CSCE 222 [Sections 504] Discrete Structures for Computing
Fall 2019 – Hyunyoung Lee

Problem Set 1

Due dates: Electronic submission of *yourLastName-yourFirstName-hw1.tex* and *yourLastName-yourFirstName-hw1.pdf* files of this homework is due on **Friday, 9/6/2019 before 10:00 p.m.** on `http://ecampus.tamu.edu`. You will see two separate links to turn in the .tex file and the .pdf file separately. Please do not archive or compress the files. **If any of the two submissions are missing, you will likely receive zero points for this homework.**

Name: Ian Stephenson

Resources. (The Not So Short Introduction to LaTeX, The Comprehensive LaTeX Symbol List, Discrete Math and Its Applications, Peer Teacher Central)

On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.

Electronic signature: Ian Stephenson

Total 100 points.

Problem 1. (2 points × 5 = 10 points) Section 1.1, Exercise 8 on page 13.

Solution. a) True
b) True
c) False
d) False
e) False

Problem 2. (4 points × 3 = 12 points) Section 1.1, Exercise 18 a), b), and c) on page 15. *Explain.* [Grading rubric: For each question, explanation 3 pts + true/false 1 pt]

Solution. a) True: $2 + 2 = 4$ is true and $1 + 1 = 2$ is true, two truths make the biconditional statement true.
b) False: $1 + 1 = 2$ is true but $2 + 3 = 4$ is false, and one false in a biconditional makes the statement false.
c) True: $1 + 1 = 3$ is false and monkeys can not fly, two false values make the biconditional true.

Problem 3. (5 points $\times$ 2 = 10 points) Section 1.1, Exercise 20 a) and c) on page 15. *Explain.* [Grading rubric: For each question, explanation 4 pts + true/false 1 pt]

Solution. a) True: $1 + 1 \neq 3$ and unicorns don't exist, therefore the statement is vacuously true
b) False: The hypothesis $1 + 1 = 2$ is true but the conclusion dogs can fly is false, so the conditional statement is false

Problem 4. (2 points $\times$ 5 = 10 points) Section 1.1, Exercise 26 a), b), c), d), and e) on page 15.

Solution. a) If you send me an email message, then I will send you the address.
b) If you were born in the United States, then you are a citizen of this country.
c) If you keep your textbook, then it will be a useful reference in your future courses.
d) If the Red Wings goalie plays well, then they will win the Stanley Cup.
e) If you got the job, then you had the best credentials.

Problem 5. (5 points $\times$ 2 = 10 points) Section 1.1, Exercise 34 c) and d) on page 16.

Solution. c)

p	q	$p \vee q$	$p \oplus (p \vee q)$
T	T	T	F
T	F	T	F
F	T	T	T
F	F	F	F

d)

p	q	$p \wedge q$	$p \vee q$	$(p \wedge q) \implies (p \vee q)$
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

Problem 6. (12 points) Section 1.2, Exercise 22 on page 24. *Explain.* [Grading rubric: Explanation 10 points + final answer 2 points]

Solution. There are two possible combinations of friends that could be invited. The first solution is to invite only Jasmine. Jasmine's attendance is not conditional on somebody else's attendance, so she will be happy even if neither of the other two friends are present. The second solution is to invite Jasmine and Kanti. Kanti will be satisfied because Jasmine is there, and Jasmine will be happy because Samir is not there. There are no situations in which you could invite Samir because he will only attend if Kanti is there, and Kanti will attend only if Jasmine is there, and Jasmine would be unhappy if Samir is there. So, you could choose to invite Jasmine, or you could choose to invite Jasmine and Kanti. Also, you could choose to not invite any of them.

Problem 7. (12 points) Section 1.2, Exercise 24 on page 25. *Explain.* [Grading rubric: Explanation 10 points + final answer 2 points]

Solution. The only answer to the question is that A is a knave and B is a knight. If you assume that both are knights, then B calling A a knave would contradict A stating that both are knights. If both are knaves, then B calling A a knave would have to be a lie, meaning that A is a knight, which contradicts the original premise that they are both knaves. If A is a knight and B is a knave, then A's statement that they are both knights is a lie, and knights can't lie. Finally, if A is a knave and B is a knight, then B stating that A is a knave is true, and A stating that they are both knights is false, but he is a knave so he is lying, which means that both person's statements evaluate to true.

Problem 8. (12 points) Section 1.3, Exercise 20 on page 38. *Solve this problem by developing a series of logical equivalences, as shown in class and also in Examples 6 and 7 on pages 31–32.* [Grading rubric: At least 6 derivation steps must be shown, and each derivation step must be accompanied by the (name of) law or logical equivalence used. Each step is worth 1pt (up to 6 pts), and explanation for each step worth 1 pt (up to 6 pts).]

