The Computation

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$$X(\theta)=\frac{1}{\sqrt{1+\left(\frac{r'}{r}\right)^2}}\begin{bmatrix}-r'\\ r\end{bmatrix}+\int_0^\theta r\sqrt{1+\left(\frac{r'}{r}\right)^2}d\theta \tag{1}$$

$$-\frac{\frac{r'}{r}\frac{r''r-(r')^2}{r^2}}{(1+\left(\frac{r'}{r}\right)^2)^{\frac{3}{2}}}r+\frac{r'}{\sqrt{1+\left(\frac{r'}{r}\right)^2}}=\alpha\left(\frac{r'^2(r''r-(r')^2)}{r^3(1+\left(\frac{r'}{r}\right)^2)^{\frac{3}{2}}}-\frac{r''}{\sqrt{1+\left(\frac{r'}{r}\right)^2}}+r\sqrt{1+\left(\frac{r'}{r}\right)^2}\right) \tag{2}$$

$$-\frac{r'\frac{r''r-(r')^2}{r^2}}{1+\left(\frac{r'}{r}\right)^2}+r'=\alpha\left(\frac{r'^2(r''r-(r')^2)}{r^3(1+\left(\frac{r'}{r}\right)^2)}-r''+r(1+\left(\frac{r'}{r}\right)^2)\right) \tag{3}$$

$$-\frac{r''r'r-r'^3}{r^2+r'^2}+r'=\alpha\left(\frac{r'^2(r''r-r'^2)}{r(r^2+r'^2)}-r''+(r^2+r'^2)1/r\right) \tag{4}$$

$$-r''r'r^2+rr'^3+r'(r^2+r'^2)=\alpha\left(r'^2(r''r-r'^2)-r''r(r^2+r'^2)+(r^2+r'^2)^2\right) \tag{5}$$

$$r''\left(\alpha r^3-r'r^2\right)+rr'^3+r'(r^2+r'^2)=\alpha\left(r'^2(-r'^2)+(r^2+r'^2)^2\right) \tag{6}$$

$$r''\left(\alpha r^3-r'r^2\right)+rr'^3+r'r^3+rr'^3=\alpha\left(r^4+2r^2r'^2\right) \tag{7}$$

$$r''=\frac{\alpha\left(r^4+2r^2r'^2\right)-r'r^3-2rr'^3}{\alpha r^3-r'r^2} \tag{8}$$

For numerical solving, we use the system:

$$r'=o \tag{9}$$

$$o'=\frac{\alpha\left(r^4+2r^2o^2\right)-or^3-2ro^3}{\alpha r^3-or^2} \tag{10}$$

The equation becomes: