

# Review of Statistical Analysis of Numerical Preclinical Radio-biological Data

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## Abstract

In this article we review the paper “Statistical analysis of numerical pre-clinical radio-biological data” [Pitt and Hill, 2016]. The work is submitted as a term project for the Graduate Level Course on Statistical Modeling and Practices at University of California Berkeley. The authors are graduate students from department of EECS and Civil&Environmental Engineering and have restricted their attention to the methods and analysis done in the paper. The review is an attempt to reproduce the tests and results presented in the paper, and discuss some other non-parametric tests and results eg. Permutation tests, that can be seen as an alternative to making certain assumptions and finding surprises in the data. No attempt has been made to look into the biological aspects and validity of certain assumptions related to them. Also we did not dive into exhaustive literature search for common practices for such data. We attempt to encourage the use of permutation tests in similar problem set-ups.

## 1 Introduction

We review the paper in the spirit of promoting reproducibility of research and attempt to replicate the authors’ work. We also discuss some more ways of identifying anomaly, and present results based on our analysis using Permutation Tests. We believe this practice is not only consistent with the aim of the original paper - promoting simple statistical tools for detecting anomaly, but also provides a validation of their results to some extent.

As a small comment on the readability, we would like to mention that the organization of the original paper could have been much better with the use of clear numberings for sections and subsections, and re-arranging some of the sections. Next, we discuss the problem set up considered by the authors, and make some remarks on the methods used. In Section 2 we replicate authors’ work and results to some extent. In Section 3, we discuss some gaps as a reader and attempt to take a step back and do some tests, and point the gaps in the work and how to address them using statistical methods. We conclude with some remarks in Section 4.

### 1.1 Problem Set Up

The paper begins by voicing a growing concern towards “Scientific fraud and Plagiarism” in the scientific community and is successful in presenting a strong message. The authors present some statistical figures and attempt to point the existence of easy statistical tools to detect fabricated data and ignorance about the tools. Moving beyond the message, we next discuss the specific problem set up. The authors in the paper analyze anomalous patterns in radio-biological data from a lab, in particular they were able to detect suspicious patterns in the data reported by one of the 10 researchers (whom we shall refer to as RTS as per their notation). They do three different tests to validate their suspicion and also validate their tests and assumptions by looking at the data obtained from three other sources.

In particular, each researcher had to report three different measurements for two different types of numbers - Colony Count and Coulter Count. Each of these numbers represents an observation of number of cells surviving some experiment, and probably three measurements are done in order to be more accurate about the observations. The concern of the authors is that it is easy to fabricate a triplet such that you get

a desired mean for that particular set of observations. One can, in fact, do that by setting the mean and then using two roughly equal constants, calculate the other two values as this initial value plus or minus the selected constants. Such a fabrication can be flagged easily by looking at the triplets and counting how many of them contain the mean as one of the three values.

Having made these observations, the authors mainly focus “on developing a method to calculate bounds and estimates for the probability that a given set of  $n$  such triplicates contains  $k$  or more triples which contain their own mean” and mention that such probability bounds should be helpful across various other areas. Under these models they show that RTS’s data is pretty surprising and that the chances of seeing such a data are astronomically low. Besides this specific set up (which requires some assumptions) they also look at some more general tests that have been used in the past to detect anomalous patterns. Namely they test for - (1) Distribution of the least significant digit, and (2) Chances of observing equal pairs of terminal digits. Ideally, for (1), we expect to see a uniform distribution over  $\{0, 1, \dots, 9\}$  unless the distribution that underlies the data suggests otherwise. Similarly, for (2), ideal chances of having an equal pair of terminal digits is 1 in 10.

