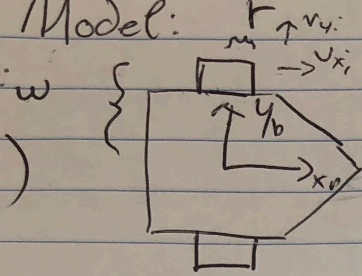
Source: Modern Robotics: Mechanics, Planning and Control, Kevin Lynch and Frank Park

D. 3 - Documentation

Forward kinematics:

Model:



(1)

We know (2) $\begin{bmatrix} v_{x_i} \\ v_{y_i} \end{bmatrix} = \begin{bmatrix} r u_i \\ 0 \end{bmatrix}$

The adjoints are at the origin of the body frame

$$q_L = \begin{bmatrix} -D & & 0 \\ & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot\theta \\ v_x \\ v_y \end{bmatrix} \quad (3) \text{ left wheel}$$

$$q_R = \begin{bmatrix} D & 0 & 0 \\ & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot\theta \\ v_x \\ v_y \end{bmatrix} \quad (4) \text{ right wheel}$$

This gives

(5) $\dot\theta_L = \dot\theta$ (6) $\dot\theta_R = \dot\theta$

(7) $r u_L = -D\dot\theta + v_x$ (8) $r u_R = D\dot\theta + v_x$

(9) $r(v_L + v_R) = 2 v_x$

Thus:

$$(10) \begin{cases} \dot\theta = -\dfrac{r}{2w} u_L + \dfrac{r}{2w} u_R \\ \dot x = \dfrac{r}{2} v_L + \dfrac{r}{2} v_R \\ \dot y = 0 \end{cases}$$

We can create an updated configuration while moving at constant angular speed with the following, that can be integrated into a new transform:

$$\text{twist} = \begin{pmatrix} \Delta\theta \\ \Delta x \\ \Delta y \end{pmatrix} = \begin{matrix} -\frac{r}{2w} \Delta\theta_L + r/_{2w}\, \Delta\theta_R \\ r/_2\, \Delta\theta_L + r/_2 + \Delta\theta_R \\ 0 \end{matrix}$$

Inverse Kinematics :

$$(11)\quad \begin{cases} \dot{\Theta} = -\frac{r}{2w} v_L + \frac{r}{2w} v_R \\ \dot{x} = \frac{r}{2} v_L + \frac{r}{2} v_R \\ \dot{y} = 0 \end{cases}$$

$$(12)\quad \begin{cases} w\dot{\Theta} = -\frac{r}{\cancel{w}2} v_L + \frac{r}{2} v_R \\ \dot{x} = \frac{r}{2} v_L + \frac{r}{2} v_R \end{cases}$$

$$(13)\quad w\dot{\Theta} + \dot{x} = r v_R \quad ; \quad v_R = \frac{w\dot{\Theta} + \dot{x}}{r} \quad (14)$$

$$\downarrow$$

$$v_L = \frac{-w\Theta + \dot{x}}{r} \quad (15)$$