CSCE 222 [Sections 503, 504] Discrete Structures for Computing
Fall 2019 – Hyunyoung Lee

Problem Set 6

Due dates: Electronic submission of *yourLastName-yourFirstName-hw6.tex* and *yourLastName-yourFirstName-hw6.pdf* files of this homework is due on **Tuesday, 10/15/2019 before 10:00 p.m.** on `http://ecampus.tamu.edu`. You will see two separate links to turn in the .tex file and the .pdf file separately. Please do not archive or compress the files. **If any of the two submissions are missing, you will likely receive zero points for this homework.**

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Resources. Discrete Math and Its Applications, 8th Edition, Rosen

On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.

Electronic Signature: **Ian Stephenson**

Total 100 points.

Problem 1. (13 points) Section 5.1, Exercise 8, page 350. Prove by induction on n.

Solution. Prove A(n): $2 - 2 \times 7 + 2 \times 7^2 - \cdots + 2(-7)^n = (1 - (-7)^{n+1})/4$

Induction Basis:
Show that A(0) holds.
$2(-7)^n = 2(-7)^0 = 2(1) = 2 = \frac{8}{4} = \frac{1-(-7)}{4} = \frac{1-(-7)^{0+1}}{4} = \frac{1-(-7)^{n+1}}{4}$
Thus, A(0) holds.

Induction Step: As the induction hypothesis, assume that A(n) holds. Then, show that A(n+1) holds.
$2 - 2 \times 7 + 2 \times 7^2 - \cdots + 2(-7)^n + 2(-7)^{n+1} = \frac{1-(-7)^{n+1}}{4} + 2(-7)^{n+1}$ by IH
$= \frac{1-(-7)^{n+1}}{4} + \frac{8(-7)^{n+1}}{4}$ by creating like bases
$= \frac{1+7(-7)^{n+1}}{4}$ by adding fractions
$= \frac{1-(-7)(-7)^{n+1}}{4}$ by rewriting 7
$= \frac{1-(-7)^{n+2}}{4}$ by factoring in 7
$= \frac{1-(-7)^{(n+1)+1}}{4}$ by rewriting $n+2$
Thus, A(n+1) holds.

Conclusion: Therefore, A(n) holds by induction on n.

Problem 2. ($7 + 13 = 20$ points) Section 5.1, Exercise 10, page 350. For b), prove by induction on n.

Solution. a)
$n = 1 : 1/2$
$n = 2 : 2/3$
$n = 3 : 3/4$
$n = 4 : 4/5$
$f(n) = \frac{n}{n+1}$

b) Prove A(n): $\frac{1}{1\times2} + \frac{1}{2\times3} + \cdots + \frac{1}{n(n+1)} = \frac{n}{n+1}$

Induction Basis:

Show that A(1) holds.

$\frac{1}{n(n+1)} = \frac{1}{1(1+1)} = \frac{1}{2} = \frac{1}{1+1} = \frac{n}{n+1}$

Induction Step:

As the induction hypothesis(IH), assume that the claim A(n) holds. Then, show that A(n+1) holds.

$\frac{1}{1\times2} + \frac{1}{2\times3} + \cdots + \frac{1}{n(n+1)} + \frac{1}{(n+1)((n+1)=1)} = \frac{n}{n+1} + \frac{1}{(n+1)(n+2)}$ by IH
$= \frac{n(n+2)+1}{(n+1)(n+2)}$ by creating like bases and adding fractions
$= \frac{n^2+2n+1}{(n+1)(n+2)}$ by multiplying in n
$= \frac{(n+1)^2}{(n+1)(n+2)}$ by factoring the polynomial
$= \frac{n+1}{n+2}$ by cancelling $n + 1$ in numerator and denominator
$= \frac{n+1}{(n+1)+1}$ by rewriting $n + 2$

Thus, A(n+1) holds.

Conclusion: Therefore, A(n) holds by induction on n.

Problem 3. (13 points) Section 5.1, Exercise 14, page 351. Prove by induction on n.

Solution. Prove A(n):For every positive integer n, $\sum_{k=1}^{n} k2^k = (n-1)2^{n+1}+2$

Induction Basis:

Show that A(1) holds.

$\sum_{k=1}^{1} k2^k = 1(2^1) = 2 = 0+2 = (1-1)2^2+2 = (n-1)2^{n+1}+2$

Thus, A(1) holds.

Induction Step:

As the induction hypothesis, assume that the claim A(n) holds. Then, show that A(n+1) holds.

$\sum_{k=1}^{n+1} k2^k = (n+1)2^{n+1} + \sum_{k=1}^{n} k2^k$ by definition of series

$= (n+1)2^{n+1} + (n-1)2^{n+1} + 2$ by IH

$= (2n)2^{n+1} + 2$ by adding coefficients of like terms

$= n(2^{n+2} + 2$ by factoring in 2

$= ((n-1)+1)2^{(n+1)+1} + 2$ by rewriting terms

Thus, A(n+1) holds.

Conclusion: Therefore, A(n) holds by induction on n.

Problem 4. (13 points) Section 5.1, Exercise 24, page 351. Prove by induction on n.

Solution. Prove A(n): $\frac{1}{2n} \leq [1 \times 3 \times 5 \times \cdots \times (2n-1)]/[2 \times 4 \times \cdots \times 2n]$ for all $n > 0, n \in Z$

Induction Basis:

Show that A(1) holds.

$\frac{1}{2n} = \frac{1}{2(1)} = \frac{1}{2} \leq \frac{1}{2} = \frac{2(1)-1}{2(1)} = \frac{2n-1}{2n}$

Thus, A(1) holds.

