Fundamentals

Preview

As mentioned in the previous chapter, the power that MATLAB brings to digital image processing is an extensive set of functions for processing multidimensional arrays of which images (two-dimensional numerical arrays) are a special case. The Image Processing Toolbox is a collection of functions that extend the capability of the MATLAB numeric computing environment. These functions, and the expressiveness of the MATLAB language, make image-processing operations easy to write in a compact, clear manner, thus providing an ideal software prototyping environment for the solution of image processing problems. In this chapter we introduce the basics of MATLAB notation, discuss a number of fundamental toolbox properties and functions, and begin a discussion of programming concepts. Thus, the material in this chapter is the foundation for most of the software-related discussions in the remainder of the book.

2.1 Digital Image Representation

An image may be defined as a two-dimensional function f(x, y), where x and y are spatial (plane) coordinates, and the amplitude of f at any pair of coordinates is called the intensity of the image at that point. The term $gray\ level$ is used often to refer to the intensity of monochrome images. Color images are formed by a combination of individual images. For example, in the RGB color system a color image consists of three individual monochrome images, referred to as the $red\ (R)$, $green\ (G)$, and $blue\ (B)\ primary\ (or\ component)\ images$. For this reason, many of the techniques developed for monochrome images can be extended to color images by processing the three component images individually. Color image processing is the topic of Chapter 7. An image may be continuous

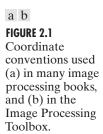
with respect to the *x*- and *y*-coordinates, and also in amplitude. Converting such an image to digital form requires that the coordinates, as well as the amplitude, be digitized. Digitizing the coordinate values is called *sampling*; digitizing the amplitude values is called *quantization*. Thus, when *x*, *y*, and the amplitude values of *f* are all finite, discrete quantities, we call the image a *digital image*.

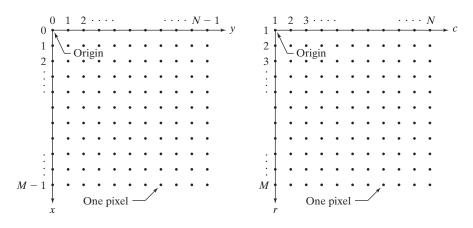
2.1.1 Coordinate Conventions

The result of sampling and quantization is a matrix of real numbers. We use two principal ways in this book to represent digital images. Assume that an image f(x, y) is sampled so that the resulting image has M rows and N columns. We say that the image is of $size\ M \times N$. The values of the coordinates are discrete quantities. For notational clarity and convenience, we use integer values for these discrete coordinates. In many image processing books, the image origin is defined to be at (x, y) = (0, 0). The next coordinate values along the first row of the image are (x, y) = (0, 1). The notation (0, 1) is used to signify the second sample along the first row. It *does not* mean that these are the *actual* values of physical coordinates when the image was sampled. Figure 2.1(a) shows this coordinate convention. Note that x ranges from 0 to M-1 and y from 0 to N-1 in integer increments.

The coordinate convention used in the Image Processing Toolbox to denote arrays is different from the preceding paragraph in two minor ways. First, instead of using (x, y), the toolbox uses the notation (r, c) to indicate rows and columns. Note, however, that the order of coordinates is the same as the order discussed in the previous paragraph, in the sense that the first element of a coordinate tuple, (a, b), refers to a row and the second to a column. The other difference is that the origin of the coordinate system is at (r, c) = (1, 1); thus, r ranges from 1 to M, and c from 1 to N, in integer increments. Figure 2.1(b) illustrates this coordinate convention.

Image Processing Toolbox documentation refers to the coordinates in Fig. 2.1(b) as *pixel coordinates*. Less frequently, the toolbox also employs another coordinate convention, called *spatial coordinates*, that uses *x* to refer to columns and *y* to refers to rows. This is the opposite of our use of variables *x* and *y*. With





a few exceptions, we do not use the toolbox's spatial coordinate convention in this book, but many MATLAB functions do, and you will definitely encounter it in toolbox and MATLAB documentation.

2.1.2 Images as Matrices

The coordinate system in Fig. 2.1(a) and the preceding discussion lead to the following representation for a digitized image:

$$f(x,y) = \begin{bmatrix} f(0,0) & f(0,1) & \cdots & f(0,N-1) \\ f(1,0) & f(1,1) & \cdots & f(1,N-1) \\ \vdots & \vdots & & \vdots \\ f(M-1,0) & f(M-1,1) & \cdots & f(M-1,N-1) \end{bmatrix}$$

The right side of this equation is a digital image by definition. Each element of this array is called an *image element*, *picture element*, *pixel*, or *pel*. The terms image and pixel are used throughout the rest of our discussions to denote a digital image and its elements.

A digital image can be represented as a MATLAB matrix:

$$f = \begin{bmatrix} f(1,1) & f(1,2) & \cdots & f(1,N) \\ f(2,1) & f(2,2) & \cdots & f(2,N) \\ \vdots & & \vdots & & \vdots \\ f(M,1) & f(M,2) & \cdots & f(M,N) \end{bmatrix}$$

MATLAB documentation uses the terms matrix and array interchangeably. However, keep in mind that a matrix is two dimensional, whereas an array can have any finite dimension.

where f(1, 1) = f(0,0) (note the use of a monospace font to denote MAT-LAB quantities). Clearly, the two representations are identical, except for the shift in origin. The notation f(p, q) denotes the element located in row p and column q. For example, f(6, 2) is the element in the sixth row and second column of matrix f. Typically, we use the letters M and N, respectively, to denote the number of rows and columns in a matrix. A 1×N matrix is called a row vector, whereas an M \times 1 matrix is called a column vector. A 1 \times 1 matrix is a scalar.

Matrices in MATLAB are stored in variables with names such as A. a. RGB. real array, and so on. Variables must begin with a letter and contain only letters, numerals, and underscores. As noted in the previous paragraph, all MATLAB quantities in this book are written using monospace characters. We use conventional Roman, italic notation, such as f(x, y), for mathematical expressions.

2.2 Reading Images

Images are read into the MATLAB environment using function imread, whose basic syntax is

imread('filename')

Recall from Section 1.6 that we use margin icons to highlight the first use of a MATLAB or toolbox function.



Here, filename is a string containing the complete name of the image file (including any applicable extension). For example, the statement





In Windows, directories are called *folders*.

reads the image from the JPEG file chestxray into image array f. Note the use of single quotes (') to delimit the string filename. The semicolon at the end of a statement is used by MATLAB for *suppressing* output. If a semicolon is not included, MATLAB displays on the screen the results of the operation(s) specified in that line. The prompt symbol (>>) designates the beginning of a command line, as it appears in the MATLAB Command Window (see Fig. 1.1).

When, as in the preceding command line, no path information is included in filename, imread reads the file from the Current Directory and, if that fails, it tries to find the file in the MATLAB search path (see Section 1.7). The simplest way to read an image from a specified directory is to include a full or relative path to that directory in filename. For example,

```
>> f = imread('D:\myimages\chestxray.jpg');
```

reads the image from a directory called myimages in the D: drive, whereas

```
>> f = imread('.\myimages\chestxray.jpg');
```

reads the image from the myimages subdirectory of the current working directory. The MATLAB Desktop displays the path to the Current Directory on the toolbar, which provides an easy way to change it. Table 2.1 lists some of the most popular image/graphics formats supported by imread and imwrite (imwrite is discussed in Section 2.4).

Typing size at the prompt gives the row and column dimensions of an image:

```
>> size(f)
ans =
1024 1024
```

More generally, for an array A having an arbitrary number of dimensions, a statement of the form

$$[D1, D2, \ldots, DK] = size(A)$$

returns the sizes of the first K dimensions of A. This function is particularly useful in programming to determine automatically the size of a 2-D image:

This syntax returns the number of rows (M) and columns (N) in the image. Similarly, the command



Format Name	Description	Recognized Extensions
BMP^\dagger	Windows Bitmap	.bmp
CUR	Windows Cursor Resources	.cur
FITS [†]	Flexible Image Transport System	.fts, .fits
GIF	Graphics Interchange Format	.gif
HDF	Hierarchical Data Format	.hdf
ICO [†]	Windows Icon Resources	.ico
JPEG	Joint Photographic Experts Group	.jpg,.jpeg
JPEG 2000 [†]	Joint Photographic Experts Group	.jp2,.jpf,.jpx, j2c,j2k
PBM	Portable Bitmap	.pbm
PGM	Portable Graymap	.pgm
PNG	Portable Network Graphics	.png
PNM	Portable Any Map	.pnm
RAS	Sun Raster	.ras
TIFF	Tagged Image File Format	.tif, .tiff
XWD	X Window Dump	. xwd

TABLE 2.1 Some of the image/graphics formats supported by imread and imwrite, starting with MATLAB 7.6. Earlier versions support a subset of these formats. See the MATLAB documentation for a complete list of supported formats.

>> M = size(f, 1);

gives the size of f along its first dimension, which is defined by MATLAB as the vertical dimension. That is, this command gives the number of rows in f. The second dimension of an array is in the horizontal direction, so the statement size(f, 2) gives the number of columns in f. A singleton dimension is any dimension, \dim , for which $size(A, \dim) = 1$.

The whos function displays additional information about an array. For instance, the statement

>> whos f

gives



The Workspace Browser in the MATLAB Desktop displays similar information. The uint8 entry shown refers to one of several MATLAB data classes discussed in Section 2.5. A semicolon at the end of a whos line has no effect, so normally one is not used.



Although not applicable in this example, attributes that might appear under Attributes include terms such as global, complex, and sparse.

[†]Supported by imread, but not by imwrite

2.3 Displaying Images

Images are displayed on the MATLAB desktop using function imshow, which has the basic syntax:



Function imshow has a number of other syntax forms for performing tasks such as controlling image magnification. Consult the help page for imshow for additional details imshow(f)

where f is an image array. Using the syntax

displays as black all values less than or equal to low, and as white all values greater than or equal to high. The values in between are displayed as intermediate intensity values. Finally, the syntax

sets variable low to the minimum value of array f and high to its maximum value. This form of imshow is useful for displaying images that have a low dynamic range or that have positive and negative values.

EXAMPLE 2.1: Reading and displaying images.

■ The following statements read from disk an image called rose_512.tif, extract information about the image, and display it using imshow:

```
>> f = imread('rose_512.tif');
>> whos f

Name Size Bytes Class Attributes
    f 512x512 262144 uint8 array
>> imshow(f)
```

A semicolon at the end of an imshow line has no effect, so normally one is not used. Figure 2.2 shows what the output looks like on the screen. The figure

FIGURE 2.2

Screen capture showing how an image appears on the MATLAB desktop. Note the figure number on the top, left of the window. In most of the examples throughout the book, only the images themselves are shown.



number appears on the top, left of the window. Note the various pull-down menus and utility buttons. They are used for processes such as scaling, saving, and exporting the contents of the display window. In particular, the **Edit** menu has functions for editing and formatting the contents before they are printed or saved to disk.

If another image, g, is displayed using imshow, MATLAB replaces the image in the figure window with the new image. To keep the first image and output a second image, use function figure, as follows:

```
>> figure, imshow(g)
```

Using the statement

```
>> imshow(f), figure, imshow(g)
```

displays both images. Note that more than one command can be written on a line, provided that different commands are delimited by commas or semicolons. As mentioned earlier, a semicolon is used whenever it is desired to suppress screen outputs from a command line.

Finally, suppose that we have just read an image, h, and find that using imshow(h) produces the image in Fig. 2.3(a). This image has a low dynamic range, a condition that can be remedied for display purposes by using the statement

```
>> imshow(h, [ ])
```

Figure 2.3(b) shows the result. The improvement is apparent.

The Image Tool in the Image Processing Toolbox provides a more interactive environment for viewing and navigating within images, displaying detailed information about pixel values, measuring distances, and other useful operations. To start the Image Tool, use the imtool function. For example, the following statements read an image from a file and then display it using imtool:

```
>> f = imread('rose 1024.tif');
>> imtool(f)
```







Function figure creates a figure window. When used without an argument, as shown here, it simply creates a new figure window. Typing figure(n) forces figure number n to become visible.



FIGURE 2.3 (a) An image, h, with low dynamic range. (b) Result of scaling by using imshow(h,[]). (Original image courtesy of Dr. David R. Pickens. Vanderbilt University Medical Center.)

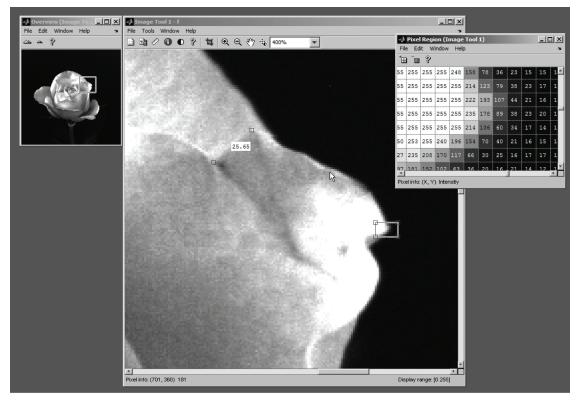


FIGURE 2.4 The Image Tool. The Overview Window, Main Window, and Pixel Region tools are shown.