However, some of the questions that were slightly untouched upon are discussed below:

- The paper begins with the RTS being labeled as anomalous and then a probability model is developed to determine the chances of seeing the mean in a triplet. The authors mentioned briefly that “Having observed what appeared to us to be an unusual frequency of triples in RTS data containing a value close to their mean, we used R to calculate the mid-ratios for all of the colony data triples that were available to us”. The authors didn’t comment how were they able to identify the particular researcher. Whether they partitioned the data into an observation set and then ran tests on the validation set is also unclear and the tables tend to hint otherwise (as the authors seem to be using almost entire data for reporting statistics). An ideal practice would be to classify the data into training and test set. The partitioning is important as *the data that raised the suspicion if used to validate it, will most likely give a very biased result.*
- The authors ran tests for the last digit and equality of the pair of terminal digits on the datasets, which can be seen as a validation of their suspicion. However all the results that are produced are of the form “RTS vs the Rest”. It would have been more convincing if the authors presented some justification or some experiment results which justified such a treatment. The ideal scenario would have been the presentation of results in a “Take - One - Out” fashion, where every individual would have been compared to the rest of them pooled together. This is the core principle behind the two-sample permutation tests, where we test the strong null hypothesis that each researcher’s data is just a random sample from the population of all the data put together. We will dive into this in Section 3.
- There was very little discussion about the huge variation in number of data points across the researchers. For some reason, the data collected by the RTS was more than twice the data put together by twelve other researchers. Such an overwhelming fraction of samples belonging to one researcher has some implications on Permutation tests as well which we explore in Section 3.

## 2 Reproducibility of Results

In this section, we replicated the statistical experiments that were conducted by the researchers. There were several mismatches in our first implementation because of subjectivity at certain places. However, with some trial and error and fine tuning we were able to replicate most of their results, obtaining similar results in the other cases. All our results and code are available at [github\[github.com/ianno/stat215a\\_project1\]](https://github.com/ianno/stat215a_project1). We first discuss specifics about the replication and then comment about the tests and methods involved.

### 2.1 Mid-Ratio Analysis

To begin with, the authors first consider the histogram of mid-ratio which is defined for a triplet  $(a, b, c)$ ,  $a < b < c$  as  $\frac{b-a}{c-a}$ , and show that the histogram of RTS concentrates abnormally around 0.4–0.6 range, compared to everyone else put together. We tried to reproduce the histogram in python using the numpy’s histogram plots (and in an early test also using Matlab) and it looked very different. Then, we tweaked the histogram to

include the right edge of the bins and it looked very similar to the Figure(1) of the paper. But the histogram still had differences, for instance, the authors get very close to 50% chance of obtaining a mid-ratio of 0.4-0.5, while we get close to 44% chance. Also, we used 1361 values for computing the histogram after removing the triplets with missing values (in fact, 1360 because one triplet had all equal values) while the authors used 1343/1361 and provided no justification for the same. Similarly, we had 595 triplets to plot the histogram for the rest of the researchers (of the same lab). However, our plots can be categorized very similar to theirs after the bin adjustment, and we categorized these differences too minor for investment of more time.

## 2.2 Probability Model

In this section, we followed the equations provided by the authors in Appendix A to calculate the probability -  $\lambda$  table. Here, first they model each triplet of observations as a three identically independent distributed (i.i.d.) Poisson random variables with mean  $\lambda$  (which could differ from triplet to triplet). Next they model the occurrence of mean (rounded off) in such a triplet as a Bernoulli random variable whose success probability is tabulated in Table 1 as a function of  $\lambda$ . They provided analytical expressions in the Appendix (which looked fine at a glance) to compute this table. We could replicate Table 1 from the paper and the trends in the values as a function of  $\lambda$ . However for large  $\lambda$  for couple of implementations we got 0 value, in place of very small values for the probabilities, and we didn't improve our implementation.

The authors used the Table 1 in two ways to choose the probability for the Bernoulli random variables. First, they used the maximum value from Table 1 as a uniform parameter for all triplets, essentially treating all triplet as i.i.d. Bernoulli(0.42), and in the second set of results, they used the Maximum Likelihood estimate (sample mean in this case) for each triplet to find the probability of success value in the table thereby treating each triplet having a different probability of success.

