CSCE 222 [Sections 503, 504] Discrete Structures for Computing
Fall 2019 – Hyunyoung Lee

Problem Set 10

Due dates: Electronic submission of *yourLastName-yourFirstName-hw10.tex* and *yourLastName-yourFirstName-hw10.pdf* files of this homework is due on **Wednesday, 11/27/2019 before 10:00 p.m.** on `https://ecampus.tamu.edu`. You will see two separate links to turn in the .tex file and the .pdf file separately. Please do not archive or compress the files. **If any of the two submissions are missing, you will likely receive zero points for this homework.**

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Resources. Discrete Math and Its Applications, Rosen, 8th Edition

On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.

Electronic Signature: **Ian Stephenson**

In this problem set, you can earn up to $100 + 10$ (extra credit) points.

Problem 1. ($5 + 5 + 10 = 20$ points) Section 13.1, Exercise 4, page 894

Solution. a) $S \implies 1S \implies 11S \implies 111S \implies 11100A \implies 111000$
b) $S \implies 1S \implies 11S \implies 1100A, A \implies 1$ is not in P, so 11001 is not in this language
c) $S \implies 1S$ gives a relationship in which we have n number of ones, or a 1^n relationship. S also generates the sequence $00A$, where $A \implies 0|0A$, leading to m number of 0's, where $m \geq 3$. Thus, $L(G) = \{1^n 0^m | n \geq 0, m \geq 3\}$.

Problem 2. (10 points) Section 13.1, Exercise 6 d), page 894

Solution. $L(G) = \{a^{2n} | n \geq 2\} or \{b^n | n \geq 2\}$

Problem 3. (10 points) Section 13.1, Exercise 14 b), page 894

Solution. $G = (V, T, S, P)$ where
$V = \{0, 1, S, A\}$
$T = \{0, 1\}$
$P = \{S \implies 00A, A \implies 1|1A\}$

Problem 4. (15 points $\times\ 2 = 30$ points) Consider the grammar $G = (V, T, E, P)$ for expressions (E for short) such that $V = \{E, a, +, *, (,)\}$, $T = \{a, +, *, (,)\}$, E is the starting symbol, and

$$P = \{E \to (E) \mid E + E \mid E * E \mid a\}.$$

a) Explain whether G is regular, context-free, or context-sensitive, respectively. Explain why or why not.

   Solution. G is not a regular grammar because if we look at the productions E+E or E*E we have a nonterminal symbol as the first symbol in the production. G is a context free grammar because for every production step, E produces a string of symbols that are in V*. The language is also context sensitive because according to Chomskys hierarchy, all context free grammars are also context sensitive.

b) Explain the language $L(G)$ that is generated by G, especially, what kind of strings belong to the language. Be specific. Also, give five shortest strings that belong to $L(G)$.

   Solution. The five shortest strings in the grammar are:
   1)$E \implies a$
   2)$E \implies (E) \implies (a)$
   3)$E \implies E + E \implies a + E \implies a + a$
   4)$E \implies E * E \implies a * E \implies a * a$
   5)$E \implies (E) \implies ((E)) \implies ((a))$

   The language L(G) can be described as producing a string of an odd number of characters. The terminal symbol a will be in every production, and may or may not have parentheses surrounding it, but will always have an equal number of parentheses on both sides. The symbol a will then be placed next to a + or a * and then at least one more a will be on the other side. If there are multiple $a's$ on the other side, then they will preceded by a + or a *, or surrounded by parentheses.

Problem 5. (5 points $\times$ 2 = 10 points) Section 13.2, Exercise 4 a) and b), page 902. *Explain* by showing the state transition and the output of each state.

Solution. a)
$1 : S_0 \implies S_2, Output : 0$
$0 : S_2 \implies S_3, Output : 0$
$0 : S_3 \implies S_1, Output : 1$
$0 : S_1 \implies S_0, Output : 1$
$1 : S_0 \implies S_2, Output : 0$
$Output : 00110$

$b)$
$1 : S_0 \implies S_2, Output : 1$
$0 : S_2 \implies S_2, Output : 1$
$0 : S_2 \implies S_2, Output : 1$
$0 : S_2 \implies S_2, Output : 1$
$1 : S_2 \implies S_0, Output : 0$
$Output : 11110$

Problem 6. (10 points $\times$ 2 = 20 points) Section 13.3, Exercise 8 e) and f), page 914. *Prove or disprove.*

Solution. e) False: $A * A \neq A*$ because $A*$ contains λ. However, $A * A$ does not contain λ because A does not contain λ.

f) False: $|A^n| \neq |A|^n$. Say that $A = \{1, 11\}$. Then A^2 would be {11, 111, 1111}. This would mean that $|A^n| = 3$, but $|A|^n = 4$.

Problem 7. (5 points $\times$ 2 = 10 points) Section 13.3, Exercise 10 b) and c) page 914. *Explain.*

Solution. b) This set is described as n number of $0's$, followed by a single 10, followed by m number of $1's$. However, 01001 has a 0 following the 10, thus the string is not in the set.

c) This set is described as n occurences of the string 010, followed by m number of $0's$, followed by l number of $1's$. The string 01001 could be comprised of one 010 concatenated with one 0 concatenated with one 1, this the string is in the set.

Checklist:
☐ Did you type in your name and UIN?
☐ Did you disclose all resources that you have used?
(This includes all people, books, websites, etc. that you have consulted)
☐ Did you electronically sign that you followed the Aggie honor code?
☐ Did you solve all problems?
☐ Did you submit both of the .tex and .pdf files of your homework separately to the correct link on eCampus?