Figure 2.4 shows some of the windows that might appear when using the Image Tool. The large, central window is the main view. In the figure, it is showing the image pixels at 400% magnification, meaning that each image pixel is rendered on a 4×4 block of screen pixels. The status text at the bottom of the main window shows the column/row location (701, 360) and value (181) of the pixel lying under the mouse cursor (the origin of the image is at the top, left). The *Measure Distance* tool is in use, showing that the distance between the two pixels enclosed by the small boxes is 25.65 units.

The *Overview Window*, on the left side of Fig. 2.4, shows the entire image in a thumbnail view. The *Main Window* view can be adjusted by dragging the rectangle in the Overview Window. The *Pixel Region Window* shows individual pixels from the small square region on the upper right tip of the rose, zoomed large enough to see the actual pixel values.

Table 2.2 summarizes the various tools and capabilities associated with the Image Tool. In addition to the these tools, the Main and Overview Window toolbars provide controls for tasks such as image zooming, panning, and scrolling.

Tool	Description	
Pixel Information	Displays information about the pixel under the mouse pointer.	
Pixel Region	Superimposes pixel values on a zoomed-in pixel view.	
Distance	Measures the distance between two pixels.	
Image Information	Displays information about images and image files.	
Adjust Contrast	Adjusts the contrast of the displayed image.	
Crop Image	Defines a crop region and crops the image.	
Display Range	Shows the display range of the image data.	
Overview	Shows the currently visible image.	

TABLE 2.2 Tools associated with the Image Tool.

Writing Images

Images are written to the Current Directory using function imwrite, which has the following basic syntax:



With this syntax, the string contained in filename must include a recognized file format extension (see Table 2.1). For example, the following command writes f to a file called patient10 run1.tif:

```
>> imwrite(f, 'patient10 run1.tif')
```

Function imwrite writes the image as a TIFF file because it recognizes the .tif extension in the filename.

Alternatively, the desired format can be specified explicitly with a third input argument. This syntax is useful when the desired file does not use one of the recognized file extensions. For example, the following command writes f to a TIFF file called patient10.run1:

```
>> imwrite(f, 'patient10.run1', 'tif')
```

Function imwrite can have other parameters, depending on the file format selected. Most of the work in the following chapters deals either with JPEG or TIFF images, so we focus attention here on these two formats. A more general imwrite syntax applicable only to JPEG images is

```
imwrite(f, 'filename.jpg', 'quality', q)
```

where q is an integer between 0 and 100 (the lower the number the higher the degradation due to JPEG compression).

EXAMPLE 2.2: Writing an image and using function imfinfo.

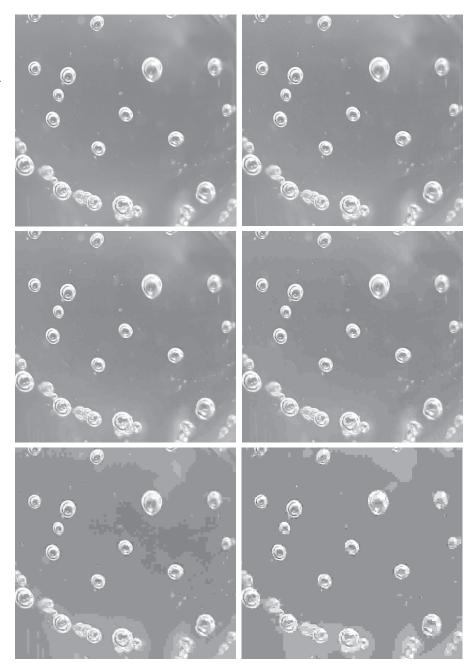
Figure 2.5(a) shows an image, f, typical of sequences of images resulting from a given chemical process. It is desired to transmit these images on a routine basis to a central site for visual and/or automated inspection. In order to reduce storage requirements and transmission time, it is important that the images be compressed as much as possible, while not degrading their visual

a b c d e f

FIGURE 2.5

(a) Original image. (b) through (f) Results of using jpg quality values q = 50, 25, 15, 5, and 0, respectively. False contouring begins to be noticeable for q = 15 [image (d)] and is quite visible for q = 5 and q = 0.

See Example 2.11 for a function that creates all the images in Fig. 2.5 using a loop.



appearance beyond a reasonable level. In this case "reasonable" means no perceptible false contouring. Figures 2.5(b) through (f) show the results obtained by writing image f to disk (in JPEG format), with q = 50, 25, 15, 5, and 0, respectively. For example, the applicable syntax for q = 25 is

```
>> imwrite(f, 'bubbles25.jpg', 'quality', 25)
```

The image for q = 15 [Fig. 2.5(d)] has false contouring that is barely visible, but this effect becomes quite pronounced for g = 5 and g = 0. Thus, an acceptable solution with some margin for error is to compress the images with q = 25. In order to get an idea of the compression achieved and to obtain other image file details, we can use function imfinfo, which has the syntax

imfinfo filename

where filename is the file name of the image stored on disk. For example,

```
>> imfinfo bubbles25.jpg
```

outputs the following information (note that some fields contain no information in this case):

```
Filename:
                   'bubbles25.jpg'
    FileModDate:
                   '04-Jan-2003 12:31:26'
       FileSize:
                   13849
         Format:
                   'ipg'
 FormatVersion:
          Width:
                   714
         Heiaht:
                   682
       BitDepth:
      ColorType:
                   'grayscale'
FormatSignature:
        Comment:
                   {}
```

Recent versions of MATLAB may show more information in the output of imfinfo, particularly for images captures using digital cameras.

where FileSize is in bytes. The number of bytes in the original image is computed by multiplying Width by Height by BitDepth and dividing the result by 8. The result is 486948. Dividing this by FileSize gives the compression ratio: (486948/13849) = 35.16. This compression ratio was achieved while maintaining image quality consistent with the requirements of the application. In addition to the obvious advantages in storage space, this reduction allows the transmission of approximately 35 times the amount of uncompressed data per unit time.

The information fields displayed by imfinfo can be captured into a socalled *structure variable* that can be used for subsequent computations. Using the preceding image as an example, and letting K denote the structure variable, we use the syntax

Structures are discussed in Section

```
>> K = imfinfo('bubbles25.jpg');
```

to store into variable K all the information generated by command imfinfo.

The information generated by imfinfo is appended to the structure variable by means of *fields*, separated from K by a dot. For example, the image height and width are now stored in structure fields K.Height and K.Width. As an illustration, consider the following use of structure variable K to compute the compression ratio for bubbles25.jpg:

```
>> K = imfinfo('bubbles25.jpg');
>> image_bytes = K.Width*K.Height*K.BitDepth/8;
>> compressed_bytes = K.FileSize;
>> compression_ratio = image_bytes/compressed_bytes
compression_ratio =
    35.1612
```

Note that imfinfo was used in two different ways. The first was to type imfinfo bubbles25.jpgatthe prompt, which resulted in the information being displayed on the screen. The second was to type K=imfinfo('bubbles25.jpg'), which resulted in the information generated by imfinfo being stored in K. These two different ways of calling imfinfo are an example of command-function duality, an important concept that is explained in more detail in the MATLAB documentation.

A more general imwrite syntax applicable only to tif images has the form

where 'parameter' can have one of the following principal values: 'none' indicates no compression; 'packbits' (the default for nonbinary images), 'lwz', 'deflate', 'jpeg', 'ccitt' (binary images only; the default), 'fax3' (binary images only), and 'fax4'. The 1 × 2 array [colres rowres] contains two integers that give the column resolution and row resolution in dots-per-unit (the default values are [72 72]). For example, if the image dimensions are in inches, colres is the number of dots (pixels) per inch (dpi) in the vertical direction, and similarly for rowres in the horizontal direction. Specifying the resolution by a single scalar, res, is equivalent to writing [res res]. As you will see in the following example, the TIFF resolution parameter can be used to modify the size of an image in printed documents.

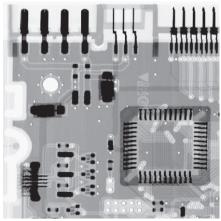
Figure 2.6(a) is an 8-bit X-ray image, f, of a circuit board generated during quality inspection. It is in jpg format, at 200 dpi. The image is of size 450×450 pixels, so its printed dimensions are 2.25×2.25 inches. We want to store this image in tif format, with no compression, under the name sf. In addition, we want to reduce the printed size of the image to 1.5×1.5 inches while keeping the pixel count at 450×450 . The following statement gives the desired result:

To learn more about command function duality, consult the help page on this topic. (See Section 1.7.2 regarding help pages.)



If a statement does not fit on one line, use an ellipsis (three periods), followed by **Return** or **Enter**, to indicate that the statement continues on the next line. There are no spaces between the periods.

EXAMPLE 2.3: Using imwrite parameters.



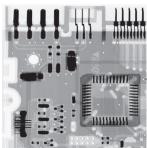




FIGURE 2.6

Effects of changing the dpi resolution while keeping the number of pixels constant. (a) A 450×450 image at 200 dpi $(size = 2.25 \times 2.25)$ inches). (b) The same image, but at 300 dpi (size = 1.5×1.5 inches). (Original image courtesy of Lixi, Inc.)

```
>> imwrite(f, 'sf.tif', 'compression', 'none', 'resolution', [300 300])
```

The values of the vector [colres rowres] were determined by multiplying 200 dpi by the ratio 2.25/1.5 which gives 300 dpi. Rather than do the computation manually, we could write

```
>> res = round(200*2.25/1.5);
>> imwrite(f, 'sf.tif', 'compression', 'none', 'resolution', res)
```



where function round rounds its argument to the nearest integer. It is important to note that the number of pixels was not changed by these commands. Only the printed size of the image changed. The original 450×450 image at 200 dpi is of size 2.25×2.25 inches. The new 300-dpi image [Fig. 2.6(b)] is identical, except that its 450×450 pixels are distributed over a 1.5×1.5 -inch area. Processes such as this are useful for controlling the size of an image in a printed document without sacrificing resolution.

Sometimes, it is necessary to export images and plots to disk the way they appear on the MATLAB desktop. The *contents* of a figure window can be exported to disk in two ways. The first is to use the **File** pull-down menu in the figure window (see Fig. 2.2) and then choose Save As. With this option, the user can select a location, file name, and format. More control over export parameters is obtained by using the print command:



where no refers to the figure number in the figure window of interest, file-format refers to one of the file formats in Table 2.1, resno is the resolution in dpi, and filename is the name we wish to assign the file. For example, to export the contents of the figure window in Fig. 2.2 as a tif file at 300 dpi, and under the name hi_res_rose, we would type

This command sends the file hi_res_rose.tif to the Current Directory. If we type print at the prompt, MATLAB prints (to the default printer) the contents of the last figure window displayed. It is possible also to specify other options with print, such as a specific printing device.

2.5 Classes

Although we work with integer coordinates, the values (intensities) of pixels are not restricted to be integers in MATLAB. Table 2.3 lists the various *classes* supported by MATLAB and the Image Processing Toolbox[†] for representing pixel values. The first eight entries in the table are referred to as *numeric* class-

TABLE 2.3
Classes used for image processing in MATLAB. The first eight entries are referred to as *numeric* classes, the ninth entry is the *char* class, and the last entry is the *logical* class.

Name	Description
double	Double-precision, floating-point numbers in the approximate range $\pm 10^{308}$ (8 bytes per element).
single	Single-precision floating-point numbers with values in the approximate range $\pm 10^{38}$ (4 bytes per element).
uint8	Unsigned 8-bit integers in the range [0, 255] (1 byte per element).
uint16	Unsigned 16-bit integers in the range [0, 65535] (2 bytes per element).
uint32	Unsigned 32-bit integers in the range [0, 4294967295] (4 bytes per element).
int8	Signed 8-bit integers in the range $[-128, 127]$ (1 byte per element).
int16	Signed 16-bit integers in the range $[-32768, 32767]$ (2 bytes per element).
int32	Signed 32-bit integers in the range $[-2147483648, 2147483647]$ (4 bytes per element).
char	Characters (2 bytes per element).
logical	Values are 0 or 1 (1 byte per element).

[†]MATLAB supports two other numeric classes not listed in Table 2.3, uint64 and int64. The toolbox does not support these classes, and MATLAB arithmetic support for them is limited.

es. The ninth entry is the *char* (character) class and, as shown, the last entry is the logical class.

Classes uint8 and logical are used extensively in image processing, and they are the usual classes encountered when reading images from image file formats such as TIFF or JPEG. These classes use 1 byte to represent each pixel. Some scientific data sources, such as medical imagery, require more dynamic range than is provided by uint8, so the uint16 and int16 classes are used often for such data. These classes use 2 bytes for each array element. The floating-point classes double and single are used for computationally intensive operations such as the Fourier transform (see Chapter 4). Double-precision floating-point uses 8 bytes per array element, whereas single-precision floating-point uses 4 bytes. The int8, uint32, and int32 classes, although supported by the toolbox, are not used commonly for image processing.

Image Types

The toolbox supports four types of images:

- Gray-scale images
- · Binary images
- Indexed images
- RGB images

Most monochrome image processing operations are carried out using binary or gray-scale images, so our initial focus is on these two image types. Indexed and RGB color images are discussed in Chapter 7.

Gray-scale images are referred to as intensity images in earlier versions of the toolbox. In the book, we use the two terms interchangeably when working with monochrome images.

2.6.1 Gray-scale Images

A gray-scale image is a data matrix whose values represent shades of gray. When the elements of a gray-scale image are of class uint8 or uint16, they have integer values in the range [0, 255] or [0, 65535], respectively. If the image is of class double or single, the values are floating-point numbers (see the first two entries in Table 2.3). Values of double and single gray-scale images normally are scaled in the range [0, 1], although other ranges can be used.

2.6.2 Binary Images

Binary images have a very specific meaning in MATLAB. A binary image is a logical array of 0s and 1s. Thus, an array of 0s and 1s whose values are of data class, say, uint8, is not considered a binary image in MATLAB. A numeric array is converted to binary using function logical. Thus, if A is a numeric array consisting of 0s and 1s, we create a logical array B using the statement

$$B = logical(A)$$

If A contains elements other than 0s and 1s, the logical function converts all nonzero quantities to logical 1s and all entries with value 0 to logical 0s. Using relational and logical operators (see Section 2.10.2) also results in logical arrays.



To test if an array is of class logical we use the islogical function:



See Table 2.9 for a list of other functions based on the is... construct.

islogical(C)

If C is a logical array, this function returns a 1. Otherwise it returns a 0. Logical arrays can be converted to numeric arrays using the class conversion functions discussed in Section 2.7.

2.6.3 A Note on Terminology

Considerable care was taken in the previous two sections to clarify the use of the terms *class* and *image type*. In general, we refer to an image as being a "class image_type image," where class is one of the entries from Table 2.3, and image_type is one of the image types defined at the beginning of this section. Thus, an image is characterized by *both* a class *and* a type. For instance, a statement discussing an "uint8 gray-scale image" is simply referring to a gray-scale image whose pixels are of class uint8. Some functions in the toolbox support all the data classes listed in Table 2.3, while others are very specific as to what constitutes a valid class.

2.7 Converting between Classes

Converting images from one class to another is a common operation. When converting between classes, keep in mind the value ranges of the classes being converted (see Table 2.3).

The general syntax for class conversion is

B = class name(A)

where class_name is one of the names in the first column of Table 2.3. For example, suppose that A is an array of class uint8. A double-precision array, B, is generated by the command B = double(A). If C is an array of class double in which all values are in the range [0, 255] (but possibly containing fractional values), it can be converted to an uint8 array with the command D = uint8(C). If an array of class double has any values outside the range [0, 255] and it is converted to class uint8 in the manner just described, MATLAB converts to 0 all values that are less than 0, and converts to 255 all values that are greater than 255. Numbers in between are rounded to the nearest integer. Thus, proper scaling of a double array so that its elements are in the range [0, 255] is necessary before converting it to uint8. As indicated in Section 2.6.2, converting any of the numeric data classes to logical creates an array with logical 1s in locations where the input array has nonzero values, and logical 0s in places where the input array contains 0s.

The toolbox provides specific functions (Table 2.4) that perform the scaling and other bookkeeping necessary to convert images from one class to another. Function im2uint8, for example, creates a unit8 image after detecting the

To simplify terminology, statements referring to values of class double are applicable also to the single class, unless stated otherwise. Both refer to floating point numbers, the only difference between them being precision and the number of bytes needed for storage.

Name	Converts Input to:	Valid Input Image Data Classes
im2uint8	uint8	logical, uint8, uint16, int16, single, and double
im2uint16	uint16	logical, uint8, uint16, int16, single, and double
im2double	double	logical, uint8, uint16, int16, single, and double
im2single	single	logical, uint8, uint16, int16, single, and double
mat2gray	double in the range $[0,1]$	logical, uint8, int8, uint16, int16, uint32, int32, single, and double
im2bw	logical	uint8, uint16, int16, single, and double

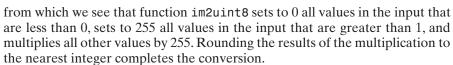
TABLE 2.4 Toolbox functions for converting images from one class to another.

data class of the input and performing all the necessary scaling for the toolbox to recognize the data as valid image data. For example, consider the following image f of class double, which could be the result of an intermediate computation:

Performing the conversion

$$>> g = im2uint8(f)$$

yields the result



Function im2double converts an input to class double. If the input is of class uint8, uint16, or logical, function im2double converts it to class double with values in the range [0, 1]. If the input is of class single, or is already of class double, im2double returns an array that is of class double, but is numerically equal to the input. For example, if an array of class double results from computations that yield values outside the range [0, 1], inputting this array into





im2double will have no effect. As explained below, function mat2gray can be used to convert an array of any of the classes in Table 2.4 to a double array with values in the range [0, 1].

As an illustration, consider the class uint8 image

Section 2.8.2 explains the use of square brackets and semicolons to specify matrices.

Performing the conversion

yields the result

0.0980 0.1961

0.4706 0.7843

from which we infer that the conversion when the input is of class uint8 is done simply by dividing each value of the input array by 255. If the input is of class uint16 the division is by 65535.

Toolbox function mat2gray converts an image of any of the classes in Table 2.4 to an array of class double *scaled* to the range [0, 1]. The calling syntax is

where image g has values in the range 0 (black) to 1 (white). The specified parameters, Amin and Amax, are such that values less than Amin in A become 0 in g, and values greater than Amax in A correspond to 1 in g. The syntax

$$g = mat2gray(A)$$

sets the values of Amin and Amax to the actual minimum and maximum values in A. The second syntax of mat2gray is a very useful tool because it scales the entire range of values in the input to the range [0,1], independently of the class of the input, thus eliminating clipping.

Finally, we consider conversion to class logical. (Recall that the Image Processing Toolbox treats logical matrices as binary images.) Function logical converts an input array to a logical array. In the process, nonzero elements in the input are converted to 1s, and 0s are converted to 0s in the output. An alternative conversion procedure that often is more useful is to use a relational operator, such as >, with a threshold value. For example, the syntax

$$g = f > T$$

produces a logical matrix containing 1s wherever the elements of f are greater than T and 0s elsewhere.

Toolbox function im2bw performs this thresholding operation in a way that automatically scales the specified threshold in different ways, depending on the class of the input image. The syntax is



See Section 2.10.2 regarding logical and relational operators.

$$g = im2bw(f, T)$$



Values specified for the threshold T must be in the range [0, 1], regardless of the class of the input. The function automatically scales the threshold value according to the input image class. For example, if f is uint8 and T is 0.4, then im2bw thresholds the pixels in f by comparing them to 255 * 0.4 = 102.

We wish to convert the following small, double image

>> f = [1 2; 3 4]f = 1 2 3 4

EXAMPLE 2.4: Converting between image classes.

to binary, such that values 1 and 2 become 0 and the other two values become 1. First we convert it to the range [0, 1]:

Then we convert it to binary using a threshold, say, of value 0.6:

As mentioned earlier, we can generate a binary array directly using relational operators. Thus we get the same result by writing

$$\Rightarrow$$
 gb = f \Rightarrow 2
gb = 0 0
1 1

Suppose now that we want to convert gb to a numerical array of 0s and 1s of class double. This is done directly:

```
>> gbd = im2double(gb)
gbd =
    0
        0
        1
    1
```

If gb had been of class uint8, applying im2double to it would have resulted in an array with values

```
0 0
0.0039 0.0039
```

because im2double would have divided all the elements by 255. This did not happen in the preceding conversion because im2double detected that the input was a logical array, whose only possible values are 0 and 1. If the input in fact had been of class uint8 and we wanted to convert it to class double while keeping the 0 and 1 values, we would have converted the array by writing

Finally, we point out that the output of one function can be passed directly as the input to another, so we could have started with image f and arrived at the same result by using the one-line statement

```
>> gbd = im2double(im2bw(mat2gray(f), 0.6));
```

or by using partial groupings of these functions. Of course, the entire process could have been done in this case with a simpler command:

```
>> gbd = double(f > 2);
```

demonstrating again the compactness of the MATLAB language.

As the first two entries in Table 2.3 show class numeric data of class double requires twice as much storage as data of class single. In most image processing applications in which numeric processing is used, single precision is perfectly adequate. Therefore, unless a specific application or a MATLAB or toolbox function requires class double, it is good practice to work with single data to conserve memory. A consistent programming pattern that you will see used throughout the book to change inputs to class single is as follows:

```
[fout, revertclass] = tofloat(f);
g = some_operation(fout)
g = revertclass(g);
```

Function tofloat (see Appendix C for the code) converts an input image f to floating-point. If f is a double or single image, then fout equals f. Otherwise, fout equals im2single(f). Output revertclass can be used to convert back to the same class as f. In other words, the idea is to convert the input

Recall from Section 1.6 that we the a margin icon to denote the first use of a function developed in the book.

tofloat

See function intrans in Section 3.2.3 for an example of how tofloat is used.

image to single, perform operations using single precision, and then, if so desired, convert the final output image to the same class as the input. The valid image classes for f are those listed in the third column of the first four entries in Table 2.4: logical, uint8, unint16, int16, double, and single.

2.8 Array Indexing

MATLAB supports a number of powerful indexing schemes that simplify array manipulation and improve the efficiency of programs. In this section we discuss and illustrate basic indexing in one and two dimensions (i.e., vectors and matrices), as well as indexing techniques useful with binary images.

2.8.1 Indexing Vectors

As discussed in Section 2.1.2, an array of dimension $1 \times N$ is called a *row vector*. The elements of such a vector can be accessed using a single index value (also called a *subscript*). Thus, v(1) is the first element of vector v, v(2) is its second element, and so forth. Vectors can be formed in MATLAB by enclosing the elements, separated by spaces or commas, within square brackets. For example,

```
>> v = [1 \ 3 \ 5 \ 7 \ 9]
     1 3 5 7 9
>> v(2)
ans =
     3
```

A row vector is converted to a column vector (and vice versa) using the transpose operator (.'):



```
>> W = V.
w =
     1
    3
     5
    7
    9
```

Using a single quote without the period computes the conjugate transpose. When the data are real, both transposes can be used interchangeably. See Table 2.5.

To access blocks of elements, we use MATLAB's colon notation. For example, to access the first three elements of v we write



```
>> v(1:3)
ans =
    1
         3
              5
```

Similarly, we can access the second through the fourth elements

or all the elements from, say, the third through the last element:



where end signifies the last element in the vector.

Indexing is not restricted to contiguous elements. For example,

The notation 1:2:end says to start at 1, count up by 2, and stop when the count reaches the last element. The steps can be negative:

```
>> v(end:-2:1)
ans =
9 5 1
```

Here, the index count started at the last element, decreased by 2, and stopped when it reached the first element.

Function linspace, with syntax



$$x = linspace(a, b, n)$$

generates a row vector x of n elements linearly-spaced between, and including, a and b. We use this function in several places in later chapters. A vector can even be used as an index into another vector. For example, we can select the first, fourth, and fifth elements of v using the command

As we show in the following section, the ability to use a vector as an index into another vector also plays a key role in matrix indexing.

2.8.2 Indexing Matrices

Matrices can be represented conveniently in MATLAB as a sequence of row vectors enclosed by square brackets and separated by semicolons. For example, typing

```
>> A = [1 2 3; 4 5 6; 7 8 9]
gives the 3 \times 3 matrix
A =
    1
         2
              3
    4
         5
              6
    7
```

Note that the use of semicolons inside square brackets is different from their use mentioned earlier to suppress output or to write multiple commands in a single line. We select elements in a matrix just as we did for vectors, but now we need two indices: one to establish a row location, and the other for the corresponding column. For example, to extract the element in the second row, third column of matrix A, we write

A submatrix of A can be extracted by specifying a vector of values for both the row and the column indices. For example, the following statement extracts the submatrix of A containing rows 1 and 2 and columns 1, 2, and 3:

```
\Rightarrow T2 = A([1 2], [1 2 3])
T2 =
     1
          2
               3
          5
               6
```

Because the expression 1:K creates a vector of integer values from 1 through K, the preceding statement could be written also as:

The row and column indices do not have to be contiguous, nor do they have to be in ascending order. For example,

```
>> E = A([1 3], [3 2])
E =
    3
        2
    9
        8
```

The notation A([a b], [c d]) selects the elements in A with coordinates (a, c), (a, d), (b, c), and (b, d). Thus, when we let $E = A([1 \ 3], [3 \ 2]),$ we are selecting the following elements in A: A(1, 3), A(1, 2), A(3, 3), and A(3, 2).

