The Computation

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$$X(\theta) = \frac{1}{\sqrt{1+\left(\frac{r'}{r}\right)^2}} \begin{bmatrix} -r' \\ r \end{bmatrix} + \int_0^\theta r\sqrt{1+\left(\frac{r'}{r}\right)^2} d\theta \tag{1}$$

$$\frac{dx}{d\theta} = \frac{r'^2(r''r - r'^2)}{(R)^3} - \frac{r''r}{R} + R \tag{2}$$

$$\frac{dy}{d\theta} = -\frac{r'r(r''r - r'^2)}{(R)^3} - \frac{r'r}{R} \tag{3}$$

$$\alpha\left(r'^2(r''r - r'^2) - r''rR^2 + R^4\right) + r'r(r''r - r'^2) + r'rR^2 = 0 \tag{4}$$

$$r''(\alpha r'^2 r - \alpha r R^2 + r'r^2) + \alpha(-r'^4 + R^4) - r'^3 r + r'rR^2 = 0 \tag{5}$$

$$-\frac{\frac{r'}{r}\frac{r''r-(r')^2}{r^2}}{(1+\left(\frac{r'}{r}\right)^2)^{\frac{3}{2}}}r + \frac{r'}{\sqrt{1+\left(\frac{r'}{r}\right)^2}} = \alpha\left(\frac{r'^2(r''r-(r')^2)}{r^3(1+\left(\frac{r'}{r}\right)^2)^{\frac{3}{2}}} - \frac{r''}{\sqrt{1+\left(\frac{r'}{r}\right)^2}} + r\sqrt{1+\left(\frac{r'}{r}\right)^2}\right) \tag{6}$$

$$-\frac{r'\frac{r''r-(r')^2}{r^2}}{1+\left(\frac{r'}{r}\right)^2} + r' = \alpha\left(\frac{r'^2(r''r-(r')^2)}{r^3(1+\left(\frac{r'}{r}\right)^2)} - r'' + r(1+\left(\frac{r'}{r}\right)^2)\right) \tag{7}$$

$$-\frac{r''r'r - r'^3}{r^2 + r'^2} + r' = \alpha\left(\frac{r'^2(r''r - r'^2)}{r(r^2+r'^2)} - r'' + (r^2 + r'^2)1/r\right) \tag{8}$$

$$-r''r'r^2 + rr'^3 + r'(r^2+r'^2) = \alpha\left(r'^2(r''r - r'^2) - r''r(r^2+r'^2) + (r^2+r'^2)^2\right) \tag{9}$$

$$r''\left(\alpha r^3 - r'r^2\right) + rr'^3 + r'(r^2+r'^2) = \alpha\left(r'^2(-r'^2) + (r^2+r'^2)^2\right) \tag{10}$$

$$r''\left(\alpha r^3 - r'r^2\right) + rr'^3 + r'r^3 + rr'^3 = \alpha\left(r^4 + 2r^2r'^2\right) \tag{11}$$

$$r'' = \frac{\alpha\left(r^4 + 2r^2r'^2\right) - r'r^3 - 2rr'^3}{\alpha r^3 - r'r^2} \tag{12}$$

For numerical solving, we use the system:

$$r' = o \tag{13}$$

$$o' = \frac{\alpha\left(r^4 + 2r^2o^2\right) - or^3 - 2ro^3}{\alpha r^3 - or^2} \tag{14}$$

The equation becomes: