CSCE 222 [Sections 503, 504] Discrete Structures for Computing
Fall 2019 – Hyunyoung Lee

Problem Set 9

Due dates: Electronic submission of *yourLastName-yourFirstName-hw9.tex* and *yourLastName-yourFirstName-hw9.pdf* files of this homework is due on **Friday, 11/15/2019 before 10:00 p.m.** on `http://ecampus.tamu.edu`. You will see two separate links to turn in the .tex file and the .pdf file separately. Please do not archive or compress the files. **If any of the two submissions are missing, you will likely receive zero points for this homework.**

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Resources. Peer Teacher Central, Discrete Math and Its Applications

On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.

Electronic Signature: **Ian Stephenson**

Total 100 + 10 (extra credit) points.

Problem 1. (10 pts $\times$ 3 = 30 points) Section 8.2, Exercise 4 b), c) and d) page 551. For each subproblem, find the closed form solution for a_n by answering the following step by step:

1. (2 points) What is the characteristic equation of the recurrence relation?
2. ($2 + 2$ points) What are the roots of the characteristic equation? Express a_n in a generic form in terms of the roots you found. For (d), you will get only one root; refer Theorem 2 in page 544 for the generic form in this case.
3. (4 points) Find the closed form solution for a_n using the initial conditions. Show your work.

Solution. b) 1) $c_x \implies x^2 - 7x + 10 = 0$
2) $(x-5)(x-2) = 0 \implies x_1 = 5, x_2 = 2$
$a_n = \alpha(5)^n + \beta(2)^n$
3) $n = 0 : a_0 = \alpha(5)^0 + \beta(2)^0 \implies \alpha + \beta = 2 \implies \beta = 2 - \alpha$
$n = 1 : a_1 = \alpha(5)^1 + \beta(2)^1 \implies 5\alpha + 2\beta = 1$
$\implies 5\alpha + 2(2-\alpha) = 1 \implies 5\alpha - 2\alpha + 4 = 1 \implies 3\alpha = -3 \implies \alpha = -1$
$-1 + \beta = 2 \implies \beta = 3$
$a_n = -1(5)^n + 3(2)^n \implies a_n = 3(2)^n - 5^n$

c) 1) $c_x \implies x^2 - 6x + 8 = 0$
2) $(x-4)(x-2) = 0, x_1 = 4, x_2 = 2$
$a_n = \alpha(4)^n + \beta(2)^n$
3) $n = 0 : a_0 = \alpha(4)^0 + \beta(2)^0 \implies \alpha + \beta = 4 \implies \beta = 4 - \alpha$
$n = 1 : a_1 = \alpha(4)^1 + \beta(2)^1 \implies 4\alpha + 2\beta = 10$
$4\alpha + 2(4-\alpha) = 10 \implies 4\alpha - 2\alpha + 8 = 10 \implies 2\alpha = 2 \implies \alpha = 1$
$1 + \beta = 4 \implies \beta = 3$
$a_n = 4^n + 3(2)^n$

d) 1) $c_x \implies x^2 - 2x + 1 = 0$
2) $(x-1)^2 = 0, x_1 = 1$
$a_n = \alpha(1)^n + \beta n(1)^n \implies a_n = \alpha + \beta n$
3) $n = 0 : a_0 = \alpha + \beta(0) \implies \alpha = 4$
$n = 1 : a_1 = \alpha + \beta(1) \implies 1 = 4 + \beta \implies \beta = -3$
$a_n = 4 - 3n$

Problem 2. (5 points $\times$ 3 = 15 points) Let $A = \{1,2,3,4,5\}$ and $B = \{0,1,2,3,4\}$.

a) List all the ordered pairs in the relation $R = \{(a,b) \mid a+b=5\}$ on A.

b) List all the ordered pairs in the relation $R = \{(a,b) \mid a<b\}$ on A.

c) List all the ordered pairs in the relation $R = \{(a,b) \mid a<b\}$ from A to B.

Solution. a) (1,4), (2, 3), (3,2), (4, 1)
b) (1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 4), (2, 5), (3, 4), (3, 5), (4, 5)
c) (1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (3, 4)

Problem 3. (8 points $\times$ 3 = 24 points) Section 9.1, Exercise 6 a), b) and d), page 608

Solution. a) Reflexive: No, x+x=0 only holds when x=0, but x is the set of all real numbers.
Symmetric: Yes, if x+y=0, then y+x=0
Antisymmetry: No, for x+y=0 to hold, y=-x must hold, but $x \neq -x$
Transitive: No, say that x=-1, y=1, z=-1. x+y=0, and y+z=0, but x+z=-2

b) Reflexive: No, x=-x does not hold
Symmetric: Yes, if $x = \pm y$, then $y = \pm x$
Antisymmetric: No, say that 5=-(-5) and -5=-5, but $5 \neq -5$
Transitive: Yes, $x = \pm y$, and $y = \pm z$, so $x = \pm(\pm z)$, or $x = \pm z$

d) Reflexive: No,
Symmetric: No,
Antisymmetric: Yes,
Transitive: No,

Problem 4. (5 points $\times$ 2 = 10 points) Let A be the set of all people and $(x, y) \in A \times A$. Is each of the following an equivalence relation? For each subproblem, explain which of the three properties of an equivalence relation – reflexivity, symmetry, and transitivity – are satisfied and which are not, by explaining why or why not. Then, answer whether it is an equivalence relation.

a) $R_1 = \{(x, y) \mid x \text{ and } y \text{ have the same parents}\}$

b) $R_2 = \{(x, y) \mid x \text{ and } y \text{ have a common grandparent}\}$

Solution. a) Reflexive: x has the same parents as x, so reflexivity holds.

