Чернов Иван БФЗ-19-1 ДЗ2

Прошу прощения за грязное оформление. Зато так вы сможете понять, что всю эту работу я провел сам.

№1 п 1

$$S = \begin{pmatrix} \cos\theta & \sin\theta\, e^{-i\varphi} \\ \sin\theta\, e^{i\varphi} & -\cos\theta \end{pmatrix}$$

$$|S - E\lambda| = \begin{vmatrix} \cos\theta - \lambda & \sin\theta\, e^{-i\varphi} \\ \sin\theta\, e^{i\varphi} & -\cos\theta - \lambda \end{vmatrix} =$$

$$= \begin{vmatrix} \cos\theta - \lambda & \sin\theta\cos(-\varphi) - i\sin\theta\cos\varphi \\ \sin\theta\cos(\varphi) + i\sin\theta\cos\varphi & -(\cos\theta + \lambda) \end{vmatrix} =$$

$$= -(\cos^2\theta - \lambda^2) - (\sin^2\theta\cos^2\varphi + i^2\sin^2\theta\cos^2\varphi) =$$

$$= -\cos^2\theta + \lambda^2 - \cos^2\theta(\sin^2\theta + \sin^2\theta) =$$

$$= -\cos^2\theta + \lambda^2 - 2\cos^2\varphi\sin^2\theta =$$

$$= \cancel{\lambda^2 - \cos^2\theta(1 - 2\sin}$$

$$-(\cos^2\theta - \lambda^2) - \sin^2\theta \frac{e^{i\varphi}}{e^{i\varphi}} =$$

$$= \lambda^2 - \cos^2\theta - \sin^2\theta = \lambda^2 - 1 \to \lambda = \pm 1$$

$$\begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

$$\begin{cases} (\cos\theta - 1)\psi_1 + \sin\theta\, e^{-i\varphi}\psi_2 = 0 \\ \psi_1 \sin\theta\, e^{i\varphi} - (\cos\theta + 1)\psi_2 = 0 \end{cases}$$

$$\cancel{\frac{(\cos\theta - 1)}{\sin\theta\, e^{i\varphi}} = \sin}$$

$$\psi_1 = -\frac{\sin\theta\, e^{-i\varphi}}{\cos\theta - 1}\psi_2$$

$$\psi_1 = -a\sin\theta\, e^{-i\varphi} = a(\sin\theta\cos\varphi - i\sin\theta\sin\varphi)$$

$$\psi_2 = a\cos\theta - 1$$

$$a^{-1} = \sqrt{\sin^2\theta + (\cos^2\theta - 1)^2} =$$

$$= \sqrt{\sin^2\theta + \cos^2\theta - 2\cos\theta + 1} = \sqrt{2 - 2\cos\theta}$$

$$|u_1\rangle = \begin{pmatrix} -\sin\theta\, e^{-i\varphi}/\sqrt{2 - 2\cos\theta} \\ (\cos\theta - 1)/\sqrt{2 - 2\cos\theta} \end{pmatrix}$$

$$\lambda = -1$$

$$(\cos\theta + 1)\,\psi_1 + \sin\theta\, e^{-i\varphi}\,\psi_2 = 0$$

$$\psi_1 \sin\theta\, e^{-i\varphi} - (\cos\theta + 1)\,\psi_2 = 0$$

$$\psi_1 = -\frac{\sin\theta\, e^{-i\varphi}}{\cos\theta + 1}\,\psi_2$$

$$\psi_1 = -a \sin\theta\, e^{-i\varphi}$$

$$\psi_2 = a(\cos\theta + 1)$$

$$a^{-1} = \sqrt{\sin^2\theta + \cos^2\theta + 2\cos\theta + 1} =$$

$$= \sqrt{2 + 2\cos\theta}$$

$$|u_2\rangle = \begin{pmatrix} -\sin\theta\, e^{-i\varphi} / \sqrt{2 + 2\cos\theta} \\ (\cos\theta + 1) / \sqrt{2 + 2\cos\theta} \end{pmatrix}$$

