The Gaussian Distribution

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November 2023

1 Introduction

Normal distribution, also known as the Gaussian distribution, is a probability distribution that is symmetric about the mean, showing that data near the mean are more frequent in occurrence than data far from the mean.

In graphical form, the normal distribution appears as a "bell curve".

Normal distributions have the same general shape: symmetric and unimodal (i.e., a single peak) with tails that appear to extend to positive and negative infinity. In a normal curve, approximately 68% of the values fall within one standard deviation of the mean, 95% fall within two standard deviations, and 99.7% fall within three standard deviations.

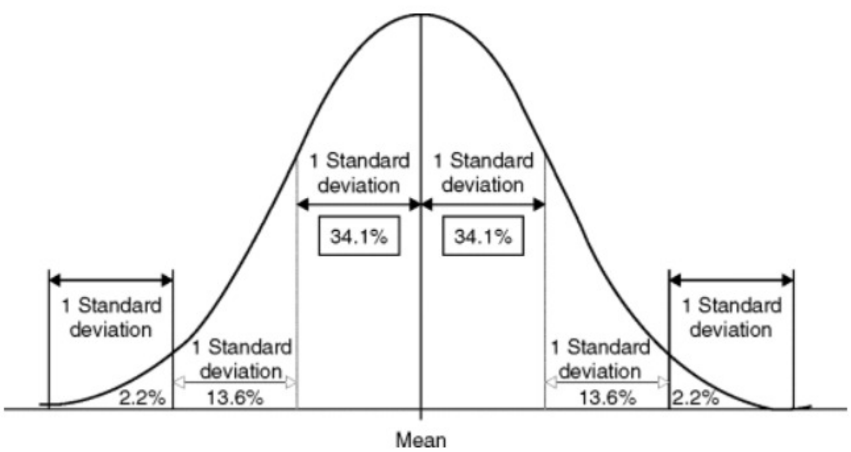

Figure 1: Gaussian distribution

2 Formula and extension

The probability density function of the univariate (one-dimensional) Gaussian distribution is:

$$p(x \mid \mu, \sigma^2) = \mathcal{N}(x; \mu, \sigma^2) = \frac{1}{Z} exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

The normalization constant Z is:

$$Z = \sqrt{2\pi\sigma^2}$$

The parameters μ and σ^2 specify the mean and variance of the distribution, respectively:

$$\mu = E[x]; \quad \sigma^2 = var[x]$$

We may extend the univariate Gaussian distribution to a distribution over d-dimensional vectors, producing a multivariate analog. The probablity density function of the multivariate Gaussian distribution is

$$p(\mathbf{x} \mid \boldsymbol{\mu}, \boldsymbol{\Sigma}) = \mathcal{N}(\mathbf{x}; \boldsymbol{\mu}, \boldsymbol{\Sigma}) = \frac{1}{Z} \exp\left(-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^\top \boldsymbol{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu})\right).$$

Figure 2: Formula

3 Simulation

An easy way to implement a Gaussian Distibution could be the following:

```python
import matplotlib.pyplot as plt
import numpy as np
from scipy.stats import norm

# Set the mean and standard deviation of the normal distribution
mu = 0
sigma = 1

# Generate values for the x-axis
x = np.linspace(mu - 3 * sigma, mu + 3 * sigma, 1000)

# Calculate the probability density function (PDF) for each x value
y = norm.pdf(x, mu, sigma)

# Plot the normal distribution
plt.plot(x, y, label='Normal Distribution')
```

```python

# Add labels and a legend
plt.xlabel('X-axis')
plt.ylabel('Probability Density Function')
plt.title('Normal Distribution')
plt.legend()

# Show the plot
plt.show()

```

This way we are going to get this graphic:

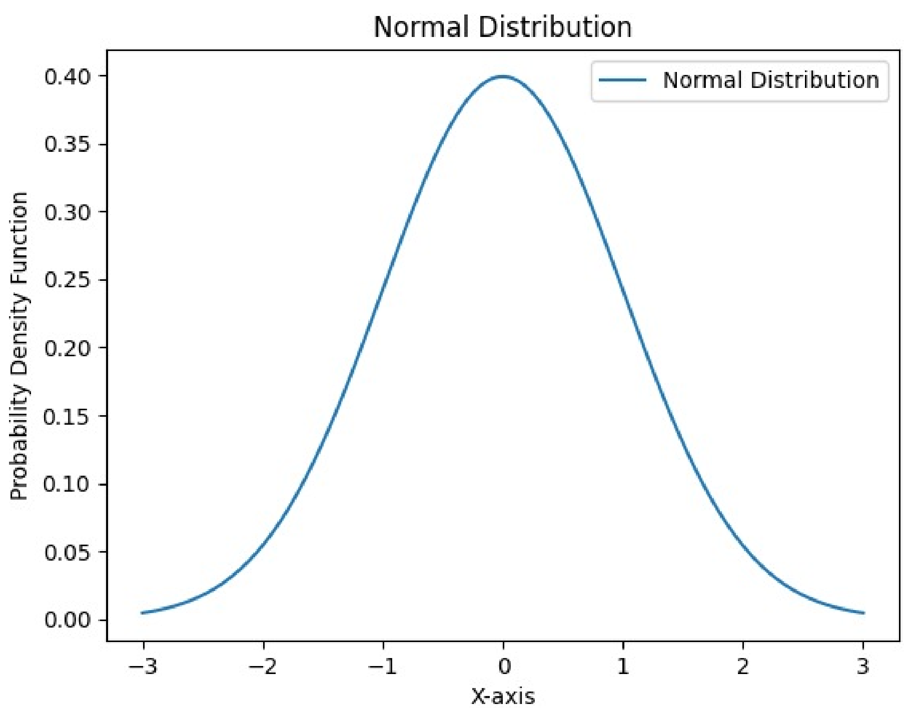

Figure 3: From GoogleColab