The row or column index can also be a single colon. A colon in the row index position is shorthand notation for selecting all rows. Similarly, a colon in the column index position selects all columns. For example, the following statement selects the entire 3rd column of A:

Similarly, this statement extracts the second row:

Any of the preceding forms of indexing can be used on the left-hand side of an assignment statement. The next two statements create a copy, B, of matrix A, and then assign the value 0 to all elements in the 3rd column of B.

The keyword end, when it appears in the row index position, is shorthand notation for the last row. When end appears in the column index position, it indicates the last column. For example, the following statement finds the element in the last row and last column of A:

```
>> A(end, end)
ans =
```

When used for indexing, the end keyword can be mixed with arithmetic operations, as well as with the colon operator. For example:

```
>> A(end, end - 2)
ans =
    7
>> A(2:end, end:-2:1)
ans =
    6
        4
    9
        7
```

2.8.3 Indexing with a Single Colon

The use of a single colon as an index into a matrix selects all the elements of the array and arranges them (in column order) into a single column vector. For example, with reference to matrix T2 in the previous section,

```
>> v = T2(:)
v =
    1
    4
    2
    5
    3
    6
```

This use of the colon is helpful when, for example, we want to find the sum of all the elements of a matrix. One approach is to call function sum twice:

```
>> col sums = sum(A)
col sums =
        15
    111
              112
```



Function sum computes the sum of each column of A, storing the results into a row vector. Then we call sum again, passing it the vector of column sums:

```
>> total sum = sum(col sums)
total sum =
    238
```

An easier procedure is to use single-colon indexing to convert A to a column vector, and pass the result to sum:

```
>> total sum = sum(A(:))
total sum =
    238
```

2.8.4 Logical Indexing

Another form of indexing that you will find quite useful is *logical indexing*. A logical indexing expression has the form A(D), where A is an array and D is a logical array of the same size as A. The expression A(D) extracts all the elements of A corresponding to the 1-valued elements of D. For example,

```
>> D = logical([1 0 0; 0 0 1; 0 0 0])
D =
    1
       0
            0
    0
      0
          1
    0 0
           0
>> A(D)
ans =
    1
    6
```

where A is as defined at the beginning of Section 2.8.2. The output of this method of logical indexing always is a column vector.

Logical indexing can be used also on the left-hand side of an assignment statement. For example, using the same D as above,

```
>> A(D) = [30 40]
A =
    30
         2
              3
        5
     4
            40
    7
        8
              9
```

In the preceding assignment, the number of elements on the right-hand side matched the number of 1-valued elements of D. Alternatively, the right-hand side can be a scalar, like this:

```
>> A(D) = 100
A =
```

```
100
                  3
          2
          5
               100
  7
          8
                  9
```

Because binary images are represented as logical arrays, they can be used directly in logical indexing expressions to extract pixel values in an image that correspond to 1-valued pixels in a binary image. You will see numerous examples later in the book that use binary images and logical indexing.

2.8.5 Linear Indexing

The final category of indexing useful for image processing is *linear indexing*. A linear indexing expression is one that uses a single subscript to index a matrix or higher-dimensional array. To illustrate the concept we will use a 4×4 *Hilbert matrix* as an example:

```
>> H = hilb(4)
H =
    1.0000
             0.5000
                      0.3333
                               0.2500
    0.5000
             0.3333
                      0.2500
                               0.2000
                      0.2000
    0.3333
             0.2500
                               0.1667
    0.2500
             0.2000
                      0.1667
                               0.1429
```



H([2 11]) is an example of a linear indexing expression:

```
>> H([2 11])
ans =
    0.5000
             0.2000
```

To see how this type of indexing works, number the elements of H from the first to the last column in the order shown:

1.0000 ¹	0.5000^{5}	0.33339	0.2500^{13}
0.5000^{2}	0.3333^{6}	0.2500^{10}	0.2000^{14}
0.3333^{3}	0.2500^{7}	0.2000^{11}	0.166715
0.25004	0.2000^{8}	0.166712	0.1429 ¹⁶

Here you can see that H([2 11]) extracts the 2nd and 11th elements of H, based on the preceding numbering scheme.

In image processing, linear indexing is useful for extracting a set of pixel values from arbitrary locations. For example, suppose we want an expression that extracts the values of H at row-column coordinates (1,3), (2,4), and (4,3):

Expression H(r, c) does not do what we want, as you can see:

Instead, we convert the *row-column* coordinates to linear index values, as follows:

MATLAB functions sub2ind and ind2sub convert back and forth between row-column subscripts and linear indices. For example,



```
>> linear_indices = sub2ind(size(H), r, c)
linear_indices =
    9   14   12
```



Linear indexing is a basic staple in vectorizing loops for program optimization, as discussed in Section 2.10.5.

EXAMPLE 2.5: Some simple

Some simple image operations using array indexing.

■ The image in Fig. 2.7(a) is a 1024×1024 gray-scale image, f, of class uint8. The image in Fig. 2.7(b) was flipped vertically using the statement

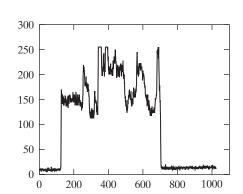
```
>> fp = f(end:-1:1, :);
```











a b c d e

FIGURE 2.7 Results obtained using array indexing. (a) Original image. (b) Image flipped vertically. (c) Cropped image. (d) Subsampled image. (e) A horizontal scan line through the

middle of the image in (a).

The image in Fig. 2.7(c) is a section out of image (a), obtained using the command

>> fc = f(257:768, 257:768);

Similarly, Fig. 2.7(d) shows a subsampled image obtained using the statement

>> fs = f(1:2:end, 1:2:end);

Finally, Fig. 2.7(e) shows a horizontal scan line through the middle of Fig. 2.7(a), obtained using the command

>> plot(f(512, :))

Function plot is discussed in Section 3.3.1.



2.8.6 Selecting Array Dimensions

Operations of the form

where operation denotes an applicable MATLAB operation, A is an array, and dim is a scalar, are used frequently in this book. For example, if A is a 2-D array, the statement

$$>> k = size(A, 1);$$

gives the size of A along its first dimension (i.e., it gives the number of rows in A). Similarly, the second dimension of an array is in the horizontal direction, so the statement size(A, 2) gives the number of columns in A. Using these concepts, we could have written the last command in Example 2.5 as

MATLAB does not restrict the number of dimensions of an array, so being able to extract the components of an array in any dimension is an important feature. For the most part, we deal with 2-D arrays, but there are several instances (as when working with color or multispectral images) when it is necessary to be able to "stack" images along a third or higher dimension. We deal with this in Chapters 7, 8, 12, and 13. Function ndims, with syntax

$$d = ndims(A)$$

gives the number of dimensions of array A. Function ndims never returns a value less than 2 because even scalars are considered two dimensional, in the sense that they are arrays of size 1×1 .

2.8.7 Sparse Matrices

When a matrix has a large number of 0s, it is advantageous to express it in *sparse* form to reduce storage requirements. Function sparse converts a matrix to sparse form by "squeezing out" all zero elements. The basic syntax for this function is

$$S = sparse(A)$$

For example, if





Then

from which we see that S contains only the (row, col) locations of nonzero elements (note that the elements are sorted by columns). To recover the original (full) matrix, we use function full:

```
>> Original = full(S)
Original =
    1
    0
        3
             4
    0
        2
```



A syntax used sometimes with function sparse has five inputs:

$$S = sparse(r, c, s, m, n)$$

where r and c are vectors containing, respectively, the row and column indices of the nonzero elements of the matrix we wish to express in sparse form. Parameter s is a vector containing the values corresponding to index pairs (r, c), and m and n are the row and column dimensions of the matrix. For instance, the preceding matrix S can be generated directly using the command

The syntax sparse(A) requires that there be enough memory to hold the entire matrix. When that is not the case, and the location and values of all nonzero elements are known, the alternate syntax shown here provides a solution for generating a sparse matrix.

```
>> S = sparse([1 2 3 2], [1 2 2 3], [1 3 2 4], 3, 3)
S =
    (1,1)
            1
    (2,2)
            3
    (3,2)
            2
    (2,3)
```

Arithmetic and other operations (Section 2.10.2) on sparse matrices are carried out in exactly the same way as with full matrices. There are a number of other syntax forms for function sparse, as detailed in the help page for this function.

Some Important Standard Arrays

Sometimes, it is useful to be able to generate image arrays with known characteristics to try out ideas and to test the syntax of functions during development. In this section we introduce eight array-generating functions that are used in

later chapters. If only one argument is included in any of the following functions, the result is a square array.

- zeros (M, N) generates an M×N matrix of 0s of class double.
- ones (M, N) generates an $M \times N$ matrix of 1s of class double.
- true(M, N) generates an M×N logical matrix of 1s.
- false(M, N) generates an M×N logical matrix of 0s.
- magic (M) generates an M×M"magic square." This is a square array in which the sum along any row, column, or main diagonal, is the same. Magic squares are useful arrays for testing purposes because they are easy to generate and their numbers are integers.
- eye (M) generates an M×M identity matrix.
- rand (M, N) generates an $M \times N$ matrix whose entries are uniformly distributed random numbers in the interval [0, 1].
- randn(M, N) generates an M×N matrix whose numbers are normally distributed (i.e., Gaussian) random numbers with mean 0 and variance 1.

For example,

```
>> A = 5*ones(3, 3)
A =
   5
       5
           5
   5
       5
           5
   5
       5
           5
>> magic(3)
ans =
   8
       1
           6
   3
       5
           7
       9
   4
           2
>> B = rand(2, 4)
B =
   0.2311 0.4860 0.7621 0.0185
   0.6068
          0.8913 0.4565
                              0.8214
```

2.10 Introduction to M-Function Programming

One of the most powerful features of MATLAB is the capability it provides users to program their own new functions. As you will learn shortly, MATLAB function programming is flexible and particularly easy to learn.

2.10.1 M-Files

M-files in MATLAB (see Section 1.3) can be scripts that simply execute a series of MATLAB statements, or they can be functions that can accept arguments and can produce one or more outputs. The focus of this section in on M-

file functions. These functions extend the capabilities of both MATLAB and the Image Processing Toolbox to address specific, user-defined applications.

M-files are created using a text editor and are stored with a name of the form filename.m, such as average.m and filter.m. The components of a function M-file are

- The function definition line
- The H1 line
- Help text
- The function body
- Comments

The function definition line has the form

```
function [outputs] = name(inputs)
```

For example, a function to compute the sum and product (two different outputs) of two images would have the form

```
function [s, p] = sumprod(f, g)
```

where f and g are the input images, s is the sum image, and p is the product image. The name sumprod is chosen arbitrarily (subject to the constraints at the end of this paragraph), but the word function always appears on the left, in the form shown. Note that the output arguments are enclosed by square brackets and the inputs are enclosed by parentheses. If the function has a single output argument, it is acceptable to list the argument without brackets. If the function has no output, only the word function is used, without brackets or equal sign. Function names must begin with a letter, and the remaining characters can be any combination of letters, numbers, and underscores. No spaces are allowed. MATLAB recognizes function names up to 63 characters long. Additional characters are ignored.

Functions can be called at the command prompt. For example,

```
>> [s, p] = sumprod(f, g);
```

or they can be used as elements of other functions, in which case they become subfunctions. As noted in the previous paragraph, if the output has a single argument, it is acceptable to write it without the brackets, as in

```
>> y = sum(x);
```

The H1 line is the first text line. It is a single comment line that follows the function definition line. There can be no blank lines or leading spaces between the H1 line and the function definition line. An example of an H1 line is

%SUMPROD Computes the sum and product of two images.

It is customary to omit the space between % and the first word in the H1 line.

The H1 line is the first text that appears when a user types

help

lookfor

>> help function name

at the MATLAB prompt. Typing lookfor keyword displays all the H1 lines containing the string keyword. This line provides important summary information about the M-file, so it should be as descriptive as possible.

Help text is a text block that follows the H1 line, without any blank lines in between the two. Help text is used to provide comments and on-screen help for the function. When a user types help function_name at the prompt, MATLAB displays all comment lines that appear between the function definition line and the first noncomment (executable or blank) line. The help system ignores any comment lines that appear after the Help text block.

The *function body* contains all the MATLAB code that performs computations and assigns values to output arguments. Several examples of MATLAB code are given later in this chapter.

