Polynomial time

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Polynomial time

$$P = \bigcup_k \mathsf{TIME}(n^k)$$

Decidable languages are languages for which there is at least one Turing machine that halts in a finite number of state table lookups for each input.

P is the set of languages that are decidable in polynomial time using a deterministic Turing machine.

Polynomial means that for a length of input n the number of steps (state table lookups) is $O(n^k)$ for some $k \in \mathbb{N}$.

TIME(n^k) is the set of languages decidable in $O(n^k)$ steps.

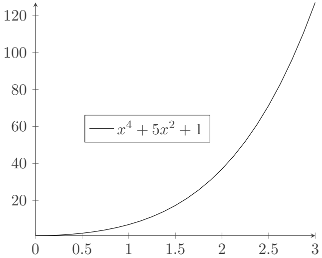
Recap on polynomials

$$a_0 + a_1 x + a_2 x^2 + \cdots + a_m x^m \text{ where } a_i \in \mathbb{R}, m \in \mathbb{N}$$

Polynomials have a fixed number of operations (add, multiply) to perform for all values of x.

Exponential constrasts with polynomial, having a variable number of operations, e.g. 2^x.

Functions that are polynomial in n are functions that take n and plug it into a polynomial to give the result.



Polynomials are closed

$$\mathbb{R}[x] = \{a_0 + a_1 x + a_2 x^2 + \cdots + a_m x^m \mid a_i \in \mathbb{R}, m \in \mathbb{N}\}$$

The set of all polynomials is closed under the following operations:

Addition: $(x^4 + 1) + (5x^3 + x) = x^4 + 5x^3 + x + 1$

Multiplication: $(x^4 + 1) \times (5x^3 + x) = 5x^7 + x^5 + 5x^3 + x$

Composition:

$$((5x^3 + x))^4 + 1 = 625x^{12} + 500x^{10} + 150x^8 + 20x^6 + x^4$$

Applying two polynomial time algorithms, one after the other, is still polynomial time.

Tractability

Cobham's thesis is that P represents the tractable problems.

Strictly exponential time problems are considered intractable.

Careful: would you consider an problem whose best known algorithm is $O(n^{100000})$ tractable?

The exponential time hypothesis is that 3-SAT can't be solved in subexponential time. 3-SAT is NP-complete.