### 2.2.1 Using $\lambda$ to obtain p-values

In this section, the researchers used their probability model calculations to compute the chance of observing the data. While replicating, it worked fine for us with the colony data as the mean of the counts  $< 100$ , and we were able to replicate their computations to minor errors. However, when we conducted the same experiments for Coulter data, due to the limitation of our implementations, we could barely come up with a reasonable probability value as the mean value of counts were a lot larger, and thus we could not replicate the values for the Coulter. We tried a regression based on the statement from the literature that when  $\lambda = 100$  we use probability  $< 0.14$ , and for  $\lambda = 2000$  we use probability = 0.032. However the take away message is hardly unaffected, and these section were not the focus of our review. For completeness we mention the interpolated probabilities for Coulter Data used for computing statistics as in Table 2 of the original paper:

Linear combination for probability values when lambda is very large. Coulter						
	mean1	probability	mean2	probability	mean3	probability
RTS Coulter	998.6	0.042	1019.2	0.039	1039.8	0.040
Others Coulter	2918.6	0.013	2966.5	0.013	3012.5	0.011
Outside Lab2 Coulter	2135.2	0.028	2454.4	0.022	2748.2	0.019
Outside Lab3 Coulter	3322.1	0.011	3383.4	0.010	3450.1	0.009

Figure 1: Approximate  $p$ -values for Coulter Data

Name	No.Complete	New "Round" value for Colony				
		No.mean	No.expected	Sd	Z	p>= k
RTS Colonies,	1343	690	207.27	13.24	23.19	0.00
Others Colonies	577	109	92.7	8.82	-1.06	0.855
Outside Lab1 Colony	48	3	8.0	2.58	-1.78	0.962
Coulter lambda is too large to calculate those statistics.						
Linear combination for probability values when lambda is very large. Coulter						
RTS Coulter	1725	176	69.58	7.37	5.89	1.01e-9
Others Coulter	928	73	11.44	3.36	4.93	4.14e-7
Outside Lab2 Coulter	95	0	2.19	1.46	-1.5	0.933
Outside Lab3 Coulter	118	1	1.18	1.08	-0.17	0.566

Figure 2: Approximate Replication of Table 2

## 2.3 Digits Analysis

To find additional confirmations on the suspect of fabricated data, the authors perform two additional tests, namely *terminal digit analysis* and *pair of equal terminal digits analysis*. Both such analyses are based on existing work (and intuition) that the least significant digit of a sample is, in general, not very informative, i.e. it is reasonable to expect it to be uniformly distributed random variable.

### 2.3.1 Terminal digit analysis

The assumption behind this test is that for experiments including counts, the last digit of a sample represented by a big number ( $> 100$ ) can be expected to be uniformly distributed. On the other hand, fabricated data often fail to show such peculiar property. The authors use the chi-square test for goodness of fit to demonstrate the fraudulent nature of RTS' samples. Our results are very similar to the ones in the paper, although not identical possibly due to the minor difference in number of data points as pointed earlier.

### 2.3.2 Equal digits analysis

This test follows from the assumptions made from the previous one, and the claim is that in case of genuine data, one should see an equal pair of terminal digits only in  $1/10$  of the samples. In this case the authors consider only big numbers ( $> 100$ ), to ensure the analysis of insignificant digits. In this scenario, however, the authors fail to state what kind of test they have performed (we assume again chi-square test for goodness) and how the data was pre-processed. This led us to obtain similar, but not identical results.

### 2.3.3 Discussion of Assumptions

We discuss the assumptions and tests in bullet points, for brevity.

- We felt that the justification for the Poisson assumption for the triplet data was given less importance. And the applicability of the model to the data was also not underlined to a desirable extent. One can possibly think of various reasons and situations where doing so is hard to justify. But, beyond our intuition we didn't investigate the validity in detail.
- Though one can argue that the parameters fitted to suspected data should not be used to test the validity of the data, we agree with the authors that such a practice only lowers the chances of the suspicion, and gives the person in question a benefit of doubt.
- The authors provide a reference for the uniformity of last insignificant digit to a work [Mosimann et al., 2002], but fail in explaining why such framework can safely be applied in this context. For instance, there might be some characteristics of the underlying biological process which prevent the last digits to be uniformly distributed. An attempt to clarify and justify this choice in the current setting would have been beneficial. The authors include here additional data, provided by three external sources (two for Coulter counts and one for Colony counts) which suffered from relatively very low number of data points. Although the authors comment on the number of these additional samples in the Discussion section, we still believe that, in the current setting, these additional samples

do not help them in making a stronger case, but instead can be misleading and definitely added to our confusion.

