Raven Solutions

Create trust with potential borrowers.

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Executive Summary.

Objectives

This report's objective is to provide comprehensive information to help first-time homebuyers understand the intricacies of a mortgage. This includes a detailed breakdown of the true costs involved, a payment chart for reference, and vital information about interest rates. By reviewing this report with a qualified financial planner, prospective homebuyers will be better equipped to make informed and financially responsible decisions when it comes to purchasing a home.

Goals

The goal of this document will be to display the details of a home loan in a way that borrowers are left satisfied that their dream of owning a home is in the best hands. Raven Solutions' visuals and thorough breakdown of the home loan experience will clearly explain what clients owe and how it is calculated. Raven Solutions will be able to show what this institution can offer potential clients and ease their minds of the common concerns that arise when making such a substantial investment. We intend to show our presentation of the home loans offered will give this institution a clear advantage over its competitors.

Conclusions

Raven Solutions will deliver the visuals and explanations needed for your loan officers to show this institution is the best fit for your clients' needs. With the assistance of Raven Solutions, anyone who walks into your office will leave with a clear understanding of how their money will be utilized to achieve their goal of owning a home.

Introduction.

Problem Identification.

Buying a home can be an intimidating experience. The process can be confusing and become overwhelming for many first-time homebuyers. Clients are often exposed to new terminology and processes that are not always initially understandable. A financial institution able to define and present these confusing topics clearly will have a leg up over the competition. Customers will always have questions regarding how their money is used. How is my monthly payment calculated? How does the interest rate affect my overall cost? What is the best mortgage to take? With this helpful guide, every client will have their questions answered and be on their way to settling into their new home.

Abstraction of problem.

Consumers have a variety of options when choosing the type of mortgage, they need to purchase a home. To help them navigate their decision, we will use data from the ***San Diego County Credit Union*** *website. SDCCD offers 10- and 30-year fixed rate mortgages at 6.535% and 6.206%. Often these numbers mean little to the average homebuyer. The higher interest rate for a shorter loan length versus a lower interest rate for a longer loan length needs to be explained in an elementary manner so the client feels they have a solid understanding of what they are getting into. The consumer may not know that they will pay more for a 30-year loan than a 10-year loan. But their monthly payment will be less and may fit into their budget better than the higher payment of a 10-year loan.*

Data and methods.

To help the client understand the big picture, let us breakdown how their payment is calculated piece by piece. The following formula is used to calculate the monthly payment.

$$x = \frac{P(1+r)^n r}{(1+r)^n - 1}$$

- *Total loan amount =* P
- *Monthly interest rate =* r
- *Length of mortgage =* n *(in months)*
- *Monthly payment =* x

It is important to note that the length of the mortgage is calculated in months. The Annual Interest Rate is advertised but this formula uses the monthly interest rate. For instance, a 30-year loan will have 360 months' worth of payments. The APR is converted to monthly interest by:

$$r_n = APR,\ r = \frac{r_n}{12}$$

The formula to calculate how much of the mortgage is left after k number of months is:

$$P_k = P(1+r)^k + x \cdot \frac{1-(1+r)^k}{r}$$

- *Principle after k months =* P_k

Derivation of formulas.

Let us breakdown how these formulas are derived. As before, let P be the total amount borrowed. A month after the loan is accepted, the amount of loan left after your first payment is calculated by

$$P_1 = P + P \cdot r - x = P(1+r) - x$$

Essentially this formula is the principal, plus interest, minus your payment. The second month is calculated by

$$\begin{aligned} P_2 &= P_1(1+r) - x \\ &= (P(1+r) - x)(1+r) - x \\ &= P(1+r)2 - (1+r)x - x \end{aligned}$$

The third month,

$$\begin{aligned} P_3 &= P_2(1+r) - x \\ &= P_2(1+r) - x \\ &= (P(1+r)^3 - (1+r)x - x)(1+r) - x \end{aligned}$$

After the kth month,

$$P_k = P(1+r)^k - (1+r)^{(k-1)}x - \cdots - (1+r)^2 x - (1+r)x - x$$

$$= P(1+r)^k - x \cdot \frac{1-(1+r)^k}{1-(1+r)}$$

$$= P(1+r)^k - x \cdot \frac{1-(1+r)^k}{r}$$

Note that when $k = n$ (the length of the loan), the payments are over and $P_n = 0.\ 0 = P(1+r)^n + x \cdot \frac{(1-(1+r)^n)}{r}$

Solving this equation for x, we discover the equation for calculating monthly payment.

$$x = \frac{P(1+r)^n r}{(1+r)^n - 1}$$

Results.

Calculations

Now, let's try to use these formulas with our data. The first loan we want to calculate is a 10-year loan of $400,000 with an APR of 6.535%. Then our variables become,

$$P = 400{,}000$$

$$n = 10\,years \cdot 12\,\frac{months}{year} = 120\,months$$

$$r = \frac{0.06535}{12} = 0.005445$$

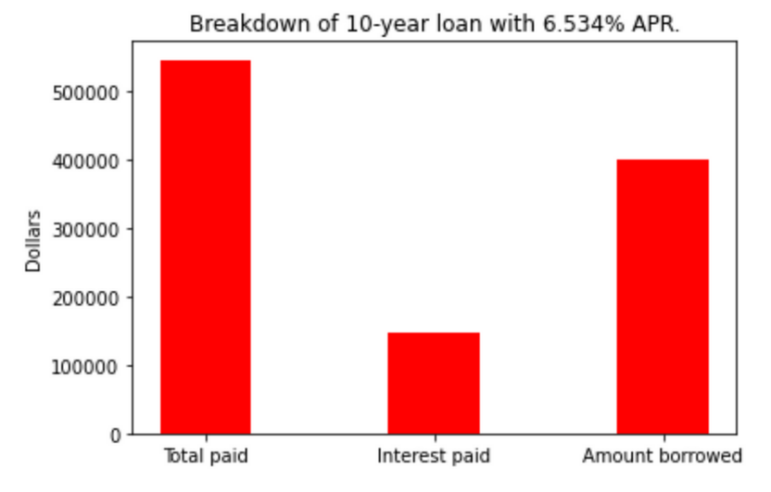
Plugging these into our equation to calculate monthly payment yields,

$$x = \frac{P(1+r)^n r}{(1+r)^n - 1}$$

$$x = \frac{400{,}000(1+0.005445)^{120} \cdot 0.005445}{(1+0.005445)^{120} - 1}$$

$$x = \$4551.29$$

Then, to pay off the mortgage within the 120 months, a payment of $4551.29 will be paid every month for the next 10 years. The total amount paid at the end of the ten years is $564,154.40. The interest paid is $146,154.40 as shown in the graph below.



We also want to calculate a 30-year loan of $400,000 with an APR of 6.206%. Then our variables are,

$$P = 400{,}000$$

$$n = 30\,years \cdot 12\,\frac{months}{year} = 360\,months$$

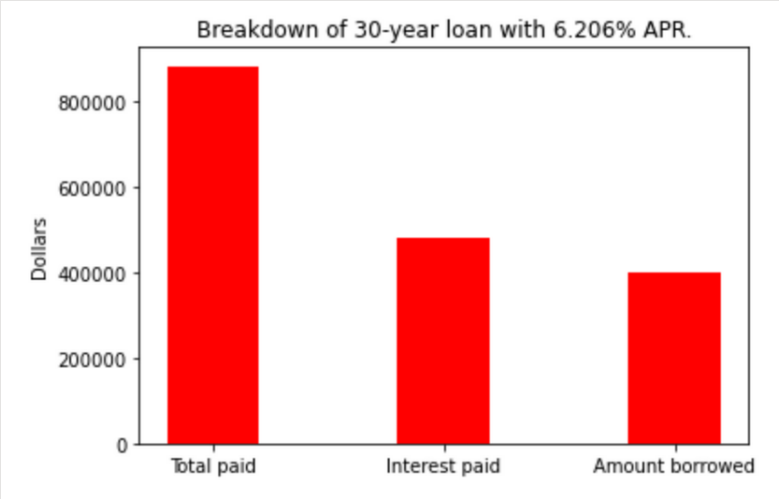
$$r = \frac{0.06206}{12} = 0.005171$$

Plugging these into our formula yields,

$$x = \frac{400000(1+0.005171)^{360} \cdot 0.005171}{(1+0.005171)^{360} - 1}$$

$$x = \$2451.23$$

To pay off the mortgage within the 360 months, a payment of $2451.23 will be paid every month for the next 30 years. The total amount paid at the end of the ten years is $882,441.29. The interest paid is $482,441.29 as shown in the graph below.



Now, what does this all mean? It's important to note the trade-offs between the 10-year and 30-year loan. The 10-year loan has a significantly higher monthly payment, $2100,06 higher to be exact. But the borrower ends up only paying $146.154.40 in interest over the loan versus $482,442.29 for the 30-year loan. If the borrower can pay more over a short period, they will save $336,288.40 in the long-haul of the mortgage. Not everyone can make such a high payment. So, a longer mortgage with a smaller monthly payment may be the right choice for their situation.

Sensitivity Analysis.

Let us see what happens to monthly payment and interest paid if we increase or decrease APR on 10-year and 30-year loans of $400,000.

10-year loan of $400,000

Interest Rate	Monthly Payment	Interest Paid
6.134%	$4,467.78	$136,134.14
6.334%	$4,508.20	$140,984.87
6.534%	$4,548.84	$145,861.01
6.734%	$4,589.68	$150,762.53
6.934%	$4,630.74	$155,689.32

30-year loan of $400,000

Interest Rate	Monthly Payment	Interest Paid
5.804%	$2,348.03	$445,291.21
6.006%	$2,399.74	$463,908.31
6.206%	$2,451.43	$482,516.05
6.406%	$2,503.59	$501,294.303
6.606%	$2,556.22	$520,239.44

From looking at these tables, it is clear small adjustments in APR can have significant changes in total interest paid over the course of the loan. For a 10-year loan, an increase of 0.400% can cost a borrower $9828.31 in the long run and increase monthly payment by $81.90. For a 30-year loan, the same increase can raise overall interest paid by $37,723.39 and increase monthly payment by $104.79. Therefore, it is important for borrowers to ensure they are receiving market level interest rates to ensure they save as much as possible.

Conclusion.

In closing, we have provided customers with a thorough breakdown of how payments are calculated and how the amount of the loan, APR and the length of the loan factor into the overall cost of the mortgage. We provided the equations used to determine monthly payment and the amount of the principle remaining after k number of payments. We defined each of the variables within the equation to further increase the clients' understanding. We also provided the derivation of the formulas, so the clients have in-depth knowledge of how the formulas are created. Demonstrations were done with current market rates to calculate monthly payment with a specific loan amount for multiple mortgage lengths. Visual aids were provided to enhance understanding of how mortgage lengths affect total interest paid. Sensitivity Analysis was done on APRs for 10-year and 30-year loans to emphasize the importance of agreeing to an appropriate interest rate when shopping mortgage terms.

The methods chosen by Raven Solutions provide comprehensive analysis for the benefit of the customer. With these materials, potential clients should feel satisfied they fully understand the agreements of the mortgage prior to entering into any contracts. However, there is potential the material here may be too technical for some homebuyers. The derivations of the formula may only serve to further confuse the client and exhaust their motivation to pursue the mortgage with this institution. It is advised the derivation is only utilized when it is specifically asked for by the client.

References.

Shen, Samuel S.P. (2017). Introduction to Modern Mathematical Modeling with R. Wiley-Interscience. Print.

Home loan basics. San Diego County Credit Union: Home. (n.d.). Retrieved April 26, 2023, from https://mortgage.sdccu.com/default.asp?siteId=A5D5972F-0E2B-48FD-881D-ED8A10&

Appendix.

Code used for formulas and plotting.

```
import numpy as np
import matplotlib.pyplot as plt

def calcPay(P,n,r,l,APR):

    # Calculate monthly payment
    num = (P*(1+r)**n)*r
    den = ((1+r)**n)-1
    x = num/den

    # Calculate interest paid and total paid
    tot_paid = x * n
    print(f"The monthly payment for a loan of ${P} with a APR of {APR}% is ${x:.2f}.")
    print(f"Total paid: {tot_paid:.2f} Interest paid: {np.round(tot_paid - P,2)}")

    # Create bar graph of principle, interest paid and total paid
    data = {'Total paid':tot_paid, 'Interest paid':tot_paid - P, 'Amount borrowed':P}
    courses = list(data.keys())
    values = list(data.values())
    plt.bar(courses, values, color ='red',
        width = 0.4)
    plt.ylabel("Dollars")
    plt.title(f"Breakdown of {l}-year loan with {APR}% APR.")
    plt.show()

#10-year loan
P = 400000
```

```
n = 120
r = 0.005455
calcPay(P,n,r,10,6.534)

#30-year loan
P = 400000
n = 360
r = 0.005171
calcPay(P,n,r,30,6.206)
```

Code used for sensitivity analysis.

```
tenYrRates = np.array([0.06134,0.06334,0.06534,0.06734,0.06934])
thirtyYrRates = np.array([0.05804,0.06006,0.06206,0.06406,0.06606])

def calcPay(n,APR):
    monthlyPayments = np.array([])
    totIntPaid = np.array([])
    P = 400000
    for i in range(5):

        # Convert APR to MPR
        r = APR[i]/12

        # Calculate monthly payment and interest paid
        num = (P*(1+r)**n)*r
        den = ((1+r)**n)-1
        x = num/den
        intPaid = x*n - P
        monthlyPayments = np.append(monthlyPayments,x)
        totIntPaid = np.append(totIntPaid,intPaid)
    return monthlyPayments, totIntPaid

calcPay(120,tenYrRates)
calcPay(360,thirtyYrRates)
```