CSCE 222 [Sections 503, 504] Discrete Structures for Computing
Fall 2019 – Hyunyoung Lee

Problem Set 7

Due dates: Electronic submission of *yourLastName-yourFirstName-hw7.tex* and *yourLastName-yourFirstName-hw7.pdf* files of this homework is due on **Friday, 10/25/2019 before 10:00 p.m.** on `http://ecampus.tamu.edu`. You will see two separate links to turn in the .tex file and the .pdf file separately. Please do not archive or compress the files. **If any of the two submissions are missing, you will likely receive zero points for this homework.**

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Resources. Discrete Math and Its Applications, 8th Edition, Rosen

On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.

Electronic Signature: **Ian Stephenson**

Total 100 + 10 (extra credit) points.

Problem 1. $(10 + 10 = 20$ points) Section 5.3, Exercise 6 b), page 378. Prove your formula using strong induction.

[Grading rubric: Finding a formula for $f(n)$ is worth 10 points and proving that your formula is valid is worth 10 points, where the base step is worth 3 points and the inductive step is worth 7 points (stating the strong induction hypothesis correctly and specifying where it is used is worth 5 points).]

Solution. $f(3) = 2, f(4) = 0, f(5) = 4, f(6) = 4, f(7) = 0, f(8) = 8,$
$f(9) = 8, f(10) = 0$

$$f(n) = \left\{ 2^{n/3} \text{ if } n\%3 = 0 \right.$$

$$0 \text{ if } n\%3 = 1$$

$$\left. 2^{(n+1)/3} \text{ if } n\%3 = 2 \right\}$$

Proof:

Induction Basis: Show that n=0, n=1, n=2 hold.

$n = 0 : 0\%3 = 0, 2^{0/3} = 2^0 = 1 = f(0)$

$n = 1 : 1\%3 = 1, 0 = f(1)$

$n = 2 : 2\%3 = 2, 2^{(2+1)/3} = 2^{3/3} = 2^1 = 2 = f(2)$

Induction Step: As the induction hypothesis, suppose that the formula for $f(n)$ holds. Then, show the $f(n + 1)$ holds.

Note: We must prove that each condition in $f(n + 1)$ holds.

Problem 2. ($5 + 10 = 15$ points) Section 5.3, Exercise 28 a) and b), page 379

Solution. a)
First Step:
(2, 3), (3, 2)
Second Step:
(4, 6), (5, 5), (6, 4)
Third Step:
(6, 9), (7, 8), (8, 7), (9, 6)
Fourth Step:
(8, 12), (9, 11), (10, 10), (11, 9), (12, 8)
Fifth Step:
(10, 15), (11, 14), (12, 13), (13, 12), (14, 11), (15, 10)

Problem 3. (5 points) Section 6.1, Exercise 12, page 417

Solution. $2^6 + 2^5 + 2^4 + 2^3 + 2^2 + 2^1 = 126$

Problem 4. (5 points) Section 6.1, Exercise 14, page 417

Solution. 2^{n-2} possible bit strings

Problem 5. (5 points $\times$ 3 = 15 points) Section 6.1, Exercise 20, page 417

Solution. a) 9, {6, 9, 12, 15, 18, 21, 24, 27, 30}
b) 6, {8, 12, 16, 20, 24, 28}
c) 2, {12, 24}

Problem 6. (5 points $\times$ 3 = 15 points) Section 6.1, Exercise 24 a), c) and e), page 417. Explain.

Solution. a) If you divide 9999 by 9, you get there are 1111 numbers between 0 and 9999, inclusive, that are divisible by 9. Then, if you divide 999/9, you get that there are 111 numbers between 0 and 1000, exclusive, that are divisible by 9. Because the values from 0 to 1000 are not included in the range but were counted when determining the number of values between 0 and 10000, you subtract them away. Therefore, the answer is $1111 - 111 = 1000$. So there are 1000 numbers between 0 and 10000 that are divisible by 9.

c) We know a few things to start the problem. First, we are dealing only with 4 digit numbers, so we need each of the four different digits to make our number. Next, we know that the first digit must be a digit between 1 and 9, inclusive, because the first number can not be zero. From this, we can say that we have 9 choices for our first digit. For the second choice, we again have 9 digits because the second digit can be zero. The third digit will have 8 choices, as it can not be either of the other two digits already chosen. The fourth digit has 7 choices. We then multiply all the values together to get that $9 \times 9 \times 8 \times 7 = 4536$. So there are 4536 numbers with distinct digits in between 1000 and 9999.