- We reiterate that treating all the other lab investigators as a single pool and singling out RTS is not sufficient, since uniformity of the pool doesn't necessarily imply a similar property for each contributors. This is the starting point of our next section.

### 3 Our Analysis

The authors begin by singling out that the histogram of RTS looks anomalous compared to the rest of them put together. They assume that one is likely to observe uniform distribution for mid-ratio, and this fact is validated by the histogram of the 9 researchers put together which looks close to uniform. The first question that came to our mind which motivated this section was - how do we single out the anomalous researcher if we don't know a priori who he/she is? If we decide on the histogram as the first test, then a simple way would be to plot histogram of the mid-ratios for the data collected by all researchers individually, and look for anomalous patterns across all these plots. For sake of similarity to the authors' set up, one will detect anomaly by contrasting each researcher's histogram with the histogram of all others put together. Such an experiment gives very interesting results and also raises an important issue with this approach.

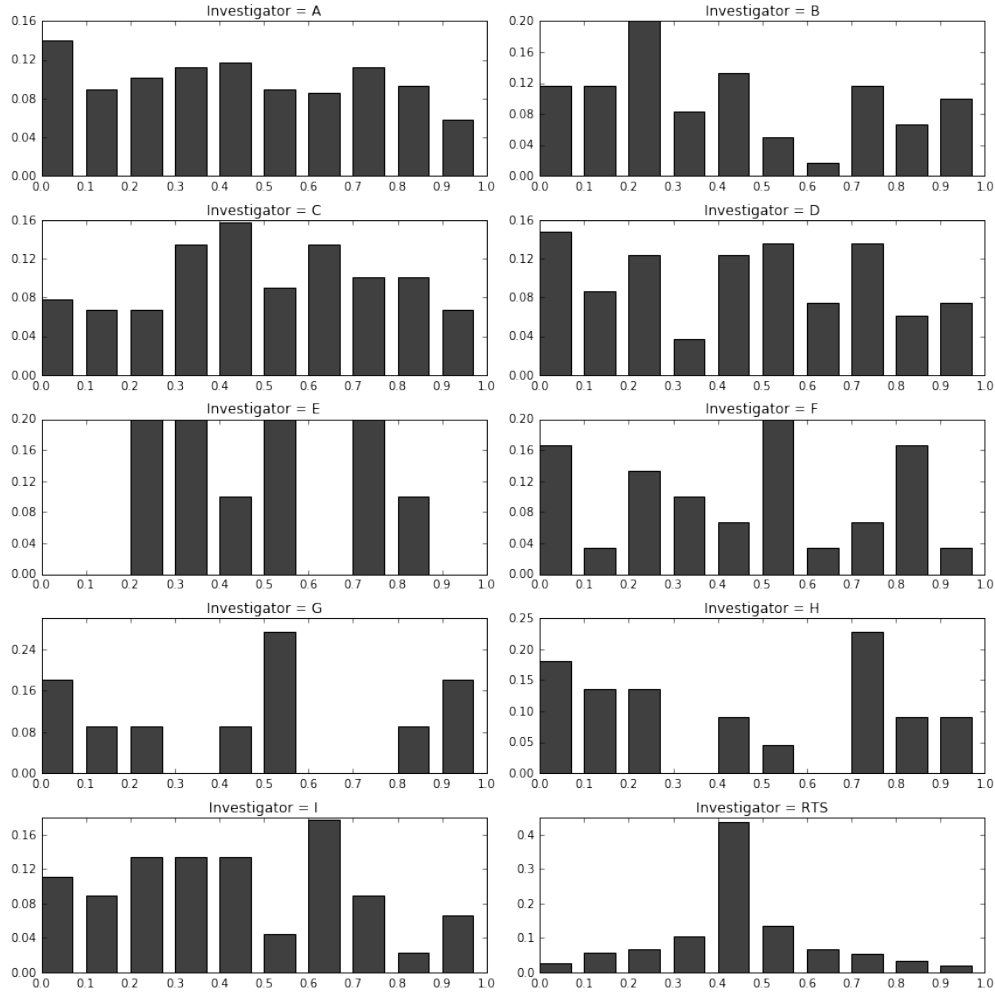


Figure 3: Individual Histograms for the Colony Data

- First, the histogram for researchers with labels “B, C, E, F, G, H, I” do not seem to be close to uniform as well. In particular, “B” and “C” have a very different histogram when contrasted with the histogram for uniform distribution. They have distinct peaks but around 0.2 and 0.4 respectively.
- Second, when we try to contrast the individual histogram of researchers with rest of them combined which includes RTS, the new “rest” histograms are dominated by RTS’s data because of the comparatively huge fraction of data collected by RTS, and so most of the other researchers look anomalous when contrasted with it.

The previous two remarks point out the limitations on the visual comparison of histogram and assumption of “uniform distribution” for mid ratios. Next we try to present a different viewpoint which has two advantages - it is free of such assumptions, and thus extends to far more general cases where even slight intuition about the data is missing.

### 3.1 Quick Primer to Permutation Tests

As discussed above, we felt that the justification for singling out the particular RTS was incomplete. So, we took a step back, and did permutation tests to identify anomalous patterns across different researchers. We briefly discuss the test set up and the philosophy of the test.

Given a treatment and control group of size  $T$  and  $C$  respectively, we want to test the hypothesis if the treatment has an effect on the population. In permutation test, the data pooled together is considered as the population (here it will have size  $N = T + C$ ). Next, one decides on a test statistic that is consistent with our hypothesis and is expected to contrast the two set of samples if the treatment has any effect. The distribution of test statistic has an exact theoretical representation but is often computationally intractable. An empirical approximation can be made by randomly partitioning the data into groups of  $T$  and  $C$  several times, and computing the test statistic contrasting the two datasets. With the distribution in hand, we can now test how surprising was the outcome that we originally had.

The conclusion that one draws when the  $p$ -values are very low is that *the two groups are different to each other* than expected had we randomly partitioned the pooled dataset, i.e., the labels of the data matters.

### 3.2 Permutation Tests for Mid-Ratio

Because we agree with the remark of the authors that it is easy to tweak the data to get a desirable triplet, we decide to set the difference in standard deviation of mid-ratios of two datasets. The choice of standard deviation as the first statistic in place of mean makes sense because uniformity as well as convenient tweaking will lead to the same expectation of 0.5; and we expect standard deviation to capture the *unintentional reduction in spread caused in data due to intentional adjustments*.

We consider each researcher equivalent to a treatment. That is, for a given researcher, eg. A with dataset  $D_A$  with size  $n_A$ , we look at test statistic computed for a random partition of the entire data (size  $N$ ) into two groups  $n_A$  and  $N - n_A$  and compute the test statistic. We repeat this experiment 1000 times to plot the empirical distribution and then compute the  $p$ -values. We obtained 0  $p$ -value for A, B, D, and RTS; and  $< 0.01$   $p$ -value for all others except E,F,G which indicates that almost all datasets are surprising with respect to this test-statistic. We would like to note that here 0  $p$ -value means that there is less than 1 in 1000 chance of observing the event, because of finite resolution owing to 1000 tests. We would also like to mention that RTS is still the most surprising if one looks at the location of the test-statistic in the tails of the distribution.

Next we look at  $\ell_1$  distance between the density, followed by  $\ell_1$  distance between the CDF of two samples for each researcher, and obtain very similar results as in the previous case, that is several researchers will be rejected by the test at significance level of even 1%. We present all these  $p$ -values in Figure 4.