Induction Step:

As the induction hypothesis(IH), assume that the claim A(n+1) holds. Then show that A(n+1) holds.

$\frac{1 \times 3 \times 5 \times \cdots \times (2n-1) \times (2(n+1)-1)}{2 \times 4 \times \cdots \times 2n \times 2(n+1)}$

$= \frac{1}{2n} \times \frac{2(n+1)-1}{2(n+1)}$ by IH

$= \frac{1}{2n} \times \frac{2n+1}{2n+2}$ by distributing and combining like terms

$= \frac{2n+1}{4n^2+4n}$ by multiplying the fractions

$= \frac{2n}{4n^2+4n} + \frac{1}{4n^2+4n}$ by seperating the terms in the numerator

$= \frac{2n}{2n(2n+2)} + \frac{1}{2n(2n+2)}$ by factoring out $2n$

$= \frac{1}{2n+2} + \frac{1}{2n(2n+2)}$ by cancelling terms in numerator and denominator

$= \frac{1}{2(n+1)} + \frac{1}{2n(2n+2)}$ by factoring out 2

$\frac{1}{2(n+1)} + \frac{1}{2n(2n+2)} \geq \frac{1}{2(n+1)}$

Thus, A(n+1) holds.

Conclusion: Therefore, A(n) holds by induction on n.

Problem 5. (13 points) Section 5.1, Exercise 34, page 351. Prove by induction on n.

Solution. Prove A(n): 6 divides $n^3 - n$ for all $n \in Z, n \geq 0$
Induction Basis:
Show that A(0) holds. $n^3 - n = 6k, 0^3 - 0 = 6k, 0 = 6k, k = 0/6, k = 0$ because $k = 0 \in Z$, A(0) holds.

Induction Step:
As the induction hypothesis(IH), assume that the claim A(n) holds. Show that A(n+1) holds.
$(n+1)^3 - (n+1) = (n^3 + 2n^2 + n + n^2 + 2n + 1) - (n+1)$ by expansion
$= n^3 + 3n^2 + 3n + 1 - n - 1$ by combining like terms
$= n^3 + 3n^2 + 2n$ by combining like terms
$= (n^3 - n) + (3n^2 + 3n)$ by seperating $3n$ and commutativity of addition
$= 6j + (3n^2 + 3n)$ by IH, note: j is some integer
$= 6j + 3n(n+1)$ by factoring out 3n
Because the integer $6j$ is always divisible by 6, and because $3n(n+1)$ is always divisible by 6, it follows that A(n+1) holds.

Conclusion: Therefore, A(n) holds by induction on n.

Problem 6. (3 pts $\times$ 5 = 15 points) Section 5.2, Exercise 4, page 363

Solution. a) P(18): 4(1) + 7(2)
P(19): 4(3) + 7(1)
P(20): 4(5) + 7(0)
P(21): 4(0) + 7(3)

b) As the induction hypothesis, assume that the statement P(n) holds for all n in 18$\leq n \leq k$.

c) Then, show that P(k+1) holds.

d) We know from the inequality that 18$\leq n \leq k$, and therefore 18$\leq$k. We can then say that 18$\leq$k+1. This statement is equivalent $(k + 1) - 4 \geq 14$. From the strong induction hypothesis, we can say that for some number of $(k+1)-4$ cents, given that it is greater than 18, we can construct postage with x number of 4 cent stamps and y number of 7 cent stamps, and we can write the equation $(k+1)-4 = 4x+7y$. Solving for $k+1$, we get that $k+1 = 4(x+1)+7y$.

e) Therefore, because you can substitute in any positve integer for x and y, and get an integer result, the claim follows by induction on n.

Problem 7. (13 points) Section 5.2, Exercise 12, page 363

Solution. Prove P(n): A positive integer $n$can be written as a sum of distinct powers of two

Induction Basis: Prove P(1), P(2)
P(1): $1 = 2^0$
P(2): $2 = 2^1$

Induction Step:
As the induction hypothesis, assume that P(n) holds. Then, show that P(n+1) holds.

Case 1(n+1 is odd): We know that we can write n as a sum of distinct powers of 2 by the induction hypothesis. From the definition of odd numbers, we can say that because $n+1$ is odd, n is even. We take the fact that n is even to say that no matter what, because we have an even integer plus one, that the plus one will always be 2^0. Then, from the induction hypothesis, we can write n as the sum of distinct powers of 2, and because n is even it will never have 0 as one of its distinct powers. Therefore, $n + 1$ can be written as the sum of arbitrary distinct powers of two plus 2^0.

Case 2(n+1 is even): From the induction hypothesis, note that n can be written as the sum of distinct powers of two. Also, by the definition of an even integer, we can say that because $n + 1$ is even, $\frac{n+1}{2}$ is an integer. Because the division by two only scales down the sum, we can still say that $\frac{n+1}{2}$ can be written as the sum of distinct powers of two. This fact will not change if we multiply the sum by two because it will just increase the distinct powers of two by one. Therefore, $k+1$ can be written as the sum of distinct powers of two.

Conclusion: Therefore, because P(n+1) holds for both the even and odd cases, P(n) holds by induction on n.

Checklist:

☐ Did you type in your name and UIN?
☐ Did you disclose all resources that you have used?
(This includes all people, books, websites, etc. that you have consulted.)
☐ Did you electronically sign that you followed the Aggie Honor Code?
☐ Did you solve all problems?
☐ Did you submit both of the .tex and .pdf files of your homework to the correct link on eCampus?