# POLS201 Spring 2019

## Interaction Effects and Difference of Means Redux

April 15

# Agenda



- Overview of Interaction Terms
- Quick review of difference of means t-test
- For Wednesday: live examples of logit regression in R
- The Nieheisel paper (focus on the tables)
- Review of the homework

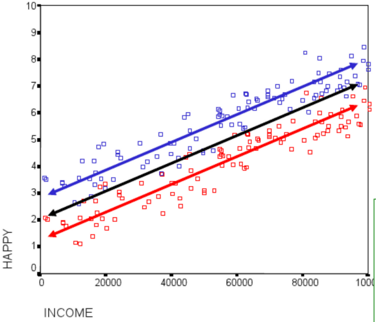
# Robustness



- The most straightforward kind of robustness test is to rerun your models with different combinations of variables.
- One IV and one control theoretically gives you four different models to run.
- Add an interaction term and you have eight, as you will soon see.

# Dummy Variables: Interpretation

- Visually: Women = blue, Men = red




Overall slope for all data points

Note: Line for men, women have same slope… but one is high other is lower. The constant differs!

If women=1, men=0: The constant (a) reflects men only. Dummy coefficient (b) reflects increase for women (relative to men)

# Introduction to Interaction Effects



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- What if slopes are different for different groups in our sample?
- Example: a policy works in rich countries but not poor countries.
- The marginal effect of the IV is conditioned on the value of some other variable.

# Interaction Terms

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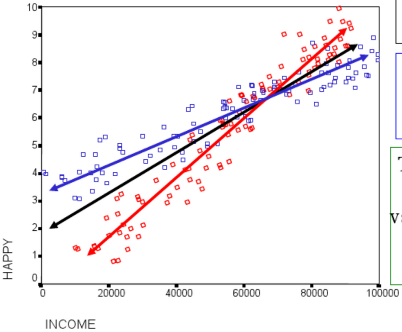


Overall slope for all data points

Note: Here, the slope for men and women differs.

The effect of income on happiness (X1 on Y) varies with gender (X2). This is called an "interaction effect"

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- All of these strategies might work:
- Run separate regressions. But this misses the interaction.
- Run a hierarchical model: a more sophisticated approach for another class
- Model an "interaction" effect

# Computationally: Interaction effects are a breeze



- Create a new variable that multiples your two IVs together

| Happiness | Income | Female | F*I |
|---|---|---|---|
| 3 | 0 | 1 | 0 |
| 6 | 66 | 1 | 66 |
| 5 | 280 | 1 | 280 |
| 2 | 100 | 0 | 0 |
| 8 | 50 | 0 | 0 |
| 6 | 120 | 0 | 0 |

# Computationally: Interaction effects are a breeze



- Create a new variable that multiples your two IVs together
- Include all three variables together in your regression

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# The syntax in lm() is easy



- happy_fit <- lm(Happiness ~ Income * Female, data = happy)
- summary(happy_fit)
- The only difference: insert a multiplcation sign “*" instead of a plus sign “+”

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- Why difficult: Variables Appear Twice!!
- $Y = \alpha + \beta 1(C) + \beta 2(D) + \beta 3(C * D) + \epsilon$

# Interpretation

- What are you interested in?
- Marginal Effects?
- Predicted Outcomes?

# Interpretation



- $Y = \alpha + \beta 1(C) + \beta 2(D) + \beta 3(C * D) + \epsilon$
- $\beta 1$ can be interpreted as the effect of C when D is zero

## Interpretation



- .0124 is the marginal effect of income on happiness when "Female" = 0 (i.e., for men)
- Include all three variables together in your regression

```
Call:
lm(formula = Happiness ~ Income * Female, data = happy)

Residuals:
    Min      1Q  Median      3Q     Max 
-2.7949 -0.7255 -0.2318  0.6724  3.8268 

Coefficients:
               Estimate Std. Error t value Pr(>|t|)   
(Intercept)    3.551443   0.984924   3.606  0.00104 **
Income         0.012434   0.003647   3.410  0.00178 **
Female        -2.629118   1.106922  -2.375  0.02370 * 
Income:Female  0.008832   0.004468   1.977  0.05675 . 
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 1.377 on 32 degrees of freedom
Multiple R-squared:  0.7775,	Adjusted R-squared:  0.7567 
F-statistic: 37.28 on 3 and 32 DF,  p-value: 1.478e-10
```