#### 3.2.1 Limitations of Permutation Test

A concern in such a test is the effect of the huge fraction of the data contributed by RTS. The  $p$ -values indicate the chance of the difference between the two groups - treatment and control, so a low  $p$ -value means that the treatment group is likely to be different than the control group. And here the control group has a

Test Stat ->		Std Dev	Density	CDF
Name	No.			
A	254	0.0000	0.0000	0.0000
B	58	0.0000	0.0060	0.0020
C	88	0.0080	0.0250	0.0070
D	80	0.0000	0.0110	0.0080
E	10	0.8940	0.1950	0.2640
F	29	0.0220	0.2620	0.1220
G	10	0.0190	0.4220	0.3200
H	21	0.0030	0.0250	0.0230
I	45	0.0080	0.0410	0.0900
RTS	1360	0.0000	0.0000	0.0000

Figure 4: Results for Permutation Tests for Mid Ratios

dominant effect from the data provided by RTS, hence a heuristic conclusion is that the data of the other lab mates is very different than the data of RTS. To be more concrete about drawing conclusions about the surprises in data about other researchers, we exclude the data provided by RTS to run the permutation tests. We will like to note that this has a bias because we ignore almost 2/3rd of the data, but doing so does give some answers that we were expecting before running these tests, which were consistent with the authors' expectations.

Test Stat ->		Std Dev	Density	CDF
Name	No.			
A	254	0.7450	0.7760	0.7350
B	58	0.5210	0.4790	0.5150
C	88	0.0450	0.0490	0.0560
D	80	0.6790	0.7090	0.6890
E	10	0.1290	0.1140	0.1250
F	29	0.9790	0.9780	0.9770
G	10	0.3130	0.2900	0.3590
H	21	0.2920	0.2860	0.3020
I	45	0.5430	0.5300	0.5750

Figure 5: Results for Permutation Tests without RTS for Mid Ratios

Owing to the high  $p$ -values, now we may say that the data provided by each individual researchers looks like a random partitioning when compared to the entire data pooled together excluding RTS, which gives some statistical evidence to RTS being the odd one out.

### 3.3 Additional Tests for Digit Analysis

For the terminal digit and equal digits tests, we extended the tests provided by the authors by considering the individual contribution of the single members of the lab and performing

- chi-square test for goodness of fit for each of the lab members and outside labs for terminal digit analysis,
- chi-square test for goodness of fit for each of the lab members and outside labs for equal digits analysis and,
- permutation tests for terminal digit analysis considering RTS and the other investigators.

### 3.3.1 Chi-square test Tests for Terminal Digit Analysis

To understand how single investigators contributions are distributed with respect to RTS and the outside labs, we decided to analyze data from all the other investigators taken one by one. To do so, we performed the chi-square test for goodness of fit for each of them. The following tables summarized our results:

Coulter Data			Colony Data		
Name	No.	P-val	Name	No.	P-val
A	1339	0.5123	A	779	0.6263
B	180	0.7510	B	174	0.1309
C	95	0.0742	C	271	0.8407
D	640	0.0094	D	250	0.4866
E	165	0.3870	E	30	0.8043
F	310	0.6405	F	90	0.8043
G	60	0.8043	G	30	0.4071
I	153	0.3781	H	63	0.0865

Figure 6: Chi Square Tests for Terminal Digits in Coulter and Colony Counts

Reading the tables, one can notice that  $p$  value for D, for Coulter Data is  $< 1\%$ .

### 3.3.2 Chi-square test Tests for Equal Digits Analysis

Also for the Equal Digits Analysis we performed the chi-square test for goodness of fit using the data of the individual investigators in the lab, in a similar fashion as before.

