CSCE 222 [Sections 503, 504] Discrete Structures for Computing
Fall 2019 – Hyunyoung Lee

Problem Set 5

Due dates: Electronic submission of *yourLastName-yourFirstName-hw5.tex* and *yourLastName-yourFirstName-hw5.pdf* files of this homework is due on **Tuesday, 10/8/2019 before 10:00 p.m.** on `http://ecampus.tamu.edu`. You will see two separate links to turn in the .tex file and the .pdf file separately. Please do not archive or compress the files. **If any of the two submissions are missing, you will likely receive zero points for this homework.**

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Resources. (Discrete Mathematics and Its Applications, 8th Edition, Rosen, tex.stackexchange.com(used to find function for cube root and logarithm))

On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.

Electronic Signature: **Ian Stephenson**

Total 100 (+ 10 extra) points.

Problem 1. (8 pts $\times$ 2 = 16 points) Section 3.3, Exercise 14, pages 242–243. *Explain.*

Solution. a) From the equation, we have that $c = 2, a_0 = 1, a_1 = 1, a_2 = 3$. The greatest exponent is two, and therefore $n = 2$. On the first iteration, we have that we know that the iteration begins at $i = 1$, and we know that $y = a_2$, so our equation will be $y = a_2 \times c + a_1$, or $y = 3(2) + 1 = 7$. The second iteration will start from $i = 2$, and thus our equation will be $y = y \times c + a_0$, where the y in the assignment is equal to the value of the previous iteration. We substitute in and get that $y = 7(2)+1 = 15$. So this program will return $y = 15$.

b) To find the total number of multiplications and additions, we must find how many multiplications and additions occur in each iteration and then determine how many iterations there are and multiply those values together. We can see in the code that each loop will execute one addition operator and one multiplication operator. We also know that the code will loop from 1 to n, inclusive, or it will loop n times. If we multiply the number of additions per loop by the number of loops, and do the same with the multiplications, we will get that there are n total multiplications and n total additions.

Problem 2. (3 pts $\times$ 8 = 24 points) Section 3.3, Exercise 16 a), b), d), e) and f), page 243. *Explain.*

Solution. a) In a given day, there are 86400 seconds. If we divide this value by the time required to complete an operation, 10^{-11} seconds, we will find that 8.64×10^{15} can be completed in a given day. We can say that $f(n) = 8.64 \times 10^{15}$, where $f(n) = log(n)$. If we exponentiate, we will get that $n = 10^{8.64 \times 10^{15}}$

b) Using our previous work, we know that there are 8.64×10^{15} operations possible in a given day. If we set that equal to our $f(n)$ we will get $1000n = 8.64 \times 10^{15}$. If we divide by 1000, we will be left with $n = 8.64 \times 10^{12}$.

d) Using our previous work, we know that there are 8.64×10^{15} operations possible in a given day. Using the given $f(n)$, we will get that $1000n^2 = 8.64 \times 10^{15}$. We divide by 1000 and are left with $n^2 = 8.64 \times 10^{12}$. When we take the square root we must also take the floor of our result because the number of operations must be an integer that is less than or equal to the greatest real number operations performed. Therefore, we get that $n = \lfloor \sqrt{8.64 \times 10^{12}} \rfloor$.

e) Using our previous work, we know that there are 8.64×10^{15} operations possible in a given day. Using the given $f(n)$, we will get that $n^3 = 8.64 \times 10^{15}$. When we take the cube root we must remember to floor the result because we need n to be an integer, and thus we are left with $n = \lfloor \sqrt[3]{8.64 \times 10^{15}} \rfloor$.

f) Using our previous work, we know that there are 8.64×10^{15} operations possible in a given day. Using the given $f(n)$, we will get that $2^n = 8.64 \times 10^{15}$. If we take the base two logarithm of both sides and then floor the result so that we get an integer result, we will be left with $n = \lfloor \log_2 8.64 \times 10^{15} \rfloor$.

Problem 3. (5 pts $\times$ 3 = 15 points) Section 2.4, Exercise 6 b), d) and h), page 177

Solution. b) $\{1, 3, 6, 10, 15, 21, 28, 36, 45, 55\}$

d) $\{\lfloor\sqrt{1}\rfloor, \lfloor\sqrt{2}\rfloor, \lfloor\sqrt{3}\rfloor, \lfloor\sqrt{4}\rfloor, \lfloor\sqrt{5}\rfloor, \lfloor\sqrt{6}\rfloor, \lfloor\sqrt{7}\rfloor, \lfloor\sqrt{8}\rfloor, \lfloor\sqrt{9}\rfloor, \lfloor\sqrt{10}\rfloor\}$
$=\{1, 1, 1, 2, 2, 2, 2, 2, 3, 3\}$

h) $\{k! \leq 1, k! \leq 2, k! \leq 3, k! \leq 4, k! \leq 5, k! \leq 6, k! \leq 7, k! \leq 8, k! \leq 9, k! \leq 10\}$
$=\{1, 2, 2, 2, 2, 3, 3, 3, 3, 3\}$