e) First, lets determine the number of values divisible by 5, which can be done by $\lfloor 9999/5 \rfloor - \lfloor 999/5 \rfloor = 1800$. Then we can find the number of values divisible by 7, which is $\lfloor 9999/7 \rfloor - \lfloor 999/7 \rfloor = 1286$. Then, we need to find the number of values that are divisible by both 5 and 7, which is to say divisible by 35, which is $\lfloor 9999/35 \rfloor - \lfloor 999/35 \rfloor = 257$. Then, from the inclusion/exclusion rule, we must subtract the intersection of values divisible by 5 and 7 away from the union of those values, as they were counted twice. Thus, we have that the number of values that are divisible by 5 or 7 is $1800 + 1286 - 257 = 2829$.

Problem 7. (5 points) Section 6.2, Exercise 6, page 426. Explain.

Solution. Using the pigeonhole principle we know that in any given class, there would need to be 102 students to guarantee that at least two students have the same grade. This is because there are 101 possible grades, and if there is one more than the total possible grades, at least two will be the same. We know that there are six professors, and each professor needs at least 102 students, so we multiply 102 by 6 to get that there must be at least 612 total students to guarantee that each professor has at least two students with the same grade.

Problem 8. (10 points) Section 6.2, Exercise 12, page 426. Explain.

Solution. Our first step will be to define the midpoint, $M = (\frac{x_a+x_b}{2}, \frac{y_a+y_b}{2})$. Then, note that there are five different coordinate pairs that will be given. We know that we will need four different forms of coordinate pairs to satisfy the pigeonhole principle, so we also note that there are four possible general pairs that we can have, namely $(2a+1, 2b+1), (2a, 2b), (2a+1, 2b), (2a, 2b+1)$, where a and b are integers, which are all the combinations of even and odd integer coordinate pairs. Because there are 5 coordinate pairs and 4 coordinate pair forms, we know that at least two of the forms will be the same based on the pigeonhole principle. Thus, we need to show that no matter what, when two of the pairs have the same form, they will yield an integer midpoint. When both are of the form $(2a+1, 2b+1)$ our midpoint will be $(\frac{2a+1+2b+1}{2}, \frac{2c+1+}{2})$, or $(a+b+1, c+d+1)$, which are integers. When both are in the form $(2a, 2b)$, our midpoint will be $(\frac{2a+2b}{2}, \frac{2c+2d}{2})$, or $(a+b, c+d)$, which are integers. When they are in the form of $(2a+1, 2b)$ and $(2c+1, 2d)$, our midpoint will be $(\frac{2a+1+2c+1}{2}, \frac{2b+2d}{2})$, or $(a+c+1, b+d)$, which are integers. Finally, when they are in the form $(2a, 2b+1)$ and $(2c, 2d+1)$, our midpoint will be $(\frac{2a+2c}{2}, \frac{2b+1+2d+1}{2})$, or $(a+c, b+d+1)$, which are both integers. Thus, for any five coordinate pairs given, there will be guaranteed at least one midpoint with two integer coordinates.

Problem 9. (10 points) Section 6.2, Exercise 14, page 426. Explain.

Solution. We know that $x mod y$ will return the division remainder of the two values. Because we are taking the modulus of x by 5, and because we are looking for the fewest number of coordinates possible to satisfy our condition, it will suffice to just look at coordinate pairs (a_1, b_1) where a_1 and b_1 are less than 5. From this, we want to find how many possible combinations of integer coordinates can be made of values less than 5. From the product rule, we know that there are 25 integer coordinates, as there are 5 coordinates for a_1 and 5 coordinates for b_1. Thus, by the pigeonhole principle, we need at least 26 coordinate pairs to guarantee that at least two of the pairs will be the same, and thus $(a_1\%5, b_1\%5) = (a_2\%5, b_2\%5)$. The pairs do not have to be limited to values less than 5, but because you are taking the modulus of the values, they will always end up as integer coordinates less than 5, and will therefore follow.

Problem 10. (10 points) Section 6.2, Exercise 48, page 428. Explain.

Solution.

Checklist:

☐ Did you type in your name and UIN?
☐ Did you disclose all resources that you have used?
(This includes all people, books, websites, etc. that you have consulted.)
☐ Did you electronically sign that you followed the Aggie Honor Code?
☐ Did you solve all problems?
☐ Did you submit both of the .tex and .pdf files of your homework to the correct link on eCampus?