$$|\varphi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\langle u_1 | u_2 \rangle = (-\sin\theta\, e^{i\varphi})(-\sin\theta\, e^{+i\varphi}) +$$

$$+ \cos^2\theta + 1^2 = \sin^2\theta + \cos^2\theta - 1 = 0$$

№1 п 2

$$|\psi\rangle = \alpha_1|1\rangle + \alpha_{-1}|-1\rangle$$

$$P_1 = |\alpha_1|^2 \qquad P_{-1} = |\alpha_{-1}|^2$$

$$\alpha_1 = \langle 1, |\psi\rangle = \frac{1}{\sqrt{2-2\cos\theta}}\frac{1}{\sqrt{2}}\begin{pmatrix} -\sin\theta e^{-i\varphi} \\ \cos\theta - 1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} =$$

$$= \frac{1}{2\sqrt{1-\cos\theta}} \cdot (-\sin\theta e^{-i\varphi} + \cos\theta - 1)$$

$$\frac{\cos\theta - 1 - \sin\theta e^{-i\varphi}}{2\sqrt{1-\cos\theta}}$$

$$P_1 = |\alpha_1|^2 = \left(\frac{\sqrt{(\cos^2\theta - 1)^2 + \sin^2\theta}}{2\sqrt{1-\cos\theta}}\right)^2 =$$

$$= \left(\frac{\sqrt{\cos^2\theta - 2\cos\theta + 1 + \sin^2\theta}}{2\sqrt{1-\cos\theta}}\right)^2 =$$

$$= \left(\frac{\sqrt{2-2\cos\theta}}{2\sqrt{1-\cos\theta}}\right) = \left(\frac{\sqrt{2}}{2}\right)^2 = \frac{1}{2}$$

$$P_{-1} = |\alpha_{-1}|^2 = \left(\frac{\sqrt{\sin^2\theta + (\cos\theta + 1)^2}}{2\sqrt{1+\cos\theta}}\right)^2 =$$

$$= \left(\frac{\sqrt{2+2\cos\theta}}{2\sqrt{1+\cos\theta}}\right)^2 = \left(\frac{\sqrt{2}}{2}\right)^2 = \frac{1}{2}$$

№2 п 1

$U(x,y) = \frac{m\omega^2}{2}(x^2+y^2)$

$T = \frac{m(\dot{x}^2+\dot{y}^2)}{2}$ $\qquad L = T - U = \frac{m}{2}\dot{x}^2 + \frac{m}{2}\dot{y}^2 - \frac{m\omega^2}{2}(x^2+y^2)$

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ур-ие движения $\qquad \frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = 0$

$\frac{\partial L}{\partial \dot{x}} = \frac{2m\dot{x}}{2} = m\dot{x}$ $\qquad \frac{\partial L}{\partial x} = -2x\frac{m\omega^2}{2} = -xm\omega^2$

$\frac{\partial L}{\partial \dot{y}} = m\dot{y}$ $\qquad \frac{\partial L}{\partial y} = -y m\omega^2$

$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}} = m\ddot{x}$

$\frac{d}{dt}\frac{\partial L}{\partial \dot{y}} = m\ddot{y}$

$\lambda^2 - \omega^2 = 0$

$\lambda = \pm\omega$

$\cancel{-m\omega^2 x = m\ddot{x}}$

$$\begin{cases} m\ddot{x} + xm\omega^2 = 0 & 1) \\ m\ddot{y} + ym\omega^2 = 0 & 2) \end{cases}$$

1) $x_{он} = C_1\cos\omega t + C_2\sin\omega t$

$\dot{x}_{он} = -\omega C_1\sin\omega t + \omega C_2\cos\omega t$

$\dot{x}_{он}(0) = 0$ т.е. $-\omega C_1 \underbrace{\sin(\omega\cdot 0)}_{0} + \omega C_2 \underbrace{\cos(\omega\cdot 0)}_{1} \Rightarrow$