All lines preceded by the symbol "%" that are not the H1 line or Help text are considered function *comment* lines and are not considered part of the Help text block. It is permissible to append comments to the end of a line of code.

M-files can be created and edited using any text editor and saved with the extension .m in a specified directory, typically in the MATLAB search path. Another way to create or edit an M-file is to use the edit function at the prompt. For example,

>> edit sumprod

opens for editing the file sumprod.m if the file exists in a directory that is in the MATLAB path or in the Current Directory. If the file cannot be found, MATLAB gives the user the option to create it. The MATLAB editor window has numerous pull-down menus for tasks such as saving, viewing, and debugging files. Because it performs some simple checks and uses color to differentiate between various elements of code, the MATLAB text editor is recommended as the tool of choice for writing and editing M-functions.

2.10.2 Operators

MATLAB operators are grouped into three main categories:

- Arithmetic operators that perform numeric computations
- Relational operators that compare operands quantitatively
- Logical operators that perform the functions AND, OR, and NOT

These are discussed in the remainder of this section.

Arithmetic Operators

MATLAB has two different types of arithmetic operations. *Matrix arithmetic operations* are defined by the rules of linear algebra. *Array arithmetic operations* are carried out element by element and can be used with multidimensional arrays. The period (dot) character (.) distinguishes array operations





from matrix operations. For example, A*B indicates matrix multiplication in the traditional sense, whereas A.*B indicates array multiplication, in the sense that the result is an array, the same size as A and B, in which each element is the product of corresponding elements of A and B. In other words, if C = A.*B, then C(I, J) = A(I, J) *B(I, J). Because matrix and array operations are the same for addition and subtraction, the character pairs .+ and .- are not used.

When writing an expression such as B = A, MATLAB makes a "note" that B is equal to A, but does not actually copy the data into B unless the contents of A change later in the program. This is an important point because using different variables to "store" the same information sometimes can enhance code clarity and readability. Thus, the fact that MATLAB does not duplicate information unless it is absolutely necessary is worth remembering when writing MATLAB code. Table 2.5 lists the MATLAB arithmetic operators, where A and B are matrices or arrays and a and b are scalars. All operands can be real or complex. The dot shown in the array operators is not necessary if the operands are scalars. Because images are 2-D arrays, which are equivalent to matrices, all the operators in the table are applicable to images.

The difference between array and matrix operations is important. For example, consider the following:

Throughout the book, we use the term array operations interchangeably with the terminology operations between pairs of corresponding elements, and also elementwise operations.

TABLE 2.5 Array and matrix arithmetic operators. Characters a and b are scalars.

Operator	Name	Comments and Examples
+	Array and matrix addition	a + b, A + B, or a + A.
-	Array and matrix subtraction	a - b, A - B, A - a, or a - A.
.*	Array multiplication	Cv = A.*B, C(I, J) = A(I, J)*B(I, J).
*	Matrix multiplication	A*B, standard matrix multiplication, or a*A, multiplication of a scalar times all elements of A.
./	Array right division [†]	C = A./B, C(I, J) = A(I, J)/B(I, J).
. \	Array left division [†]	$C = A.\B, C(I, J) = B(I, J)/A(I, J).$
/	Matrix right division	A/B is the preferred way to compute $A*inv(B)$.
\	Matrix left division	$A\B$ is the preferred way to compute inv(A) *B.
.^	Array power	If $C = A.^B$, then $C(I, J) = A(I, J)^B(I, J)$.
^	Matrix power	See help for a discussion of this operator.
. '	Vector and matrix transpose	A. ', standard vector and matrix transpose.
I	Vector and matrix complex conjugate transpose	$A{}^{\shortmid}$, standard vector and matrix conjugate transpose. When A is real $A{}^{\backprime}$ = $A{}^{\backprime}$.
+	Unary plus	+A is the same as 0 + A.
_	Unary minus	-A is the same as $0 - A$ or $-1*A$.
:	Colon	Discussed in Section 2.8.1.

[†]In division, if the denominator is 0, MATLAB reports the result as Inf (denoting infinity). If both the numerator and denominator are 0, the result is reported as NaN (Not a Number).

$$A = \begin{bmatrix} a1 & a2 \\ a3 & a4 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} b1 & b2 \\ b3 & b4 \end{bmatrix}$$

The array product of A and B gives the result

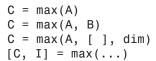
$$A.*B = \begin{bmatrix} a1b1 & a2b2 \\ a3b3 & a4b4 \end{bmatrix}$$

whereas the matrix product yields the familiar result:

$$A * B = \begin{bmatrix} a1b1 + a2b3 & a1b2 + a2b4 \\ a3b1 + a4b3 & a3b2 + a4b4 \end{bmatrix}$$

Most of the arithmetic, relational, and logical operations involving images are array operations.

Example 2.6, to follow, uses functions \max and \min . The former function has the syntax forms



In the first form, if A is a vector, max(A) returns its largest element; if A is a matrix, then max(A) treats the columns of A as vectors and returns a row vector containing the maximum element from each column. In the second form, max(A, B) returns an array the same size as A and B with the largest elements taken from A or B. In the third form, max(A, [], dim) returns the largest elements along the dimension of A specified by scalar dim. For example, max(A, [], 1) produces the maximum values along the first dimension (the rows) of A. Finally, [C, I] = max(...) also finds the indices of the maximum values of A, and returns them in output vector I. If there are duplicate maximum values, the index of the first one found is returned. The dots indicate the syntax used on the right of any of the previous three forms. Function min has the same syntax forms just described for max.

EXAMPLE 2.6: Illustration of arithmetic

max

The syntax forms shown

for max apply also to function min.

Illustration of arithmetic operators and functions max and min.

■ Suppose that we want to write an M-function, call it imblend, that forms a new image as an equally-weighted sum of two input images. The function should output the new image, as well as the maximum and minimum values of the new image. Using the MATLAB editor we write the desired function as follows:

function [w, wmax, wmin] = imblend(f, g)
%IMBLEND Weighted sum of two images.

- % [W, WMAX, WMIN] = IMBLEND(F, G) computes a weighted sum (W) of
- % two input images, F and G. IMBLEND also computes the maximum
- % (WMAX) and minimum (WMIN) values of W. F and G must be of
- % the same size and numeric class. The output image is of the
- % same class as the input images.

```
w1 = 0.5 * f;
w2 = 0.5 * g;
w = w1 + w2;
wmax = max(w(:));
wmin = min(w(:));
```

Observe the use of single-colon indexing, as discussed in Section 2.8.1, to compute the minimum and maximum values. Suppose that f = [12; 34] and g = [12; 21]. Calling imblend with these inputs results in the following output:

```
>> [w, wmax, wmin] = imblend(f, g)
w =
    1.0000
             2,0000
    2.5000
             2.5000
wmax =
    2,5000
wmin =
    1
```

Note in the code for imblend that the input images, f and g, were multiplied by the weights (0.5) first before being added together. Instead, we could have used the statement

```
>> w = 0.5 * (f + g);
```

However, this expression does not work well for integer classes because when MATLAB evaluates the subexpression (f + g), it saturates any values that overflow the range of the class of f and g. For example, consider the following scalars:

```
>> f = uint8(100);
>> g = uint8(200);
>> t = f + g
t =
    255
```

Instead of getting a sum of 300, the computed sum saturated to the maximum value for the uint8 class. So, when we multiply the sum by 0.5, we get an incorrect result:

```
>> d = 0.5 * t
d =
    128
```

Compare this with the result when we multiply by the weights first before adding:

```
>> e1 = 0.5 * f
e1 =
    50
>> e2 = 0.5 * q
e2 =
    100
>> e = w1 + w2
e =
    150
```

A good alternative is to use the image arithmetic function imlincomb, which computes a weighted sum of images, for any set of weights and any number of images. The calling syntax for this function is



$$g = imlincomb(k1, f1, k2, f2,...)$$

For example, using the previous scalar values,

```
\gg w = imlincomb(0.5, f, 0.5, g)
w =
    150
```

Typing help imblend at the command prompt results in the following output:

%IMBLEND Weighted sum of two images.

```
[W, WMAX, WMIN] = IMBLEND(F, G) computes a weighted sum (W) of
two input images, F and G. IMBLEND also computes the maximum
(WMAX) and minimum (WMIN) values of W. F and G must be of
the same size and numeric class. The output image is of the
same class as the input images.
```

Relational Operators

MATLAB's relational operators are listed in Table 2.6. These are array operators; that is, they compare corresponding pairs of elements in arrays of equal dimensions.

EXAMPLE 2.7: Relational operators.

Although the key use of relational operators is in flow control (e.g., in if statements), which is discussed in Section 2.10.3, we illustrate briefly how these operators can be used directly on arrays. Consider the following:

```
>> A = [1 2 3; 4 5 6; 7 8 9]
```

Operator	Name
<	Less than
<=	Less than or equal to
>	Greater than
>=	Greater than or equal to
==	Equal to
~=	Not equal to

TABLE 2.6 Relational operators.

```
A =
    1
         2
             3
    4
         5
             6
    7
         8
             9
>> B = [0 2 4; 3 5 6; 3 4 9]
B =
    0
         2
             4
    3
         5
             6
    3
             9
>> A == B
ans =
    0
         1
             0
    0
         1
             1
    0
         0
             1
```

We see that the operation A == B produces a logical array of the same dimensions as A and B, with 1s in locations where the corresponding elements of A and B match, and 0s elsewhere. As another illustration, the statement

produces a logical array with 1s where the elements of A are greater than or equal to the corresponding elements of B, and 0s elsewhere.

In relational operators, both operands must have the same dimensions unless one operand is a scalar. In this case, MATLAB tests the scalar against every element of the other operand, yielding a logical array of the same size as the operand, with 1s in locations where the specified relation is satisfied and 0s elsewhere. If both operands are scalars, the result is a 1 if the specified relation is satisfied and 0 otherwise.

Logical Operators and Functions

Table 2.7 lists MATLAB's logical operators, and the following example illustrates some of their properties. Unlike most common interpretations of logical operators, the operators in Table 2.7 can operate on both logical *and* numeric data. MATLAB treats a logical 1 or nonzero numeric quantity as true, and a logical 0 or numeric 0 as false in all logical tests. For instance, the AND of two operands is 1 if both operands are logical 1s or both are nonzero numbers. The AND operation is 0 if either of its operands is logically or numerically 0, or if they both are logically or numerically 0.

The operators & and | operate on arrays; they compute AND and OR, respectively, on corresponding elements of their inputs. The operators && and || operate only on scalars. They are used primarily with the various forms of if, and with while and for loops, all of which are discussed in Section 2.10.3.

EXAMPLE 2.8: Logical operators.

Consider the AND operation on the following numeric arrays:

```
>> A = [1 2 0; 0 4 5];
>> B = [1 -2 3; 0 1 1];
>> A & B
ans =
1 1 0
0 1 1
```

We see that the & operator produces a logical array that is of the same size as the input arrays and has a 1 at locations where both operands are nonzero and 0s elsewhere. Again, note that all operations are done on pairs of corresponding elements of the arrays. The | operator works in a similar manner. An | expression is true if either operand is a logical 1 or nonzero numerical quantity, or if they both are logical 1s or nonzero numbers; otherwise it is

TABLE 2.7 Logical operators.

Description
Elementwise AND
Elementwise OR
Elementwise and scalar NOT
Scalar AND
Scalar OR

false. The ~ operator works with a single operand. Logically, if the operand is true, the ~ operator converts it to false. When using ~ with numeric data, any nonzero operand becomes 0, and any zero operand becomes 1. If you try to use the scalar logical operators && or || with nonscalar operands, MATLAB will issue an error.

MATLAB also supports the logical functions summarized in Table 2.8. The all and any functions are particularly useful in programming.

Consider the arrays $A = [1 \ 2 \ 3; \ 4 \ 5 \ 6]$ and $B = [0 \ -1 \ 1; \ 0 \ 0 \ 2]$. Substituting these arrays into the functions in Table 2.8 yield the following results:

EXAMPLE 2.9: Logical functions.

```
>> xor(A, B)
ans =
    1
        0
             0
    1
         1
             0
>> all(A)
ans =
    1
>> any(A)
ans =
    1
        1
             1
>> all(B)
ans =
    0
        0
             1
>> any(B)
ans =
    0
        1
             1
```

Operator	Comments
xor (exclusive OR)	The xor function returns a 1 only if both operands are logically different; otherwise xor returns a 0.
all	The all function returns a 1 if all the elements in a vector are nonzero; otherwise all returns a 0. This function operates columnwise on matrices.
any	The any function returns a 1 if any of the elements in a vector is nonzero; otherwise any returns a 0. This function operates columnwise on matrices.

TABLE 2.8 Logical functions.

Note how functions all and any operate on columns of A and B. For instance, the first two elements of the vector produced by all(B) are 0 because each of the first two columns of B contains at least one 0; the last element is 1 because all elements in the last column of B are nonzero.