Coulter Counts:					
Name	Eq. digits	No.	Ratio	Chi-square	P
A	132	1318	0.1002	0.0003	0.9853
B	16	180	0.0889	0.2469	0.6193
C	8	95	0.0842	0.2632	0.6080
D	62	638	0.0972	0.0564	0.8122
E	13	134	0.0970	0.0133	0.9083
F	40	309	0.1294	2.9777	0.0844
G	4	60	0.0667	0.7407	0.3894
I	11	153	0.0719	1.3428	0.2465
Colony Counts:					
Name	Eq. digits	No.	Ratio	Chi-square	P
A	28	263	0.1065	0.1221	0.7268
B	4	48	0.0833	0.1481	0.7003
C	1	28	0.0357	1.2857	0.2568
D	7	41	0.1707	2.2791	0.1311
E	1	16	0.0625	0.2500	0.6171
F	2	31	0.0645	0.4337	0.5102
H	4	33	0.1212	0.1650	0.6846
I	6	47	0.1277	0.3995	0.5273

Figure 7: Chi Square Tests for Equal Terminal Pair in Coulter and Colony Counts

Here none of the  $p$ -values look abnormally low. One can argue that for A it is very high, but going by the practice of deciding thresholds before seeing the results none of the results are surprising.

### 3.3.3 Permutation Test for Terminal Digit Analysis

The following tables illustrate the permutation test results using the same test statistics as for mid-ratios:



Coulter Counts				
Test Stat ->		Density	CDF	Std Dev
Name	No.			
A	1215	0.3270	0.0000	0.1110
B	180	0.5250	0.4260	0.7680
C	75	0.0000	0.0440	0.1120
D	633	0.6040	0.0000	0.0220
E	165	0.3220	0.5190	0.6680
F	306	0.1680	0.0110	0.1700
G	60	0.2120	0.5010	0.8030
I	153	0.1250	0.0170	0.1090
RTS	5185	0.0000	0.0000	0.0000

Colony Counts				
Test Stat ->		Density	CDF	Std Dev
Name	No.			
A	765	0.0220	0.0010	0.1420
B	174	0.2890	0.0260	0.2320
C	267	0.0000	0.0520	0.1560
D	240	0.1610	0.6780	0.5150
E	30	0.1750	0.6770	0.7180
F	87	0.0550	0.3690	0.6170
G	30	0.1120	0.1360	0.3400
H	63	0.0480	0.0190	0.3240
RTS	4085	0.0000	0.0000	0.0330

Figure 8: Permutation Tests for Terminal Digit Analysis, Coulter counts

In all the above cases, it is possible to see how RTS data is consistently suspicious, which is a confirmation of the authors' suspects. And as pointed before, the huge fraction of data contributed by RTS contributes towards the low  $p$ -values for other individual researchers as well. We tried permutation tests after excluding RTS data and get better  $p$ -values as before, for brevity we do not mention the values here.

## 4 Conclusion

Data fraud is an extremely critical issue in science, engineering and many other fields. Methods to detect manipulated data are needed to identify fraudulent research behaviors. Detecting frauds, however, is a delicate matter. Challenging the credibility of a researcher or of a scientific work, in fact, can have heavy consequences for all the parties involved in the process. Methodologies and techniques used in this kind of work need to be clear and widely accepted, and they need to produce results which leave minimal (ideally no) space to ambiguity. Independently, reproducibility of results is a fundamental element to rule out any doubts that could arise at any time.

In our review, we carefully analyzed the authors' results and conclusions by: reproducing all the results that have been discussed in the paper and proposing and implementing additional tests to clarify doubts and suggesting additional possibilities to the authors.

We found out that authors' results are correct, although it has not been possible to reproduce exactly all the experiments due to lack of some key pieces of information (for instance how data has been pre-processed). Moreover, we encourage the use of stronger tools like permutation tests and our demonstration can be considered as a promotion of the same. Such tests help the analysis to get *rid of assumptions*, thereby shifting the focus from debate on assumptions to actual anomalies present and to better understanding of individual investigator's data (besides the RTS) as to how do they compare to the general data pool.

At the end of our review, we do believe that there is a significant evidence that RTS has suspicious data, but we suggest the authors to collect additional material and investigate more, since some of our tests

suggest that other investigator’s data have anomalies as well if we do not discount the huge fraction of data given by RTS.

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