CSCE 222 [Sections 503, 504] Discrete Structures for Computing
Fall 2019 – Hyunyoung Lee

Problem Set 3

Due dates: Electronic submission of *yourLastName-yourFirstName-hw3.tex* and *yourLastName-yourFirstName-hw3.pdf* files of this homework is due on **Friday, 9/20/2019 before 10:00 p.m.** on `http://ecampus.tamu.edu`. You will see two separate links to turn in the .tex file and the .pdf file separately. Please do not archive or compress the files. **If any of the two submissions are missing, you will likely receive zero points for this homework.**

Name: Ian Stephenson **Section: Section 504**

Resources. Discrete Math and Its Applications, 8th Edition, Rosen

On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.

Electronic Signature: **Ian Stephenson**

Total: 100 (+ 5 extra) points

***** Please make sure that you are solving the correct problems from the 8th Edition of the Rosen book, not the 7th Edition! *****

Problem 1. (2 points $\times$ 6 subproblems = 12 points) Section 2.1, Exercise 10, page 132.

Solution. a) No
b) No
c) Yes
d) Yes
e) Yes
f) No

Problem 2. (3 points $\times$ 4 subproblems = 12 points) Section 2.1, Exercise 26, page 132.

Solution. a) No
b) Yes
c) No
d) Yes

Problem 3. (10 points) Section 2.1, Exercise 28, page 132. *Use definitions and justify each step of your argument.*

Solution. By the definition of a cartesian product, we can say that A $\times$ B means that there is an element (a,b) such that a $\in$ A and b $\in$ B. We could also define a subset to be that for A $\subseteq$ B, every element contained in the set A is also an element contained in the set B. Given the subsets from the hypothesis, A $\subseteq$ C and B $\subseteq$ D, it would follow that the element a $\in$ C because a $\in$ and A is a subset of C, and that the element b $\in$ D because b $\in$ B and B is a subset of D. Returning to the definition of a cartesian product, because a $\in$ C and b $\in$ D, it would follow that (a,b) $\in$ C $\times$ D. Because every element (a,b) must be in the domain of A $\times$ B and C $\times$ D, we could say that every element of A $\times$ B must be in C $\times$ D, or that A $\times$ B $\subseteq$ C $\times$ D.

Problem 4. (2 points $\times$ 4 subproblems = 8 points) Section 2.2, Exercise 4, page 144.

Solution. a) A $\cup$ B = $\{a, b, c, d, e, f, g, h\}$
b) A $\cap$ B = $\{a, b, c, d, e\}$
c) A - B = $\{\emptyset\}$
d) B - A = $\{f, g, h\}$

Problem 5. (5 points $\times$ 2 subproblems = 10 points) Section 2.2, Exercise 16 c) and d), page 144. *Use definitions, and explain each step using definitions and/or laws.*

Solution. c) A - B $\subseteq$ A
By the definition of A-B, we could write the statement to be (x $\in$ A $\land$ x$\notin$ B) $\subseteq$ A. By the definition of a subset, we could say that $\forall$ x((x $\in$ A $\land$ x$\notin$ B)$\rightarrow$ x$\in$ A). The hypothesis can be simplified to x $\in$ A. The statement will then say that $\forall$ x(x $\in$ A $\rightarrow$ x $\in$ A). This shows that A - B $\in$ A, or that A - B $\subseteq$ A.

d) By the definition of B - A, A $\cap$ (x $\in$ B $\land$ x $\notin$ A). Then, by the definition of intersection, x $\in$ A $\land$ x $\in$ B $\land$ x $\notin$ A. The x $\in$ A and x $\notin$ A will always be false, so this whole statement is false. This false, in context of the problem, could be written as x $\in\emptyset$ because this statement will always be will. Therefore, A $\cap$ (B - A) = $\emptyset$.

Problem 6. (5 points $\times$ 2 subproblems = 10 points) Section 2.2, Exercise 56 a) and c), page 145.

Solution. a) Union: $\{1, 2, 3, \ldots\}$
Intersection: $\emptyset$
b) Union: (0,∞)
Intersection: (0,1)

Problem 7. (3 points $\times$ 4 subproblems = 12 points) Section 2.3, Exercise 12, page 162.

Solution. a) f(n) = n - 1 is a one to one function
b) f(n) = n^2 + 1 is not a one to one function
c) f(n) = n^3 is a one to one function
d) f(n) = $\lfloor n/2 \rfloor$ is not a one to one function

Problem 8. (3 points $\times$ 2 subproblems = 6 points) Section 2.3, Exercise 14 a) and b), page 162.

Solution. a) f(m,n) = 2m - n is an onto function
b) f(m,n) = m^2 - n^2 is not an onto function

Problem 9. (2.5 points $\times$ 4 subproblems = 10 points) Section 2.3, Exercise 60, page 164.

Solution. a) 1 byte
b) 2 bytes
c) 63 bytes
d) 375 bytes

Problem 10. (15 points) Prove that

$$\left\lceil \left\lceil \frac{x}{2} \right\rceil /2 \right\rceil = \left\lceil \frac{x}{4} \right\rceil$$

holds for all real numbers x. *Use the definition of the ceiling function as we discussed in class.*

Solution. Let's say that n = $\left\lceil \left\lceil \frac{x}{2} \right\rceil /2 \right\rceil$. Therefore n - 1 < $\left\lceil \frac{x}{2} \right\rceil /2 \leq$ n by the definition of a ceiling function. Multiplying all the terms by 2 we get that
2n - 2 < $\left\lceil \frac{x}{2} \right\rceil \leq$ 2n. Because $\frac{x}{2}$ must be in the range (2n-2,2n] for $\left\lceil \frac{x}{2} \right\rceil$ to be in the range, it follows that 2n-2 < $\frac{x}{2} \leq$ 2n is true for all values of n. If we take this inequality and divide all the terms by 2, we get that n-1 < $\frac{x}{4} \leq$ n. This inequality is equivalent to $\left\lceil \frac{x}{4} \right\rceil$. Therefore, $\left\lceil \left\lceil \frac{x}{2} \right\rceil /2 \right\rceil = \left\lceil \frac{x}{4} \right\rceil$.

Checklist:
□ Did you type in your name and section?
□ Did you disclose all resources that you have used?
(This includes all people, books, websites, etc. that you have consulted.)
□ Did you electronically sign that you followed the Aggie Honor Code?

☐ Did you try to solve all problems?
☐ Did you submit both of the .tex and .pdf files of your homework to the correct link on eCampus?

LaTeX symbols for sets and functions

1. Set of integers that are less than or equal to n: $\{x \in \mathbf{Z} \mid x \leq n\}$
2. x is a real number: $x \in \mathbf{R}$
3. x is not an integer: $x \notin \mathbf{Z}$
4. Cardinality of set A: $|A|$
5. Union of set A and set B: $A \cup B$
6. Generalized union: $\bigcup_{i=1}^{\infty} A_i$
7. Intersection of set A and set B: $A \cap B$
8. Generalized intersection: $\bigcap_{i=1}^{\infty} A_i$
9. The empty set: $\emptyset$
10. Set A is a subset of set B: $A \subseteq B$
11. Set A is a proper subset of set B: $A \subset B$
12. Cartesian product of set A and set B: $A \times B$
13. Complement of set A: A^C or $\overline{A}$
14. Ellipsis: $\ldots$ or $\cdots$
15. Ceiling function: $\lceil 3.14 \rceil = 4$
16. Floor function: $\lfloor 3.14 \rfloor = 3$
17. Square root: $\sqrt{b^2 - 4ac}$