$x = C_1\cos\omega t$ $\qquad \Rightarrow C_2 = 0$

$x(0) = a \qquad a = C_1 \underbrace{\cos(\omega 0)}_{1} \Rightarrow C_1 = a$

$x = a\cos(\omega t)$

аналогично с y:

$y_{он} = C_1\cos\omega t + C_2\sin\omega t \qquad y(0) = 0 \Rightarrow C_1 =$

$\dot{y}_{он} = -\omega C_1\sin\omega t + \omega C_2\cos(\omega t)$

$\dot{y}_{он}(0) = v_0 \Rightarrow \cancel{C_2 = \omega v_0} \quad \omega C_2 = v_0 \Rightarrow$

$$C_2 = v_0/\omega$$

$$y = v_0/\omega \cdot \sin(\omega t)$$

$$y = \frac{v_0}{\omega} \sin(\omega t)$$

$$x = a \cos(\omega t)$$

$$\sin^2 + \cos^2 = 1$$

$$\sin(\omega t) = \frac{y\omega}{v_0}$$

$$\cos(\omega t) = \frac{x}{a}$$

$$\left(\frac{y\omega}{v_0}\right)^2 + \left(\frac{x}{a}\right)^2 = 1$$ – траектория движения

№2 п 2

№2.2

$$S_0 = \int (p_x\,dx + p_y\,dy)$$

$$L = \frac{m\dot{x}^2}{2} + \frac{m\dot{y}^2}{2} - \frac{m\omega^2}{2}(x^2+y^2)$$

$$p_x = m\dot{x} \qquad p_y = m\dot{y}$$

$$H = p_x\dot{x} + p_y\dot{y} - L = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{m\omega^2}{2}(x^2+y^2)$$

$$H = E = \frac{m}{2}\frac{dx^2+dy^2}{dt^2} + \frac{m\omega^2}{2}(x^2+y^2)$$

$$dt = \sqrt{\frac{m}{2}}\sqrt{\frac{dx^2+dy^2}{E - \frac{m\omega^2}{2}}}$$

$$p_x = \frac{dx}{dt} = dx \cdot \sqrt{\frac{2}{m}}\sqrt{\frac{E - \frac{m\omega^2}{2}}{dx^2+dy^2}}$$

$$p_y = dy \sqrt{\frac{2}{m} \cdot \frac{E - \frac{m\omega^2}{2}}{dx^2+dy^2}}$$

$$S_0 = \int \sqrt{2m}\sqrt{E - \frac{m\omega^2}{2}(x^2+y^2)}\sqrt{dx^2+dy^2} =$$

$$= \int \sqrt{2m}\sqrt{E - \frac{m\omega^2}{2}(x^2+y^2)}\sqrt{1+y'^2}\,dx$$

$$S_0 = \sqrt{2m} \int \sqrt{E - \frac{m\omega^2}{2}(x^2+y^2)}\,\sqrt{dx^2+dy^2}$$

$$S_0 = \sqrt{2m} \int \sqrt{E - a(x^2+y^2)}\,\sqrt{1+\dot{y}^2}\,dx$$

$L(y, \dot{y}, x)$ где x - время, y - коорд, $\dot{y}$ - скорость

$$L = \sqrt{2m}\sqrt{E - a(x^2+y^2)}\,\sqrt{1+\dot{y}^2}$$

$$\frac{\partial L}{\partial y} = \sqrt{1+\dot{y}^2}\,\frac{-a\cancel{2}y\,\sqrt{2m}}{\cancel{2}\sqrt{E-a(x^2+y^2)}}$$

$$\frac{\partial L}{\partial \dot{y}} = \sqrt{E-a(x^2+y^2)}\cdot\frac{\sqrt{2m}\,\dot{y}}{\sqrt{1+\dot{y}^2}}$$

