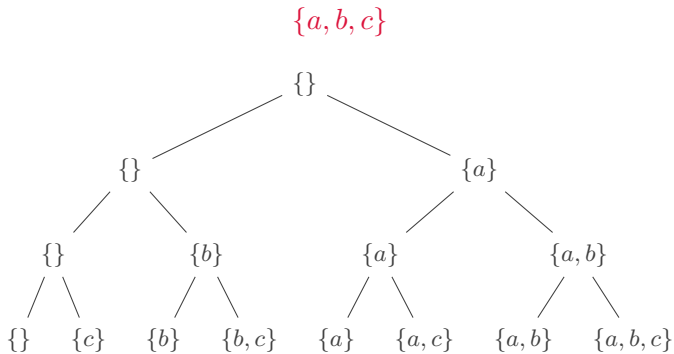
**SUBSETSUM** is a language, as above.

**Angle brackets** denote encoding their contents as a string over some alphabet.

**Encoding** can be done in many ways, and Turing machines can be used to translate between different encodings, albeit with a computational cost.

$t = 0$  gives a subset of SUBSETSUM.

## Counting subsets



## SUBSETSUM and NP

$2^n$  is the number of subsets. Exponential.

**Note** that  $2^n$  is also the number of settings of  $n$  Boolean variables.

**Correspondence** can be seen in terms of 0's and 1's. In SUBSET-SUM the elements from the set that are included in a given subset are represented by 1's.

**SUBSET-SUM** is NP-complete.

**Usual** proof that SUBSET-SUM is NP-complete is done by reduction to 3-SAT.