CSCE 222 [Sections 503, 504] Discrete Structures for Computing
Fall 2019 – Hyunyoung Lee

Problem Set 8

Due dates: Electronic submission of *yourLastName-yourFirstName-hw8.tex* and *yourLastName-yourFirstName-hw8.pdf* files of this homework is due on **Friday, 11/8/2019 before 10:00 p.m.** on `http://ecampus.tamu.edu`. You will see two separate links to turn in the .tex file and the .pdf file separately. Please do not archive or compress the files. **If any of the two submissions are missing, you will likely receive zero points for this homework.**

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Resources. Peer Teacher Central, Discrete Mathematics and Its Applications by Rosen

On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.

Electronic Signature: **Ian Stephenson**

Total 100 points.

Problem 1. (2.5 points $\times$ 4 = 10 points) Section 6.3, Exercise 20, page 435

Solution. a) 120
b) 386
c) 176
d) 968

Problem 2. (2 points $\times$ 5 = 10 points) Section 6.3, Exercise 22 b), c), d), e), and f), page 435

Solution. b) 720
c) 120
d) 120
e) 24
f) 0

Problem 3. (5 points $\times$ 2 = 10 points) Section 6.4, Exercise 12 a) and b), page 444

Solution. a) 20000
b) 960

Problem 4. $(10+3+7=20$ points) Section 8.1, Exercise 10, page 537. For a) and c), explain and show your work.

Solution. a) Say that a_n is a number that represents the number of possible n length bit strings that contain the substring '01'. We must look at a few possible different test cases. First, lets say that there is a string that ends in the characters '01'. This means that there are 2^{n-2} possible combinations by the power rule of counting. Next, lets say that there a i zeroes that precede a 1. Thus, the first i characters are set, and so there are 2^{n-i-1} different bit strings. Then, there is a case where '01' is located at some arbitrary place in the string. This would mean that there are a_{n-1} possible strings, because you are removing one set of characters from the bit string, which removes one possible combination. If we combine these different cases we get that $a_n = a_{n-1} + 2^{n-2} + 2^{n-i-1}$. This will simplify down to $a_n = a_{n-1} + 2^{n-1} - 1$.

b) $a_0 = a_1 = 0$, bit strings must be at least two characters to contain '01'.

c) $a_0 = 0$
$a_1 = 0$
$a_2 = 1$
$a_3 = 4$
$a_4 = 11$
$a_5 = 26$
$a_6 = 57$
$a_7 = 120$

Problem 5. (20 points) Section 8.1, Exercise 28, page 538. This problem has two parts as below.

Solution.
a) (10 points) *Show that the Fibonacci numbers satisfy ...*

$fib_5 = 8$
$fib_6 = 13$
$fib_7 = 21$

$f_5 = 5(1) + 3 = 8$
$f_6 = 5(2) + 3(1) = 13$
$f_7 = 5(3) + 3(2) = 21$

b) (10 points) *Use this recurrence relation to show that ...* (Prove by induction on n.)
Induction Basis: Show that f_0 holds.
$f_{5(0)} = f_0 = 0 = 0/5$, therefore f_0 holds.
Induction hypothesis: As the induction hypothesis, suppose that $P(n) : f_{5n} = 5k$, where k is an integer. Then, show that $f5(n+1)$ holds.

For Problems 6 and 7, use Table 1 on page 568.

Problem 6. (5 points $\times$ 3 = 15 points) Section 8.4, Exercise 6 b)–d), page 575

Solution. b) $\frac{2x}{1-2x}$
c) $\frac{2x-1}{(1-x)^2}$
d) $\frac{e^x-1}{x}$

Problem 7. (5 points $\times$ 3 = 15 points) Section 8.4, Exercise 8 b)–d), page 575.

Solution. b) $a_0 = -1, a_1 = 9, a_2 = -27, a_3 = 27, a_{n>3} = 0$
c) $a_n = 2^n$
d) $a_0 = 0, a_1 = 1, a_n = \frac{n(n-1)}{2}$

Checklist:
- ☐ Did you type in your name and UIN?
- ☐ Did you disclose all resources that you have used?
  (This includes all people, books, websites, etc. that you have consulted.)
- ☐ Did you electronically sign that you followed the Aggie Honor Code?
- ☐ Did you solve all problems?
- ☐ Did you submit both of the .tex and .pdf files of your homework to the correct link on eCampus?