Problem 4. (5 pts $\times$ 3 = 15 points) Section 2.4, Exercise 10 a), b) and e), page 177

Solution. a) $a_0 = -1$,
$a_1 = -2(-1) = 2$,
$a_2 = -2(2) = -4$,
$a_3 = -2(-4) = 8$,
$a_4 = -2(8) = -16$,
$a_5 = -2(-16) = 32$

b) $a_0 = 2$,
$a_1 = -1$,
$a_2 = -1 - 2 = -3$,
$a_3 = -3 - (-1) = -2$,
$a_4 = -2 - (-3) = 1$,
$a_5 = 1 - (-2) = 3$

e) $a_0 = 1$,
$a_1 = 1$,
$a_2 = 2$,
$a_3 = 2 - 1 + 1 = 2$,
$a_4 = 2 - 2 + 1 = 1$,
$a_5 = 1 - 2 + 2 = 1$

Problem 5. (5 pts $\times$ 4 = 20 points) Section 2.4, Exercise 16 a), b), c) and d), page 178. *Explain.*

Solution. a) The first few terms in the sequence are $a_1 = -5, a_2 = 5, a_3 = -5$, and so on. The sequence will always oscillate between 5 and -5, thus the sequence will be solved by either $5(-1)^n$ or $5(-1)^{n+1}$. We can see that the -5 is always occurring in the odd terms of the sequence. Because of this, we choose $5(-1)^n$ because the 1 will be positive when n is even which makes our even sequence terms positive, as in our given sequence. Therefore the sequence is solved by $5(-1)^n$.

b) The first few terms in the sequence are $a_1 = 1+3, a_2 = 4+3, a_3 = 7+3, a_4 = 10+3$, and so on. As we can see, the first term in the sum is always a multiple of 3, which leads us to believe that it is going to involve $3n$. We notice that when $n = 1$, the first term in the sum of a_1 is 1, which leads us to believe that we will be dealing with $3(n-1)$ and not $3n$. To get to a value of one for the first term, we must then add 1, making this $3(n-1)+1$. Then we look at the second term, which is always an addition of 3. If we distribute the 3 and then add 3, we will get that $a_n = 3n+1$.

c) Because we have falling terms, we can use the method of backwards substitution to solve our sequence. So we begin with our $a_n = a_{n-1} - n$. Then, $a_{n-1} - n = a_{n-2} - (n-1) - n = a_{n-3} - (n-2) - (n-1) - n$. If we expand out each one of these terms we will get that $a_n = a_0 - n = a0 - 2n + 1 = a0 - 3n + 2 + 1 = a0 - 4n + 3 + 2 + 1$. From this expansion, we can see that the terms are always going to be $a_0 - nn + \sum_{k=1}^{n-1} k$, or $a_0 - n^2 + \sum_{k=1}^{n-1} k$, or $4 - n^2 + \sum_{k=1}^{n-1} k$. We know that the sum can be rewritten in the form of $n(n+1)/2$. Because our summation goes to $n-1$, our rewritten summation will be $(n-1)n/2$. We can substitute this back into our expanded sequence to get $a_n = 4 - n^2 + \frac{(n-1)n}{2}$.

d) Because we have falling terms, we can use backwards substitution to solve our sequence. We can say that $a_n = 2a_{n-1} - 3 = 2(2a_{n-2}) - 3 - 3(2) = 2(2(2a_{n-3}) - 3 - 3(2) - 3(4)$. From this sequence, we can see that we always have a $2a_0$ which is equal to -2. This -2 is being multiplied by 2^{n-1}, so we can combine them to be -2^n. We also are subtracting a summation of $3(2^n)$, denoted by $-3\sum_{k=0}^{n-1} 2^k$. This summation is given as a geometric series where $a = 1, r = 2, j = n$, so the series can be written as $\frac{1(2)^n - 1}{2-1}$, or $2^n - 1$. We can rewrite the equation to be $-2^n - 3(2^n - 1)$, or $a_n = -2^n - 3(2^n) + 3$.

Problem 6. (5 pts $\times$ 4 = 20 points) Section 2.4, Exercise 34, page 179

Solution. a) 3
b) 78
c) 9
d) 180

Checklist:
☐ Did you type in your name and UIN?
☐ Did you disclose all resources that you have used?
(This includes all people, books, websites, etc. that you have consulted.)
☐ Did you electronically sign that you followed the Aggie Honor Code?
☐ Did you solve all problems?
☐ Did you submit both of the .tex and .pdf files of your homework to the correct link on eCampus?