In addition to the functions listed in Table 2.8, MATLAB provides a number of other functions that test for the existence of specific conditions or values and return logical results. Some of these functions are listed in Table 2.9. A few of them deal with terms and concepts discussed earlier in this chapter; others are used in subsequent discussions. The functions in Table 2.9 return a logical 1 when the condition being tested is true; otherwise they return a logical 0. When the argument is an array, some of the functions in Table 2.9 yield an array the same size as the argument containing logical 1s in the locations that satisfy

TABLE 2.9
Some functions
that return a
logical 1 or a
logical 0,
depending on
whether the value
or condition in
their arguments
is true or false.
Type is* in the
help
documentation
for a complete list.

Function	Description
iscell(C)	True if C is a cell array.
iscellstr(s)	True if s is a cell array of strings.
ischar(s)	True if s is a character string.
isempty(A)	True if A is the empty array, [].
isequal(A, B)	True if A and B have identical elements and dimensions.
isfield(S, 'name')	True if 'name' is a field of structure S.
isfinite(A)	True in the locations of array A that are finite.
isinf(A)	True in the locations of array A that are infinite.
isinteger(A)	True if A is an integer array.
isletter(A)	True in the locations of A that are letters of the alphabet.
islogical(A)	True if A is a logical array.
ismember(A, B)	True in locations where elements of A are also in B.
isnan(A)	True in the locations of A that are NaNs (see Table 2.10 for a definition of NaN).
isnumeric(A)	True if A is a numeric array.
isprime(A)	True in locations of A that are prime numbers.
isreal(A)	True if the elements of A have no imaginary parts.
isscalar(A)	True if A has exactly one element.
isspace(A)	True at locations where the elements of A are whitespace characters.
issparse(A)	True if A is a sparse matrix.
isstruct(S)	True if S is a structure.
isvector(A)	True if A is a row or column vector.

the test performed by the function, and logical 0s elsewhere. For example, if $A = [1 \ 2; \ 3 \ 1/0]$, the function is finite (A) returns the matrix $[1 \ 1; \ 1 \ 0]$, where the O (false) entry indicates that the last element of A is not finite.

Some Important Values

The functions in Table 2.10 return values that are used extensively in MATLAB programming. For example, eps typically is added to denominators in expressions to prevent overflow when a denominator becomes zero.

Floating-Point Number Representation

MATLAB uses conventional decimal notation, with an optional decimal point and leading plus or minus sign, for numbers. Scientific notation uses the letter e to specify a power-of-ten scale factor. Imaginary numbers use either i or j as a suffix. Some examples of valid number representations are

3	-99	0.0001
9.6397238	1.60210e-20	6.02252e23
1i	-3.14159j	3e5i

By default, numbers are stored internally using the long format specified by the Institute of Electrical and Electronics Engineers (IEEE) floating-point standard. Often, this format is called double-precision floating point, and corresponds to the MATLAB class double. As discussed in Section 2.5 (see Table 2.3), double-precision floating-point numbers have a precision of 16 significant decimal digits and a range of approximately $\pm 10^{+308}$. Single-precision floatingpoint numbers have a precision of 7 significant decimal digits and a range of approximately $\pm 10^{+38}$.

Function	Value Returned
ans	Most recent answer (variable). If no output variable is assigned to an expression, MATLAB automatically stores the result in ans.
eps	Floating-point relative accuracy. This is the distance between 1.0 and the next largest number representable using double-precision floating point.
i (or j)	Imaginary unit, as in 1 + 2i.
NaN or nan	Stands for Not-a-Number (e.g., 0/0).
pi	3.14159265358979
realmax	The largest floating-point number that your computer can represent.
realmin	The smallest positive floating-point number that your computer can represent.
computer	Your computer type.
version	Version number for MATLAB.
ver	Version information for all installed MATLAB products.

TABLE 2.10 Some important functions and constants.

Formats

The format function, with the following forms



```
format
format type
format('type')
```

is used to control how numerical data is displayed in the Command Window (only the display is affected, not how MATLAB computes and stores numerical data). The first form changes the output format to the default appropriate for the class of data being used; the second changes the format to the specified *type*; and the third form is the function form of the syntax. Table 2.11 shows the format types of interest in this book, and the following examples illustrate their use by displaying pi in various formats.

To determine the format currently in use, we write

```
get
```

The syntax get(0, 'Format') returns the type of Format currently in use (see Table 2.12). Also, see Section 7.4 for another syntax form of function get.

```
>> get(0,'Format')
ans =
    short
```

When the format is set to short, both pi and single(pi) display as 5-digit values:

```
>> pi
ans =
3.1416
>> single(pi)
ans =
3.1416
```

If we set the format to long, then

To use exponential notation we type

Туре	Result
short	Scaled fixed point format, with 4 digits after the decimal point. For example, 3.1416.
long	Scaled fixed point format with 14 to 15 digits after the decimal point for double, and 7 digits after the decimal point for single. For example, 3.141592653589793.
short e	Floating point format, with 4 digits after the decimal point. For example, 3.1416e+000.
long e	Floating point format, with 14 to 15 digits after the decimal point for double, and 7 digits after the decimal point for single. For example, 3.141592653589793e+000.
short g	Best (in terms of shorter output) of fixed or floating point, with 4 digits after the decimal point. For example, 3.1416.
long g	Best (in terms of shorter output) of fixed or floating point, with 14 to 15 digits after the decimal point for double, and 7 digits after the decimal point for single. For example, 3.14159265358979.
short eng	Engineering format that has 4 digits after the decimal point, and a power that is a multiple of three. For example, 3.1416e+000.
long eng	Engineering format that has exactly 16 significant digits and a power that is a multiple of three. For example, 3.14159265358979e+000.

TABLE 2.11 Format types. The examples are based on constant рi.

```
>> format short e
>> pi
ans =
    3.1416e+000
```

or, we could have used the function form of the syntax:

```
>> format('short', 'e')
```

and the result would have been the same. As an exercise, you should look up the help page for function format and experiment with the other format types.

2.10.3 Flow Control

The ability to control the flow of operations based on a set of predefined conditions is at the heart of all programming languages. In fact, conditional branching was one of two key developments that led to the formulation of general-purpose computers in the 1940s (the other development was the use of memory to hold stored programs and data). MATLAB provides the eight flow control statements summarized in Table 2.12. Keep in mind the observation made in the previous section that MATLAB treats a logical 1 or nonzero number as true, and a logical or numeric 0 as false.

TABLE 2.12 Flow control statements.

Statement	Description
if	if, together with else and elseif, executes a group of statements based on a specified logical condition.
for	Executes a group of statements a fixed (specified) number of times.
while	Executes a group of statements an indefinite number of times, based on a specified logical condition.
break	Terminates execution of a for or while loop.
continue	Passes control to the next iteration of a for or while loop, skipping any remaining statements in the body of the loop.
switch	switch, together with case and otherwise, executes different groups of statements, depending on a specified value or string.
return	Causes execution to return to the invoking function.
trycatch	Changes flow control if an error is detected during execution.

if, else, and elseif

Conditional statement if has the syntax

if expression statements end

As discussed in connection with Table 2.7, logical AND and OR operators appearing inside expression should be the scalar logical operators && and ||.

The expression is evaluated and, if the evaluation yields true, MATLAB executes one or more commands, denoted here as statements, between the if and end lines. If expression is false, MATLAB skips all the statements between the if and end lines and resumes execution at the line following the end line. When nesting ifs, each if must be paired with a matching end.

The else and elseif statements further conditionalize the if statement. The general syntax is

if expression1
 statements1
elseif expression2
 statements2
else
 statements3
end

If expression1 is true, statements1 are executed and control is transferred to the end statement. If expression1 evaluates to false, then expression2 is evaluated. If this expression evaluates to true, then statements2 are executed and control is transferred to the end statement. Otherwise (else) statements3 are executed. Note that the else statement has no condition.

The else and elseif statements can appear by themselves after an if statement; they do not need to appear in pairs, as shown in the preceding general syntax. It is acceptable to have multiple elseif statements.

Suppose that we want to write a function that computes the average intensity of an image. As explained in Section 2.8.3, a two-dimensional array f can be converted to a column vector, v, by letting v = f(:). Therefore, we want our function to be able to work with both vector and image inputs. The program should produce an error if the input is not a one- or two-dimensional array.

EXAMPLE 2.10: Conditional branching.

```
function av = average(A)
%AVERAGE Computes the average value of an array.
    AV = AVERAGE(A) computes the average value of input A,
    which must be a 1-D or 2-D array.
%
    Check the validity of the input.
if ndims(A) > 2
   error('The dimensions of the input cannot exceed 2.')
end
% Compute the average
av = sum(A(:))/length(A(:));
```

Note that the input is converted to a 1-D array by using A(:). In general, length(A) returns the size of the longest dimension of an array, A. In this example, because A(:) is a vector, length(A) gives the number of elements of A. This eliminates the need for a separate test to determine whether the input is a vector or a 2-D array. Another way to obtain the number of elements in an array directly is to use function numel, whose syntax is

```
n = numel(A)
```



Thus, if A is an image, numel (A) gives its number of pixels. Using this function, the last line of the previous program becomes

```
av = sum(A(:))/numel(A);
```

Finally, note that the error function terminates execution of the program and outputs the message contained within the parentheses (the quotes shown are required).

for

A for loop executes a group of statements a specified number of times. The syntax is

```
for index = start:increment:end
   statements
end
```

It is possible to nest two or more for loops, as follows:

```
for index1 = start1:increment1:end
    statements1
    for index2 = start2:increment2:end
        statements2
    end
    additional loop1 statements
end
```

For example, the following loop executes 11 times:

```
count = 0;
for k = 0:2:20
    count = count + 1;
end
```

If the loop increment is omitted, it is taken to be 1. Loop increments also can be negative, as in k = 0:-1:-10. Note that no semicolon is necessary at the end of a for line. MATLAB automatically suppresses printing the values of a loop index. As discussed in detail in Section 2.10.5, improvements in program execution speed sometimes can be achieved by replacing for loops with so-called *vectorized code* whenever possible.

EXAMPLE 2.11: Using a for loop to write multiple images to file.

Example 2.2 compared several images using different JPEG quality values. Here, we show how to write those files to disk using a for loop. Suppose that we have an image, f, and we want to write it to a series of JPEG files with quality factors ranging from 0 to 100 in increments of 5. Further, suppose that we want to write the JPEG files with filenames of the form series_xxx.jpg, where xxx is the quality factor. We can accomplish this using the following for loop:

```
for q = 0:5:100
   filename = sprintf('series_%3d.jpg', q);
   imwrite(f, filename, 'quality', q);
end
```

Function sprintf, whose syntax in this case is



See the help page for sprintf for other useful syntax forms.

```
s = sprintf('characters1%ndcharacters2', q)
```

writes formatted data as a string, s. In this syntax form, characters1 and characters2 are character strings, and %nd denotes a decimal number (specified by q) with n digits. In this example, characters1 is series_, the value of n is 3, characters2 is . jpg, and q has the values specified in the loop.

while

A while loop executes a group of statements for as long as the expression controlling the loop is true. The syntax is

```
while expression
   statements
end
```

As with the if statement, logical AND and OR operators appearing inside expression should be the scalar logical operators && and ||. As in the case of for, while loops can be nested:

```
while expression1
   statements1
   while expression2
      statements2
   end
   additional loop1 statements
end
```

For example, the following nested while loops terminate when both a and b have been reduced to 0:

```
a = 10;
b = 5;
while a
   a = a - 1;
   while b
      b = b - 1;
   end
end
```

Note that to control the loops we used MATLAB's convention of treating a numerical value in a logical context as true when it is nonzero and as false when it is 0. In other words, while a and while b evaluate to true as long as a and b are nonzero. As in the case of for loops, gains in program execution speed sometimes can be achieved by replacing while loops with vectorized code (Section 2.10.5).

break

As its name implies, break terminates the execution of a for or while loop. When a break statement is encountered, execution continues with the next statement outside the loop. In nested loops, break exits only from the innermost loop that contains it.

continue

The continue statement passes control to the next iteration of the for or while loop in which it appears, skipping any remaining statements in the body of the loop. In nested loops, continue passes control to the next iteration of the innermost loop enclosing it.

switch

This is the statement of choice for controlling the flow of an M-function based on different types of inputs. The syntax is

```
switch switch_expression
   case case_expression
     statement(s)
   case {case_expression1, case_expression2,...}
     statement(s)
   otherwise
     statement(s)
end
```

The switch construct executes groups of statements based on the value of a variable or expression. The keywords case and otherwise delineate the groups. Only the first matching case is executed. There must always be an end to match the switch statement. The curly braces are used when multiple expressions are included in the same case statement. As an example, suppose that an M-function accepts an image f and converts it to a specified class, call it newclass. Only three image classes are acceptable for the conversion: uint8, uint16, and double. The following code fragment performs the desired conversion and outputs an error if the class of the input image is not one of the acceptable classes:

```
switch newclass
  case 'uint8'
    g = im2uint8(f);
  case 'uint16'
    g = im2uint16(f);
  case 'double'
    g = im2double(f);
  otherwise
    error('Unknown or improper image class.')
end
```

The switch construct is used extensively throughout the book.

