Ch 10 linear algebra supplement: matrix multiplication again

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Matrix Multiplication ... revisited.

$$\underset{m}{\boxed{C}}^{\,n} = \underset{m}{\boxed{A}}^{\,k} \cdot \underset{k}{\boxed{B}}^{\,n}$$

Formula: $\underbrace{c_{ij} = \sum_{\ell=1}^{k} a_{i\ell}\, b_{\ell j}}_{\text{avoid using if possible!}}$

Higher-level ways to think of it:

① $\begin{bmatrix} -\vec{c}_1^{\,T}- \\ \vdots \\ -\vec{c}_m^{\,T}- \end{bmatrix} = \begin{bmatrix} -\vec{a}_1^{\,T}- \\ \vdots \\ -\vec{a}_m^{\,T}- \end{bmatrix} \boxed{B}$ $\quad \vec{c}_i^{\,T} = \vec{a}_i^{\,T} \cdot B$

② $\begin{bmatrix} c_{ij} \end{bmatrix} = \begin{bmatrix} -\vec{a}_1^{\,T}- \\ \vdots \\ -\vec{a}_m^{\,T}- \end{bmatrix} \begin{bmatrix} | & & | \\ \vec{b}_1 & \cdots & \vec{b}_n \\ | & & | \end{bmatrix}$ $\quad c_{ij} = \vec{a}_i^{\,T} \cdot \vec{b}_j$

dot product (or matrix multiplication, same thing here)

③ $\begin{bmatrix} | & & | \\ \vec{c}_1 & \cdots & \vec{c}_n \\ | & & | \end{bmatrix} = \boxed{A} \cdot \begin{bmatrix} | & & | \\ \vec{b}_1 & \cdots & \vec{b}_n \\ | & & | \end{bmatrix}$ $\quad \vec{c}_j = A \cdot \vec{b}_j$

④ $\boxed{C} = \begin{bmatrix} | & & | \\ \vec{a}_1 & \cdots & \vec{a}_k \\ | & & | \end{bmatrix} \cdot \begin{bmatrix} -\vec{b}_1^{\,T}- \\ \vdots \\ -\vec{b}_k^{\,T}- \end{bmatrix}$ $\quad C = \sum_{\ell=1}^{k} \vec{a}_\ell \cdot \vec{b}_\ell^{\,T}$

Sum of k rank-1 matrices

⚠ These "$\vec{a}_i$" not the same as "$\vec{a}_i$" from ① and ②

Special Cases

$D = \text{diag}(d_1, \dots, d_n) = \begin{bmatrix} d_1 & & \\ & \ddots & \\ & & d_n \end{bmatrix}$,

$A \cdot D$ scales column i of A by d_i

$D \cdot A$ scales row i of A by d_i.

Diagonal · Diagonal is diagonal, $\begin{bmatrix} d_1 & & \\ & \ddots & \\ & & d_n \end{bmatrix} \cdot \begin{bmatrix} c_1 & & \\ & \ddots & \\ & & c_n \end{bmatrix} = \begin{bmatrix} d_1c_1 & & \\ & \ddots & \\ & & d_nc_n \end{bmatrix}$. Nice!

Determinant of diagonal matrix is $\prod_{i=1}^{n} d_i$

A diagonal matrix is pos. def. iff all $d_i > 0$.