Subset-sum

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Subset-sum problem

Problem
Given a set of integers S, is there a non-empty subset whose elements sum to zero?

Example
Does $\{1, 3, 7, -5, -13, 2, 9, -8\}$ have such a subset?

Note
If somebody suggests a solution, it is very quick to check it. Being able to quickly verify a solution is a characteristic of NP problems.

SUBSET-SUM

$$\{\langle S, t\rangle \mid S = \{x_1, x_2, \ldots, x_k\} \mid \exists\{y_i\} \subseteq \{x_j\} \mid \sum_i y_i = t\}$$

SUBSETSUM is a language, as above.

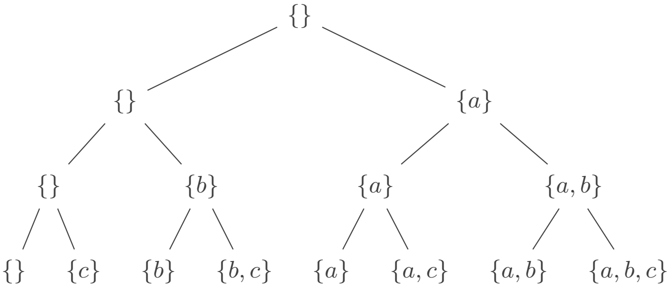
Angle brackets denote encoding their contents as a string over some alphabet.

Encoding can be done in many ways, and Turing machines can be used to translate between different encodings, albeit with a computational cost.

$t = 0$ gives a subset of SUBSETSUM.

Counting subsets

$$\{a, b, c\}$$



SUBSETSUM and NP

2^n is the number of subsets. Exponential.

Note that 2^n is also the number of settings of n Boolean variables.

Correspondence can be seen in terms of 0's and 1's. In SUBSET-SUM the elements from the set that are included in a given subset are represented by 1's.

SUBSET-SUM is NP-complete.

Usual proof that SUBSET-SUM is NP-complete is done by reduction to 3-SAT.