EXAMPLE 2.12: Extracting a subimage from a given image.

■ In this example we write an M-function (based on for loops) to extract a rectangular subimage from an image. Although we could do the extraction using a single MATLAB statement (do it as an exercise after you read about vectorized code in Section 2.10.5), the objective here is to illustrate for loops. The inputs to the function are an image, the size (number of rows and columns) of the subimage we want to extract, and the coordinates of the top, left corner of the subimage. Keep in mind that the image origin in MATLAB

[†]Unlike the C language switch construct, MATLAB's switch does not "fall through." That is, switch executes only the first matching case; subsequent matching cases do not execute. Therefore, break statements are not used.

is at (1, 1), as discussed in Section 2.1.1.

```
function s = subim(f, m, n, rx, cy)
%SUBIM Extracts a subimage, s, from a given image, f.
    The subimage is of size m-by-n, and the coordinates of its top,
    left corner are (rx, cy).
s = zeros(m, n);
for r = 1:m
   for c = 1:n
      s(r,c) = f(r + rx - 1, c + cy - 1);
   end
end
```

As an exercise, you should implement the preceding program using while, instead of for, loops.

2.10.4 Function Handles

A function handle is a MATLAB data type that contains information used in referencing a function. One of the principal advantages of using function handles is that you can pass a function handle as an argument in a call to another function. As you will see in the next section, the fact that a function handle carries all the information needed for MATLAB to evaluate the function can lead to simpler program implementation. Function handles also can improve performance in repeated operations, and, in addition to being passed to other functions, they can be saved in data structures or files for later use.

There are two different types of function handles, both of which are created using the function handle operator, @. The first function handle type is the named (also called simple) function handle. To create a named function handle, follow the @ operator with the name of the desired function. For example:



```
>> f = @sin
f =
    @sin
```

Function sin can be called indirectly by calling the function handle, f:

```
>> f(pi/4)
ans =
    0.7071
>> sin(pi/4)
ans =
    0.7071
```

The second function handle type is the anonymous function handle, which is formed from a MATLAB expression instead of a function name. The general format for constructing an anonymous function is:

For example, the following anonymous function handle squares its input:

$$>> g = @(x) x.^2;$$

and the following handle computes the square root of the sum of two squared variables:

$$>> r = @(x, y) sqrt(x.^2 + y.^2);$$

Anonymous function handles can be called just like named function handles:

Many MATLAB and Image Processing Toolbox functions take function handles as input arguments. For instance, the quad function performs numerical integration. The function to be integrated is specified by passing a function handle as an input argument to quad. For example, the following statement computes the definite integral of the sin function over the interval [0, pi/4] (recall from the discussion above that f = @sin):

```
>> quad(f, 0, pi/4)
ans =
    0.2929
```

where f is as defined above. Anonymous function handles can be passed to other functions in exactly the same manner. The following statement computes the definite integral of x^2 over the interval [0, 1]:

```
\rightarrow quad(g, 0, 1)
ans =
     0.3333
```

where g is as defined above. We give additional examples of function handles in the following section and in later chapters.



Function quad performs numerical integration using an adaptive Simpson quadrature approach.

2.10.5 Code Optimization

As discussed in some detail in Section 1.3, MATLAB is a programming language designed specifically for array operations. Taking advantage of this fact whenever possible can result in significant increases in computational speed. In this section we discuss two important approaches for MATLAB code optimization: preallocating arrays and vectorizing loops.

Preallocating Arrays

0

Preallocation refers to initializing arrays before entering a for loop that computes the elements of the array. To illustrate why preallocation can be important, we start with a simple experiment. Suppose that we want to create a MATLAB function that computes

$$f(x) = \sin(x/100\pi)$$

for x = 0, 1, 2, ..., M - 1. Here is our first version of the function:

```
function y = sinfun1(M)
x = 0:M - 1;
for k = 1:numel(x)
   y(k) = \sin(x(k) / (100*pi));
end
The output for M = 5 is
>> sinfun1(5)
ans =
```

MATLAB functions tic and toc can be used to measure how long a function takes to execute. We call tic, then call the function, and then call toc:

0.0095 0.0127

```
>> tic; sinfun1(100); toc
Elapsed time is 0.001205 seconds.
```

0.0032 0.0064



(If you type the preceding three statements in separate lines, the time measured will include the time required for you to type the second two lines.)

Timing functions using calls as in the preceding paragraph can produce large variations in the measured time, especially when done at the command prompt. For example, repeating the previous call gives a different result:

```
>> tic; sinfun1(100); toc
Elapsed time is 0.001197 seconds.
```

Function timeit can be used to obtain reliable, repeatable time measurements of function calls. The calling syntax for timeit † is

timeit

```
s = timeit(f)
```

where f is a function handle for the function to be timed, and s is the measured time, in seconds, required to call f. The function handle f is called with no input arguments. We can use timeit as follows to time sinfun1 for M = 100:

```
>> M = 100;
>> f = @() sinfun1(M);
>> timeit(f)
ans =
    8.2718e-005
```

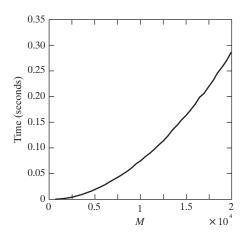
This call to function timeit is an excellent illustration of the power of the concept of function handles introduced in the previous section. Because it accepts a function handle with no inputs, function timeit is *independent* of the parameters of the function we wish to time. Instead, we delegate that task to the creation of the function handle itself. In this case, only one parameter, M, was necessary. But you can imagine more complex functions with numerous parameters. Because a function handle stores all the information needed to evaluate the function for which it is defined, it is possible for timeit to require a single input, and yet be capable of timing any function, independently of its complexity or number of parameters. This is a very useful programming feature.

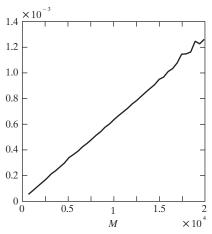
Continuing with our experiment, we use timeit to measure how long sinfun1 takes for M = 500, 1000, 1500, ..., 20000:

```
M = 500:500:20000;
for k = 1:numel(M)
    f = @() sinfun1(M(k));
    t(k) = timeit(f);
end
```

Although we might expect the time required to compute sinfun1(M) to be proportional to M, Fig. 2.8(a) shows that the time required actually grows as a function of M^2 instead. The reason is that in sinfun1.m the output variable y grows in size by one element each time through the loop. MATLAB can handle this implicit array growth automatically, but it has to reallocate new memory space and copy the previous array elements every time the array grows. This frequent memory reallocation and copying is expensive, requiring much more time than the sin computation itself.

[†]It is not practical to provide a listing of function timeit in the book because this function contains hundreds of tedious, repeated lines of code designed to accurately determine time-measurement overhead. You can obtain a listing from: http://www.mathworks.com/matlabcentral/fileexchange/18798.





a b

FIGURE 2.8

(a) Approximate execution times for function sinfun1 as a function of M. (b) Approximate times for function sinfun2. The glitches were caused by interval variations in memory paging. The time scales in (a) and (b) are different.

The solution to this performance problem is suggested by the MATLAB Editor, which reports for sinfun1.m that:

'y' might be growing inside a loop. Consider preallocating for speed.

Preallocating y means initializing it to the expected output size before beginning the loop. Usually, preallocation is done using a call to function zeros (see Section 2.9). Our second version of the function, sinfun2.m, uses preallocation:

As mentioned in Section 1.7.1, the MATLAB editor analyzes code and makes improvement suggestions. In the case of sinfun1, the y inside the for loop would be shown underlined in red. Putting the cursor over y would display the message shown here.

```
function y = sinfun2(M)
x = 0:M-1;
y = zeros(1, numel(x));
for k = 1:numel(x)
   y(k) = \sin(x(k) / (100*pi));
end
```

Compare the time required for sinfun1(20000) and sinfun2(20000):

```
>> timeit(@() sinfun1(20000))
ans =
    0.2852
>> timeit(@() sinfun2(20000))
ans =
    0.0013
```

Execution times depend on the machine used. The important quantity here is the *ratio* of the execution times. The version using preallocation runs about 220 times faster. Figure 2.8(b) shows that the time required to run sinfun2 is proportional to M. [Note that the time scale is different for Figs. 2.8(a) and (b).]

Vectorizing Loops

Vectorization in MATLAB refers to techniques for eliminating loops altogether, using a combination of matrix/vector operators, indexing techniques, and existing MATLAB or toolbox functions. As an example, we revisit the sinfun functions discussed in the previous section. Our third version of sinfun exploits the fact that sin can operate elementwise on an array input, not just on a scalar input. Function sinfun3 has no for loops:

```
function y = sinfun3(M)

x = 0:M-1;

y = sin(x ./ (100*pi));
```

In older versions of MATLAB, eliminating loops by using matrix and vector operators almost always resulted in significant increases in speed. However, recent versions of MATLAB can compile simple for loops automatically, such as the one in sinfun2, to fast machine code. As a result, many for loops that were slow in older versions of MATLAB are no longer slower than the vectorized versions. We can see here, in fact, that sinfun3, with no loops, runs at about the same speed as sinfun2, which has a loop:

```
>> timeit(@() sinfun2(20000))
ans =
          0.0013
>> timeit(@() sinfun3(20000))
ans =
          0.0018
```

As the following example shows, gains in speed still are possible using vectorization, but the gains are not as dramatic as they used to be in earlier versions of MATLAB.

EXAMPLE 2.13: An illustration of vectorization, and introduction of function meshgrid.

■ In this example, we write two versions of a MATLAB function that creates a synthetic image based on the equation:

$$f(x,y) = A\sin(u_0x + v_0y)$$

The first function, twodsin1, uses two nested for loops to compute f:

```
function f = twodsin1(A, u0, v0, M, N)
f = zeros(M, N);
for c = 1:N
    v0y = v0 * (c - 1);
    for r = 1:M
```

```
u0x = u0 * (r - 1);
      f(r, c) = A*sin(u0x + v0y);
   end
end
```

Observe the preallocation step, f = zeros (M, N), before the for loops. We use timeit to see how long this function takes to create a sinusoidal image of size 512×512 pixels:

```
>> timeit(@() twodsin1(1, 1/(4*pi), 1/(4*pi), 512, 512))
   0.0471
```

Without preallocation, this function would run approximately 42 times slower, taking 1.9826 s to execute with the same input parameters.

We can display the resulting image using the auto-range syntax ([]) of imshow:

```
\Rightarrow f = twodsin1(1, 1/(4*pi), 1/(4*pi), 512, 512);
>> imshow(f, [ ])
```

Figure 2.9 shows the result.

In our second version of the function, we vectorize it (that is, we rewrite it without using for loops) by using a very useful MATLAB function called meshgrid, with syntax

```
[C, R] = meshgrid(c, r)
```

The input arguments c and r are vectors of horizontal (column) and vertical (row) coordinates, respectively (note that columns are listed *first*). Function meshgrid transforms the coordinate vectors into two arrays C and R that can be used to compute a function of two variables. For example, the following



As detailed in help, meshgrid has a 3-D formulation useful for evaluating functions of three variables and for constructing volumetric plots.



FIGURE 2.9 Sinusoidal image generated in Example 2.13.

commands use meshgrid to evaluate the function z = x + y for integer values of x ranging from 1 to 3, and for integer values of y ranging from 10 to 14:

```
>> [X, Y] = meshgrid(1:3, 10:14)
X =
    1
         2
             3
         2
             3
    1
    1
         2
             3
    1
         2
             3
    1
         2
             3
Y =
    10
          10
                10
    11
          11
                11
    12
          12
                12
    13
          13
                13
    14
          14
                14
>> Z = X + Y
Z =
    11
          12
                13
    12
          13
               14
    13
          14
                15
    14
          15
               16
                17
    15
          16
```

Finally, we use meshgrid to rewrite the 2-D sine function without loops:

```
function f = twodsin2(A, u0, v0, M, N)
r = 0:M - 1; % Row coordinates.
c = 0:N - 1; % Column coordinates.
[C, R] = meshgrid(c, r);
f = A*sin(u0*R + v0*C);
```

As before, we use timeit to measure its speed:

```
>> timeit(@() twodsin2(1, 1/(4*pi), 1/(4*pi), 512, 512))
ans =
    0.0126
```

The vectorized version takes roughly 50% less time to run.

