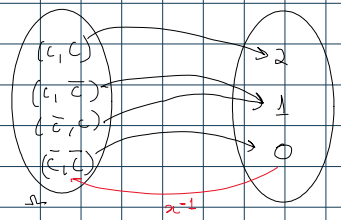
T - 7/3/2023

11 March 2023 00:06

Lançar uma moeda: $X =$ "nº de caras em 2 lançamentos"



$P(\Omega) = 1 \Rightarrow \underbrace{(\exists\, 0, 1, 2\,\{)}_{S} = 1$

x	0	1	2	cc
$P(X=x)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	0

$S = \{$ conjunto dos valores que X assume com probabilidade positiva $\}$

$P(X=0) = P(x^{-1}(0)) = P((\bar{c}, \bar{c})) = \frac{1}{4}$

$P(X=1) = P(x^{-1}(1)) = P((\bar{c}, c)) + P((c, \bar{c})) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

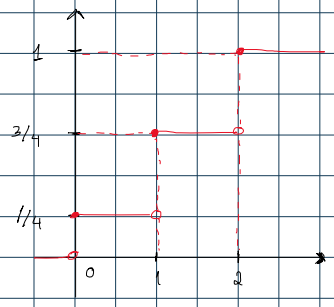
$P(X=2) = P(x^{-1}(2)) = P((c, c)) = \frac{1}{4}$

↳ função distribuição

$F: \mathbb{R} \to [0, 1]$

$x \to F(x) = P(X \le x)$

$$= \begin{cases} 0, & x < 0 \\ 1/4, & 0 \le x < 1 \\ 3/4, & 1 \le x < 2 \\ 1, & x \ge 2 \end{cases}$$



$F(x) = P(X \le x) = \sum_{a \in S \cap]-\infty, x]} P(X = a)$

$P(0 \le X < 2) = P(X < 2) - P(X < 0) = 3/4 - 0 = 3/4$

$P(X \le 1)$

$P(0 \le X < 2 / X > 0) = \frac{P(0 \le X < 2 \cap X > 0)}{P(X > 0)} = \frac{P(X = 1)}{1 - P(X \le 0)} = \frac{1/2}{1 - 1/4} = \frac{2}{3}$

$E(X) = \sum_{x \in S} x \cdot P(X = x) = 0 \times \frac{1}{4} + 1 \times \frac{1}{2} + 2 \times \frac{1}{4} = 1$ → se o suporte for infinito, $E(X)$ só existe se a série for convergente.

- $E(X+1) = E(X) + 1$

$V(X) = E(X^2) - E^2(X) = 0^2 \times \frac{1}{4} + 1^2 \times \frac{1}{2} + 2^2 \times \frac{1}{4} - 1^2 = \frac{1}{2}$

$\sigma(X) = \sqrt{V(X)} = \sqrt{\frac{1}{2}}$