$$\underbrace{\sqrt{1+\dot{y}^2}}_{B_y}\,\frac{-ay\sqrt{2m}}{\underbrace{\sqrt{E-a(x^2+y^2)}}_{A_{xy}}} = \frac{d}{dx}\frac{\overbrace{\sqrt{E-a(x^2+y^2)}}^{A_{xy}\cdot\sqrt{2m}}\,\dot{y}}{\underbrace{\sqrt{1+\dot{y}^2}}_{B_y}}$$

$$-ay\sqrt{2m}\,\frac{B_y}{A_{xy}} = \frac{d}{dx}\,\frac{A_{xy}}{B_y}\,\dot{y}$$

$$\frac{A_{xy}}{B_y}\cdot\dot{y} = \Phi \qquad \sqrt{2m}\sqrt{E \ldots (x^2+y^2)}\,\sqrt{1+\dot{y}^2}$$

$$-ay\dot{y}\sqrt{2m}\,\frac{1}{\Phi} = \frac{d\Phi}{dx}$$

$$-a\sqrt{2m}\,y\dot{y} = \Phi\Phi'\cdot\sqrt{2m}$$

$y($

$$\frac{m\omega^2}{2}\,y\dot{y} = \Phi\Phi'$$

$$(-m\omega^2)\,y\dot{y} = 2\Phi\Phi'$$

$$b\,y\dot{y} = 2\Phi\Phi'$$

$$-b\, y(x)\, y'(x) = 2\, \Phi(x)\, \Phi'(x)$$

$$-b\, y \frac{dy}{dx} = 2\, \Phi \frac{d\Phi}{dx}$$

$$-b\, y\, dy\, dx = 2\, \Phi\, d\Phi\, dx$$

$$-b \int y\, dy = 2 \int \Phi\, d\Phi$$

$$-b \frac{y^2}{2} = \cancel{2} \frac{\Phi^2}{\cancel{2}}$$

$$-m\omega^2 y^2 = \left(\frac{A_{xy}}{B_y}\right)^2 y' + C$$

$$-m\omega^2 y^2 =$$

$$-b y^2 = c y' + \tilde{C}$$

$$c v_0 + \tilde{C} = 0$$

$$\tilde{C} = -c v_0$$

$$-\frac{m\omega^2}{2} \frac{y^2}{2} = \left(\frac{\sqrt{E - \frac{m\omega^2}{2}(x^2 + y^2)}}{\sqrt{1 + y'^2}}\right)^2 y' + \tilde{C}$$

$$-\frac{m\omega^2}{2} y^2 = \frac{2E - m\omega^2 (x^2 + y^2)}{1 + y'^2} y' + \tilde{C}$$

при сатлении (0,0)

$$0 = \frac{2E - m\omega^2 a^2}{1 + v_0^2} v_0^2 + \tilde{C}$$

$$\tilde{C} = -\frac{2E - m\omega^2 a^2}{1 + v_0^2} v_0^2$$

$$\bar{b} \qquad \bar{a}$$

$$-m\omega^2 y^2 = \left(\frac{2E - m\omega^2 (x^2 + y^2)}{\cancel{1+y^2}}\right) \frac{y'^2}{1 + y'^2} + \bar{C}$$

$$-\bar{b} = \frac{\bar{a}\, y'^2}{1 + y'^2} + \bar{C}$$

$$-\bar{b} - \bar{b} y'^2 = \bar{a} y'^2 + \bar{C} + \bar{C} y'^2$$

$$(\bar{a} + \bar{c} + \bar{b}) y'^2 + \bar{c} + \bar{b} = 0$$

$$y'^2 = \sqrt{\frac{-\bar{c} - \bar{b}}{\bar{a} + \bar{c} + \bar{b}}}$$

$$\sqrt{\frac{-\tilde{c} + m\omega^2 y^2}{2E - m\omega^2(x^2 + y^2) + \tilde{c} + m\omega^2 y^2}} =$$

$$\cancel{\left(\frac{2E - m\omega^2 a^2}{1 - v_0^2} v_0^2\right)^2}$$

$$= \sqrt{\frac{-\tilde{c} - m\omega^2 y^2}{2E - m\omega^2 x^2 - \cancel{m\omega^2 y^2} + \cancel{m\omega^2 y^2} + \tilde{c}}} =$$