Symmetric: Say that x has the same parents as y. That would also mean that y has the same parents as x. Therefore, symmetry holds.

Transitive: Say that x has the same parents as y, and say that y has the same parents as z. It would reason that x, y, and z all have the same parents, and therefore x and z have the same parents. Thus, transitivity holds.
$\therefore R_1$ is an equivalence relation

b) Reflexive: x has a common grandparent with x, so reflexivity holds.

Symmetric: Say that x has a common grandparent with y. It would reason that y also has a common grandparent with x. Therefore, symmetry holds.

Transitive: Say that x and y have a common grandparent, and that y and z have a common grandparent. This does not imply that x and z have a common grandparent, and therefore transitivity does not apply.
$\therefore R_2$ is not an equivalence relation because it is not transitive

Problem 5. (10 points) We define on the set $\mathbf{N}_1 = \{1, 2, 3, \cdots\}$ of positive integers a relation $\sim$ such that two positive integers x and y satisfy $x \sim y$ if and only if $x/y = 2^k$ for some integer k. Show that $\sim$ is an equivalence relation.

Solution. Reflexive: Say that y=x, and therefore $x/x = 2^k$, where k is some integer. Thus, $1 = 2^k, 1 = 2^0$. 0 is an integer, and so reflexivity holds.

Symmetric: For to be symmetric, we must show that if $x/y = 2^k$, then $y/x = 2^l$ holds. Take $x/y = 2^k$. By the laws of negative exponents, this would mean that $y/x = 2^{-k}$. Because $-k$ is an integer, $\sim$ is symmetric.

Transitive: For to be transitive, $x/y = 2^k$ and $y/z = 2^l$ implies that $x/z = 2^m$. If you multiply x/y and y/z, you get that $x/z = 2^{k+l}$. When you add two integers k and l you are left with an integer, and therefore $\sim$ is transitive.

$\therefore$ x$\sim$y is an equivalence relation

Problem 6. (7 points $\times$ 3 = 21 points) Section 9.6, Exercise 2 b), c) and e), page 662. For each subproblem, explain which of the three properties of a partial ordering – reflexivity, antisymmetry, and transitivity – are satisfied and which are not, by explaining why or why not. Then, answer whether it is a partial ordering.

Solution. b) Reflexive: For every element $a \in A$, (a, a) is in R. For example, $0 \in A$, and $(0, 0) \in R$. Thus, reflexivity holds.

Antisymmetry: If $(a, b) \in R$ and $(b, a) \in R$, then $a = b$ must hold. We have that, for example, (0, 0) is in R, and 0=0, so antisymmetry holds for this. As a counterexample, we have that (0, 2) is in R, but (2, 0) is not in R. Because $a \neq b$, antisymmetry holds. Thus, antisymmetry holds.

Transitive: If (a, b) is in R, and (b, c) is in R, then (a, c) must also be in R. Most elements fall into the category of a=b=c, where (a, b), (b, c), and (a, c) are all the same element. The elements (2, 0) and (2, 3) do not fall under this category. For both of these, a=2 and b=0 or b=3. For transitivity to hold, the sets b=c for (b, c) must hold. Because the sets (0, 0) and (3, 3) are in R, transitivity holds.
$\therefore$ The set is a partial ordering

c) Reflexive: For every element $a \in A, (a, a) \in R$. Thus, reflexivity holds.

Antisymmetry: If $(a, b) \in R$ and $(b, a) \in R$, then $a = b$ must hold. The sets (0, 0), (1, 1), (2, 2), (3, 3) are all in R and satisfy the conditional. For all sets (a, b) where $a \neq b$ and (a, b) is in R, (b, a) is not in R. Therefore, antisymmetry holds.

Transitive: We have that (3, 1) is in R, and that (1, 2) is in R. However, the element (3, 2) is not in R. The conditional statement (a, b) in R and (b, c) in R implies that (a, c) is in R, does not hold. Thus transitivity does not hold.
$\therefore$ the set is not a partial ordering because it is not transitive.

e) Reflexive: For every element $a \in A, (a, a) \in R$. Thus, reflexivity holds.

Antisymmetry: If $(a, b) \in R$ and $(b, a) \in R$, then $a = b$ must hold. However, (0, 1) is in R, and (1, 0) is in R, but $0 \neq 1$. Thus antisymmetry does not hold.

Transitive: If (a, b) is in R, and (b, c) is in R, then (a, c) must also be in R. However, (2, 0) is in R, and (0, 3) is in R, but (2, 3) is not in R. Thus, transitivity does not hold.
$\therefore$ the set is not a partial ordering because it is not transitive or antisymmetric.

Checklist:
☐ Did you type in your name and UIN?
☐ Did you disclose all resources that you have used?
(This includes all people, books, websites, etc. that you have consulted.)
☐ Did you electronically sign that you followed the Aggie Honor Code?
☐ Did you solve all problems?
☐ Did you submit both of the .tex and .pdf files of your homework to the correct link on eCampus?