[†]Function meshgrid assumes that x and y are horizontal (column) and vertical (row) coordinates, respectively. This is an example of the comments in Section 2.1.1 regarding the fact that MATLAB and the Image Processing Toolbox sometimes use different coordinate system conventions.

Because each new release of MATLAB tends to have improved ability to run loops faster, it is difficult to give general guidelines about when to vectorize MATLAB code. For many mathematically trained users who are familiar with matrix and vector notation, vectorized code is often more readable (it looks more "mathematical") than code based on loops. For example, compare this line from function twodsin2:

```
f = A*sin(u0*R + v0*C):
```

with these lines from twodsin1 for performing the same operation:

```
for c = 1:N
   v0y = v0*(c - 1);
   for r = 1:M
      u0x = u0 * (r - 1);
      f(r, c) = A*sin(u0x + v0y);
    end
end
```

Clearly the first formulation is more concise, but the mechanics of what actually is taking place are clearer in the second.

One should strive first to write code that is correct and understandable. Then, if the code does not run fast enough, use the MATLAB Profiler (see Section 1.7.1) to identify possible performance trouble spots. If any of these trouble spots are for loops, make sure that there are no preallocation issues and then consider using vectorization techniques. The MATLAB documentation contains further guidance about performance; search the documentation for the section titled "Techniques for Improving Performance."

2.10.6 Interactive I/O

In this section we establish a foundation for writing interactive M-functions that display information and instructions to users and accept inputs from a keyboard.

Function disp is used to display information on the screen. Its syntax is

```
disp(argument)
```



If argument is an array, disp displays its contents. If argument is a text string, then disp displays the characters in the string. For example,

```
>> A = [1 2; 3 4];
>> disp(A)
    1
        2
    3
        4
>> sc = 'Digital Image Processing.';
>> disp(sc)
```

```
Digital Image Processing.
>> disp('This is another way to display text.')
This is another way to display text.
```

Note that only the contents of argument are displayed, without words such as ans =, which we are accustomed to seeing on the screen when the value of a variable is displayed by omitting a semicolon at the end of a command line.

Function input is used for inputting data into an M-function. The basic syntax is



```
t = input('message')
```

This function outputs the words contained in message and waits for an input from the user, followed by a **Return** (Enter), and stores the input in t. The input can be a single number, a character string (enclosed by single quotes), a vector (enclosed by square brackets and elements separated by spaces or commas), a matrix (enclosed by square brackets and rows separated by semicolons), or any other valid MATLAB data structure. For example,

```
>> t = input('Enter your data:')
Enter your data: 25
t =
    25
>> class(t)
ans =
    double
>> t = input('Enter your data:')
Enter your data: 'abc'
t =
    abc
>> class(t)
ans =
    char
>> t = input('Enter your data:')
Enter your data: [0 1 2 3]
t =
    0
        1
            2
                3
>> size(t)
ans =
    1
        4
```

If the entries are a mixture of characters and numbers, then we use one of MATLAB's string processing functions. Of particular interest in the present discussion is function strread, which has the syntax

```
[a, b, c, ...] = strread(cstr, 'format', 'param', 'value')
```



This function reads data from the character string cstr, using a specified format and param/value combinations. In this chapter the formats of interest are %f and %g, to denote floating-point numbers and character strings, respectively. For param we use delimiter to denote that the entities identified in format will be delimited by a character specified in value (typically a comma or space). For example, suppose that we have the string

```
See the help page for
strread for a list of
the numerous syntax
forms applicable to this
```

```
>> t = '12.6, x2y, z';
```

To read the elements of this input into three variables a, b, and c, we write

```
>> [a, b, c] = strread(t, '%f%q%q', 'delimiter', ',')
    12,6000
b =
    'x2v'
C =
    ' Z '
```

Output a is of class double. The quotes around outputs x2y and z indicate that b and c are cell arrays, which are discussed in the next section. We convert them to character arrays simply by letting

```
>> d = char(b)
d =
    x2y
```



and similarly for c. The number (and order) of elements in the format string must match the number and type of expected output variables on the left. In this case we expect three inputs: one floating-point number followed by two character strings.

Function strcmp is used to compare strings. For example, suppose that we wish to write an M-function, g = imnorm(f, param), that accepts an image, f, and a parameter param than can have one of two forms: 'norm1' and 'norm255'. In the first instance, f is to be scaled to the range [0, 1]; in the second, it is to be scaled to the range [0, 255]. The output should be of class double in both cases. The following code fragment accomplishes the required normalization:



Function strcmp compares two stringsand returns a logical true (1) if the strings are equal or a logical false (0) if they are not.

```
f = mat2gray(f);
if strcmp(param, 'norm1')
   g = f;
elseif strcmp(param, 'norm255')
    g = 255*f;
else
   error('Unknown value of param.')
end
```

An error would occur if the value specified in param is not 'norm1' or 'norm255'. Also, an error would be issued if other than all lowercase characters are used for either normalization factor. We can modify the function to accept either lower or uppercase characters by using function strcmpi, which performs case-insensitive string comparisons.

strcmpi

2.10.7 An Introduction to Cell Arrays and Structures

We conclude this chapter with a discussion of cell arrays and structures. As you will see in subsequent chapters, are used extensively in M-function programming.

Cell arrays

Cell arrays provide a way to combine a mixed set of objects (e.g., numbers, characters, matrices, other cell arrays) under one variable name. For example, suppose that we are working with (1) an uint8 image, f, of size 512×512 pixels; (2) a sequence of 2-D coordinates in the form of rows of a 188×2 array, b; and (3) a cell array containing two character names, char array = {'area', 'centroid'} (curly braces are used to enclose the contents of a cell array). These three dissimilar entities can be organized into a single variable, C, using cell arrays:

```
C = \{f, b, char array\}
```

Typing C at the prompt would output the following results:

```
>> C
C =
   [512x512 uint8]
                     [188x2 double]
                                      {1x2 cell}
```

In other words, the outputs shown are not the values of the various variables, but a description of some of their properties instead. To see the complete contents of an element of the cell, we enclose the numerical location of that element in curly braces. For instance, to see the contents of char array we type

```
>> C{3}
ans =
    'area' 'centroid'
```

or we can use function celldisp:

```
>> celldisp(C{3})
ans{1} =
    area
ans{2} =
    centroid
```



Using parentheses instead of curly braces on an element of C gives a description of the variable:

```
>> C(3)
ans =
    {1x2 cell}
```

We can work with specified contents of a cell array by transferring them to a numeric or other pertinent form of array. For instance, to extract f from C we use

```
>> f = C\{1\};
```

Function size gives the size of a cell array:

```
>> size(C)
ans =
    1
         3
```

Function cellfun, with syntax

```
D = cellfun('fname', C)
```



applies the function fname to the elements of cell array C and returns the results in the double array D. Each element of D contains the value returned by fname for the corresponding element in C. The output array D is the same size as the cell array C. For example,

```
>> D = cellfun('length', C)
D =
    512
           188
                   2
```

In other words, length(f) = 512, length(b) = 188 and length(char array) = 2. Recall from Section 2.10.3 that length (A) gives the size of the longest dimension of a multidimensional array A.

Finally, we point out that cell arrays contain *copies* of the arguments, not pointers to those arguments. Thus, if any of the arguments of C in the preceding example were to change after C was created, that change would not be reflected in C.

EXAMPLE 2.14: Using cell arrays.

Suppose that we want to write a function that outputs the average intensity of an image, its dimensions, the average intensity of its rows, and the average intensity of its columns. We can do it in the "standard" way by writing a function of the form

```
mean2
mean
```

mean2(A) computes the mean (average) value of the elements of the 2-D array A.

If v is a vector, mean(v) returns the mean value of the elements of v. If A is a matrix, mean (A) treats the columns of A as vectors, returning a row vector of mean values. If A is a multidimensional array, mean(A, dim) returns the mean value of the elements along the dimension specified by scalar dim.

```
function [AI, dm, AIrows, AIcols] = image_stats(f)
dm = size(f);
AI = mean2(f);
AIrows = mean(f, 2);
AIcols = mean(f, 1);
```

where f is the input image and the output variables correspond to the quantities just mentioned. Using cells arrays, we would write

```
function G = image stats(f)
G\{1\} = size(f);
G\{2\} = mean2(f);
G{3} = mean(f, 2);
G{4} = mean(f, 1);
```

Writing $G(1) = \{size(f)\}\$, and similarly for the other terms, also is acceptable. Cell arrays can be multidimensional. For instance, the previous function could be written also as

```
function H = image stats2(f)
H(1, 1) = {size(f)};
H(1, 2) = \{mean2(f)\};
H(2, 1) = \{mean(f, 2)\};
H(2, 2) = \{mean(f, 1)\};
```

Or, we could have used $H\{1,1\} = size(f)$, and so on for the other variables. Additional dimensions are handled in a similar manner.

Suppose that f is of size 512×512 . Typing G and H at the prompt would give

```
>> G = image stats(f);
>> G
G =
                          [512x1 double]
    [1x2 double]
                   [1]
                                            [1x512 double]
>> H = image stats2(f);
>> H
```

```
H =
      1x2 double]
                                   1]
    [512x1 double]
                       [1x512 double]
```

If we want to work with any of the variables contained in G, we extract it by addressing a specific element of the cell array, as before. For instance, if we want to work with the size of f, we write

```
>> v = G\{1\}
or
>> v = H\{1,1\}
```

where v is a 1×2 vector. Note that we did not use the familiar command $[M, N] = G\{1\}$ to obtain the size of the image. This would cause an error because only functions can produce multiple outputs. To obtain M and N we would use M = v(1) and N = v(2).

The economy of notation evident in the preceding example becomes even more obvious when the number of outputs is large. One drawback is the loss of clarity in the use of numerical addressing, as opposed to assigning names to the outputs. Using structures helps in this regard.

Structures

Structures are similar to cell arrays in that they allow grouping of a collection of dissimilar data into a single variable. However, unlike cell arrays, in which cells are addressed by numbers, the elements of structures are addressed by user-defined names called fields.

Continuing with the theme of Example 2.14 will clarify these concepts. Using structures, we write

EXAMPLE 2.15: Using structures.

```
function s = image_stats(f)
s.dm = size(f);
s.AI = mean2(f);
s.AIrows = mean(f, 2);
s.AIcols = mean(f, 1);
```

where s is a structure. The fields of the structure in this case are dm (a 1×2 vector), AI (a scalar), AIrows (an $M \times 1$ vector), and AIcols (a $1 \times N$ vector), where M and N are the number of rows and columns of the image. Note the use of a dot to separate the structure from its various fields. The field names are arbitrary, but they must begin with a nonnumeric character.

Using the same image as in Example 2.14 and typing s and size(s) at the prompt gives the following output:

Note that s itself is a scalar, with four fields associated with it in this case.

We see in this example that the logic of the code is the same as before, but the organization of the output data is much clearer. As in the case of cell arrays, the advantage of using structures would become even more evident if we were dealing with a larger number of outputs.

The preceding illustration used a single structure. If, instead of one image, we had Q images organized in the form of an $M \times N \times Q$ array, the function would become

```
function s = image_stats(f)
K = size(f);
for k = 1:K(3)
    s(k).dim = size(f(:, :, k));
    s(k).AI = mean2(f(:, :, k));
    s(k).AIrows = mean(f(:, :, k), 2);
    s(k).AIcols = mean(f(:, :, k), 1);
end
```

In other words, structures themselves can be indexed. Although, as with cell arrays, structures can have any number of dimensions, their most common form is a vector, as in the preceding function.

Extracting data from a field requires that the dimensions of both s and the field be kept in mind. For example, the following statement extracts all the values of AIrows and stores them in v:

```
for k = 1:length(s)
    v(:, k) = s(k).AIrows;
end
```

Note that the colon is in the first dimension of v and that k is in the second because s is of dimension $1 \times Q$ and AI rows is of dimension $M \times 1$. Thus, because k goes

from 1 to O, v is of dimension $M \times O$. Had we been interested in extracting the values of AIcols instead, we would have used v(k,:) in the loop.

Square brackets can be used to extract the information into a vector or matrix if the field of a structure contains scalars. For example, suppose that D. Area contains the area of each of 20 regions in an image. Writing

```
>> w = [D.Area];
```

creates a 1×20 vector w in which each element is the area of one of the regions.

As with cell arrays, when a value is assigned to a structure field, MATLAB makes a copy of that value in the structure. If the original value is changed at a later time, the change is not reflected in the structure.

Summary

The material in this chapter is the foundation for the discussions that follow. At this point, you should be able to retrieve an image from disk, process it via simple manipulations, display the result, and save it to disk. It is important to note that the key lesson from this chapter is how to combine MATLAB and Image Processing Toolbox functions with programming constructs to generate solutions that expand the capabilities of those functions. In fact, this is the model of how material is presented in the following chapters. By combining standard functions with new code, we show prototypic solutions to a broad spectrum of problems of interest in digital image processing.