$$y'^2 = \frac{-\tilde{c} - m\omega^2 y^2}{2E - m\omega^2 x^2 + \tilde{c}}$$

$$y'^2 \quad \left(\frac{dy}{dx}\right)^2 = \ldots$$

$$\frac{dy}{dx} = \sqrt{\frac{-\tilde{c} - m\omega^2 y^2}{2E - m\omega^2 x^2 + \tilde{c}}} =$$

$$y = \int \frac{\gamma}{\sqrt{\frac{2E+\tilde{C}}{m\omega^2} - x^2}}\, dx \cdot \frac{1}{\sqrt{m\omega^2}} =$$

$$y = \frac{\cancel{A}\,\gamma}{\sqrt{m\omega^2}} \arcsin\left(\frac{x\sqrt{m\omega^2}}{\sqrt{2E+\tilde{C}}}\right) + C$$

$$y = \frac{\sqrt{-\tilde{C} - m\omega^2 y^2}}{\sqrt{m\omega^2}} \arcsin\left(\frac{x\sqrt{m\omega^2}}{\sqrt{2E+\tilde{C}}}\right) + C$$

$$C = -\sqrt{\frac{-\tilde{C} - m\omega^2 g^2}{m\omega^2}} \arcsin \frac{a\sqrt{m\omega^2}}{\sqrt{2E+\tilde{C}}}$$

$$C = -\sqrt{\frac{-\tilde{C}}{\sqrt{m\omega^2}}} \arcsin \frac{a\sqrt{m\omega^2}}{\sqrt{2E+\tilde{C}}}$$

$$y = \frac{\sqrt{-\tilde{C} - m\omega^2 y^2}}{\sqrt{m\omega^2}} \arcsin\left(\frac{x\sqrt{m\omega^2}}{\sqrt{2E+\tilde{C}}}\right) -$$

$$- \frac{-\tilde{C}}{\sqrt{m\omega^2}} \arcsin\left(\frac{a\sqrt{m\omega^2}}{\sqrt{2E+\tilde{C}}}\right)$$

где $\tilde{C} = -\frac{2E - m\omega^2 a^2}{1+v_0^2} v_0^2$

№4 п 1

№ 4. $\psi(x) = const\ x^2 e^{-x/\lambda}$ $x \geq 0$

$$\int_0^{+\infty} |C x^2 e^{-x/\lambda}|^2 dx = \frac{3a^5C^2}{4} = 1$$

$$C = \sqrt{\frac{4}{3}\frac{1}{a^5}}$$

$$\int_0^{+\infty} x^4 e^{-2x/\lambda} \qquad 2/\lambda = a$$

$$\int_0 x^4 e^{-ax} dx = \left| \begin{matrix} x^4 = u & du = 4x^3 \\ e^{-ax} dv & v = \frac{e^{-ax}}{-a} \end{matrix} \right| =$$

$$= \frac{x^4 e^{-ax}}{-a}\Big|_0^{+\infty} + \frac{4}{a}\int_0^{+\infty} x^3 e^{-ax} dx = \left| \begin{matrix} x^3 = u & du = 3x^2 \\ dv = e^{-ax} & v = \frac{e^{-ax}}{-a} \end{matrix} \right|$$

$$= \frac{x^3 e^{-ax}}{-a}\Big|_0^{+\infty} + \frac{3}{a}\int x^2 e^{-ax} dx = \left| \begin{matrix} u = x^2 & du = 2x \\ dv = e^{-ax} & v = \frac{e^{-ax}}{-a} \end{matrix} \right|$$

$$= \frac{x^2 e^{-ax}}{-a}\Big|_0^{\infty} + \frac{2}{a}\int_0^{\infty} x e^{-ax} dx = \left| \begin{matrix} u = x & du = dx \\ dv = e^{-ax} & v = \frac{e^{-ax}}{-a} \end{matrix} \right|$$