Solution.

$$
\begin{aligned}
p \leftrightarrow q &\equiv (p \implies q) \wedge (q \rightarrow p) && \text{Definition of Biconditional} \\
&\equiv (\neg p \vee q) \wedge (\neg q \vee p) && A \rightarrow B \equiv \neg A \vee B \\
&\equiv ((\neg p \vee q) \wedge \neg q) \wedge ((\neg p \vee q) \wedge p) && \text{Distributive Laws} \\
&\equiv ((\neg p \wedge \neg q) \vee (\neg q \wedge q)) \vee ((\neg p \wedge p) \vee (p \wedge q)) && \text{Distributive Laws} \\
&\equiv ((\neg p \wedge \neg q) \vee F) \vee ((\neg p \wedge p) \vee F) && \text{by } \neg A \wedge A \equiv F, NegationLaws \\
&\equiv (\neg p \wedge \neg q) \vee (p \wedge q) && \text{by } A \vee F \equiv A, IdentityLaws
\end{aligned}
$$

Problem 9. (12 points) Section 1.3, Exercise 34 on page 38. *Solve this problem by developing a series of logical equivalences, as shown in class and also in Example 8 on page 32.* [Grading rubric: At least 8 derivation steps must be shown, and each derivation step must be accompanied by the (name of) law or logical equivalence used. Each step is worth 1pt (up to 8 pts) and explanation for each step worth 0.5 pts (up to 4 pts).]

Solution.

$$
\begin{aligned}
((p \vee q) \wedge (\neg p \vee r)) \rightarrow (q \vee r) &\equiv (((p \vee q) \wedge \neg p) \vee ((p \vee q) \wedge r)) \rightarrow (q \vee r) \quad \text{Distributive Laws} \\
&\equiv ((p \wedge \neg p) \vee (q \wedge \neg p) \vee (p \wedge r) \vee (q \wedge r)) \rightarrow (q \vee r) \quad \text{Distributive Laws} \\
&\equiv ((q \wedge \neg p) \vee (q \wedge \neg p) \vee (p \wedge r) \vee (q \wedge r)) \rightarrow (q \vee r) \text{ by } A \wedge \neg A \equiv F, A \vee F \equiv A \\
&\equiv ((q \wedge \neg p) \vee p) \wedge ((q \wedge \neg p) \vee (q \wedge r)) \rightarrow (q \vee r) \quad \text{Distributive Laws} \\
&\equiv ((p \wedge q) \vee (p \wedge \neg p)) \wedge ((r \vee \neg p) \wedge (r \vee q)) \vee (q \vee r) \rightarrow (q \vee r) \text{Distributive Laws} \\
&\equiv (p \wedge q) \wedge ((r \vee \neg p) \wedge (r \vee q)) \vee (q \vee r) \rightarrow (q \vee r) \quad \text{by } A \wedge \neg A \equiv F, A \vee F \equiv A \\
&\equiv (p \wedge q) \wedge (r \vee \neg p) \wedge (q \vee r) \rightarrow (q \vee r) \quad \text{by } A \vee A \equiv A \\
&\equiv \neg((p \wedge q) \wedge (r \vee \neg p) \wedge (q \vee r)) \vee (q \vee r) \quad \text{by } A \rightarrow B \equiv \neg A \vee B \\
&\equiv \neg(p \wedge q) \vee \neg(r \vee \neg p) \vee \neg(q \vee r) \vee (q \vee r) \quad \text{De Morgans Law} \\
&\equiv \neg(p \wedge q) \vee \neg(r \vee \neg p) \vee T \quad \text{by } A \vee \neg A \equiv T \\
&\equiv T \quad \text{by } A \vee T \equiv T
\end{aligned}
$$

Problem 10. (5 points $\times$ 2 = 10 points) Section 1.4, Exercise 20 c) and d), page 57.

Solution. c) P(-5) $\wedge$ P(-3) $\wedge$ P(-1) $\wedge$ P(3) $\wedge$ P(5)
d) P(1) $\vee$ P(3) $\vee$ P(5)

Checklist:

- ☐ Did you type in your name and section?
- ☐ Did you disclose all resources that you have used?
  (This includes all people, books, websites, etc. that you have consulted.)
- ☐ Did you sign that you followed the Aggie Honor Code?
- ☐ Did you solve all problems?
- ☐ Did you submit both of the .tex and .pdf files of your homework to the correct link on eCampus?