CSCE 222 [Sections 503, 504] Discrete Structures for Computing
Fall 2019 – Hyunyoung Lee

Problem Set 2

Due dates: Electronic submission of *yourLastName-yourFirstName-hw2.tex* and *yourLastName-yourFirstName-hw2.pdf* files of this homework is due on **Friday, 9/13/2019, before 10:00 p.m.** on `http://ecampus.tamu.edu`. You will see two separate links to turn in the .tex file and the .pdf file separately. Please do not archive or compress the files. **If any of the two files are missing, you will likely receive zero points for this homework.**

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Resources. (Discrete Math and Its Applications, 8th Edition, Rosen)

On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.

Electronic signature: **(Ian Stephenson)**

***** Please make sure that you are solving the correct problems from the 8th Edition of the Rosen book, not the 7th Edition! *****

Total 100 points.

Problem 1. (5 points $\times$ 2 = 10 points) Section 1.4, Exercise 36 b) and d), page 58.

Solution. b) $P(0) \wedge P(1) \wedge P(2) \wedge P(3) \wedge P(4)$
d) $P(-4) \vee P(-3) \vee P(-2)$

Problem 2. (6 points $\times$ 3 = 18 points) Section 1.5, Exercise 28 f), g), and i), page 71. *Justify your answer or give a counterexample.* [Grading rubric: For each subproblem, saying true / false correctly is 2 points and justifying your answer or giving a counterexample is worth 4 points.]

Solution. f) False: This statement will be true only if x=1 and y=1. However, the domain of y is all real numbers, and the statement claims that xy=1 for all values of y, which is false. Using a counterexample, say that x=2 and y=2 and substitute that into xy=1. This would mean that 4=1, which is false.

g) True: The statement reads that for all x, there exists some y such that x+y=1. Given any arbitrary x value, this statement would be true for all y values where y=1-x. Given any x value, the statement that x+(1-x)=1 will always yield 1=1.

i) False: The statement reads that for all x values there exists some values such that x+y=2 and 2x-y=1. If you solve for y in the first equation, you will that y=2-x. This can be substituted into the second equation to get that 2x-(2-x)=1. This statement should be true for all values of x. However, given that x=0, this statement would read that 0-2=1, or that -2=1, which is false. Therefore, this statement is not valid for all values of x.

Problem 3. (10 points $\times$ 2 = 20 points) Section 1.6, Exercise 14 c) and d), page 83.

Solution. c) Because all movies by John Sayles are wonderful, and there is a movie about coal miners by John Sayles, then by universal instantiation there is a movie by John Sayles about coal miners that is wonderful. So, there exists a John Sayles movie about coal miners, and if there is a John Sayles movie about coal miners, then there is a wonderful movie about coal miners. Therefore, by modus ponens, there is a wonderful movie about coal miners.

d) Because everybody who goes to France goes to the Louvre, then by universal instantiation it would follow that if any person goes to France, then they will visit the Louvre. Because someone in the class is in the domain of all people, and this person has been to France, and because if any person goes to France, then they will visit the Louvre, it would follow by modus ponens that someone in the class has visited the Louvre.

Problem 4. (10 points) Section 1.7, Exercise 6, page 95.

Solution. We define an odd integer such that the integer, n, is equal to 2k+1, where k is also an integer. We could define two odd integers, n_1, and n_2, where n_1=2k+1 and n_2=2j+1. The product of n_1 and n_2 is equivalent to (2k+1)(2j+1). Written out, this product would be 4kj+2k+2j+1, or as 2(2kj+k+j)+1. Because this follows the definition of an odd integer, n=2k+1, this proves that the product of two integers is odd.

Problem 5. (9 points $\times$ 2 = 18 points) Section 1.7, Exercise 20, page 95.

Solution. a) If we assume that the proposition, if 3n+2 is even then n is even, is false, it would follow that if n is odd then 3n+2 is odd. We can say that n, by the definition of an odd integer, is equal to 2k+1. We could then substitute 2k+1 into 3n+2 and get 6k+5. 6k+5 could be broken down to 6k+4+1, which is equivalent to 2(3k+2)+1. By the definition of an odd integer, the value 2(3k+2)+1 will be odd. Because the contrapositive, if n is odd then 3n+2 is odd, holds true, it would follow that the original theorem has been also been proven true by contraposition.

b) Seeking a contradiction, let's assume that 3n+2 is even and that n is odd are both true. By the definition of an odd integer, it can be said that n=2k+1. Substituting this n value back into 3n+2, we will get 6k+5, which can be broken down to be 2(3k+2)+1. This satisfies the definition of an odd integer and shows that 3n+2 is odd. Because we assumed that 3n+2 is even is true, and we showed that 3n+2 was odd is also true, we have created a contradiction. Therefore, the original proposition, if 3n+2 is even then n is even, is proven true by contradiction.

Problem 6. (12 points) Prove by *contradiction* that if $n \geq 1$ is a perfect square, then $n + 2$ is not a perfect square.

Solution. From the prosition, if $n \geq 1$ is a perfect square, then n+2 is not a perfect square, let p represent the first statement and let q represent the second statement. Seeking a contradiction, let's assume that p and ¬ q are true. That is that $n \geq 1$ is a perfect square and that n+2 is a perfect square. By definition, a perfect sqaure is a number, a, such that a=b^2. We can substitute n+2 into a to get that n+2=b^2. This would mean that n=$b^2 - 2$. Bringing this back to the statement p, we would get that $b^2 - 2 \geq 1$, or that $b^2 \geq 3$. This would mean that $|b| \geq \sqrt{3}$. So p, $|b| \geq \sqrt{3}$, is true for some values of b. However, ¬p, $|b| < \sqrt{3}$, is also true for some values of b. Because p and ¬p are both true for some values of b, it is proven by contradiction that if $n \geq 1$ is a perfect square, then n+2 is not a perfect square.

Problem 7. (12 points) Prove by *contradiction* that at least three of any 25 days chosen must fall in the same month of the year.

Solution. Let's take p to be the proposition that that at least 3 of any 25 days chosen must fall in the same month. Seeking a contradiction, let's assume that $\neg p$ is true, or that at most 2 of the 25 days chosen must fall in the same month. In any given year there are 12 months, and at most two of the chosen days can fall in the same month. This implies that 24 days were chosen because two days per each of the twelve months could be chosen, and $2 \times 12 = 24$. This creates a contradiction, because the statement said that 25 days were chosen, but we showed that at most 24 days could be chosen. This proves that $\neg p$ implies the contradiction that 25 days were chosen and 24 days were chosen. This proves by contradiction that at least 3 of any 25 days chosen must fall in the same month.

Checklist:

- ☐ Did you type in your name and section?
- ☐ Did you disclose all resources that you have used?
  (This includes all people, books, websites, etc. that you have consulted.)
- ☐ Did you sign that you followed the Aggie Honor Code?
- ☐ Did you solve all problems?
- ☐ Did you submit the .tex and .pdf files of your homework to the correct link on eCampus?