$$= \frac{x e^{-ax}}{-a}\Big|_0^{+\infty} + \frac{1}{a}\underbrace{\int_0^{+\infty} e^{-ax} dx}_{= \frac{e^{-ax}}{-a}}$$

$$\frac{x^4 e^{-ax}}{-a} + \frac{4}{a}\left(\frac{x^3 e^{-ax}}{-a} + \frac{3}{a}\left(\frac{x^2 e^{-ax}}{-a} + \frac{2}{a}\left(\frac{x e^{-ax}}{-a} - \frac{1 e^{-ax}}{a^2}\right)\right)\right)\Bigg|_0^{+\infty}$$

$$-\frac{x^4 e^{-ax}}{a} - \frac{4x^3 e^{-ax}}{a^2} - \frac{12x^2 e^{-ax}}{a^3} - \frac{24 x e^{-ax}}{a^4} - \frac{12 e^{-ax}}{a^5} =$$

$\times 6$ $\qquad a = \frac{2}{\lambda}$

$$= C^2\left(-\frac{\lambda e^{-2x/\lambda} x^4}{2} - \lambda^2 x^3 e^{-2x/\lambda} - \right.$$

$$\left. - \frac{3\lambda^3 x^2 e^{-2x/\lambda}}{2} - \frac{3\lambda^4 x e^{-2x/\lambda}}{2} - \frac{3\lambda^5 e^{-2x/\lambda}}{4}\right)\Bigg|_0^{+\infty}$$

$$\cancel{= -C^2 (2\lambda e^{-2x/\lambda}}$$

$$= -C^2 e^{-2x/\lambda} \frac{(\lambda 2x^4 + 4\lambda^2 x^3 + 6\lambda^3 x^2 + 6\lambda^4 x + 3\lambda^5)}{4}$$

$$= -C^2 e^{-2x/\lambda} \frac{\lambda(2x^4 + 4\lambda x^3 + 6\lambda^2 x^2 + 6\lambda^3 x + 3\lambda^4)}{4}\Bigg|_0^{+\infty}$$

$$= -C \frac{(ax^4 + bx^3 + cx^2 + dx + e)}{e^x}$$

при $x \to \infty$ e^x рост гораздо быстрее чем у многочлена 4 степени $\Rightarrow$

$\Rightarrow = 0$

тогда $\int_0^{+\infty} \left|C x^2 e^{-x/\lambda}\right|^2 dx = \frac{3C^2}{4}\lambda^5 = 1 \Rightarrow$

$\Rightarrow C = \sqrt{\frac{4}{3}\frac{1}{\lambda^5}}$ Ответ.

$$\psi(x) = \sqrt{\frac{4}{3\lambda^5}}\, x^2 e^{-x/\lambda}$$

№4 п 2-3

2) $\rho = |\psi(x)|^2 dx = \frac{4}{3\lambda^5} x^4 e^{-2x/\lambda} dx$

3) $\langle x \rangle = \int \left(\frac{4}{3\lambda^5} x^4 e^{-2x/\lambda}\right) \cdot x \, dx =$

$$= \frac{4}{3} \frac{1}{\lambda^5} \int_0^{+\infty} x^5 e^{-2x/\lambda} dx = \dots ($$

$$\frac{4}{3} \frac{1}{\lambda^5} \cdot \left(-\frac{x^5 \cancel{(-2)}}{\cancel{\lambda}\, 2} \lambda \Big|_0^{+\infty} + \frac{\lambda 5}{2} \int_0^{+\infty} x^4 e^{-2x/\lambda} dx = \right.$$

$$= \frac{4}{3} \frac{1}{\lambda^5} \left(\underbrace{\left(-\frac{x^5}{2} \lambda e^{-2x/\lambda} \right) \Big|_0^{+\infty}}_{\to 0} + \frac{15}{8} \lambda^6 \right) =$$

$$= \frac{60}{24} \lambda = \frac{5}{2} \lambda$$