# Interpretation



- $Y = \alpha + \beta 1(C) + \beta 2(D) + \beta 3(C * D) + \epsilon$
- $\beta 2$ can be interpreted as the effect of D when C is zero

# Interpretation



- -2.629 is the marginal effect of female on happiness when "Income" $= 0 >$- Include all three variables together in your regression

```
Call:
lm(formula = Happiness ~ Income * Female, data = happy)

Residuals:
    Min      1Q  Median      3Q     Max
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# Interpretation

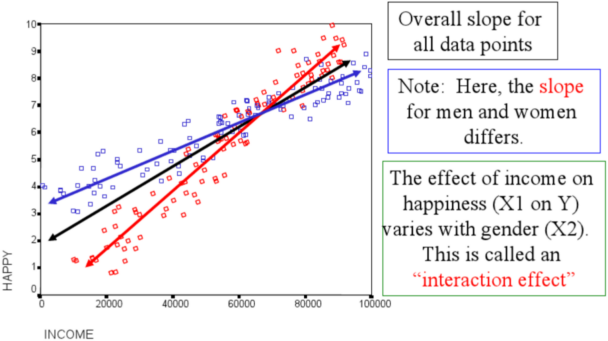


- $Y = \alpha + \beta 1(C) + \beta 2(D) + \beta 3(C * D) + \epsilon$
- $\beta 3$ is the difference in the **MARGINAL** effect of C when D goes from 0 to 1
- That is, the difference in the marginal effect of income for men v. women
- For any level of income

# Interpretation



- Suppose women = Red line and men = blue line
- Black is the overall slope for all data points
- Notice that the **slope** for men differs from the **slope** for women
- The effect of income on happiness varies by gender. This is the interaction effect



# Interaction Effects

-The Marginal Effects of Interaction Terms gets tricky. -It is sometimes more straightforward to predict Y, and then back out marginal effects!

# Calculating Predicted Values



- $Y = \alpha + \beta 1(C) + \beta 2(D) + \beta 3(C * D) + \epsilon$
- $\alpha$ = female = 0 and income = 0, i,.e., men without income

# Calculating Predicted Values



- $Y = \alpha + \beta 1(C) + \beta 2(D) + \beta 3(C * D) + \epsilon$
- $\alpha + \beta 2 =$ female $= 1$ and income $= 0$, i,.e., women without income

# Calculating Predicted Values

- $Y = \alpha + \beta 1(C) + \beta 2(D) + \beta 3(C * D) + \epsilon$
- $\alpha + \beta 1 =$ female $= 0$ and income $=$ C, i,.e., men with income of C

# The predict(model_name) function. . .



- . . . automatically calculates these values for every observation
- or you could see the effect with a simple line graph for each gender

# Can you interact two dummy variables? Yes



- You end up with four specific predictions for the combinations of zero and one
- The coefficient of your interaction term = the scenario where both dummies equal one.
- $Y = \alpha + \beta 1(C) + \beta 2(D) + \beta 3(C * D) + \epsilon$

## Best Practices



1. Use multiplicative interaction models whenever one's hypothesis is conditional in nature.
   - e.g. "Religiosity drives conservative political ideology, but only in cities."
2. Include all constitutive terms in the model specification. Just use the multiplcation sign instead of the plus sign and you will be fine.
3. Do not interpret the coefficients on constitutive terms as if they are unconditional marginal effects.
4. Do not forget to calculate substantively meaningful marginal effects and standard errors.
   - the individiual componnents may not be significant by themselves, but the interactions might be, and vice versa.

# Question



- So why would separate regressions might be preferable to creating an interaction term?

# Possible Answers



- Separate regressions do generate a lot of information for each value of the IV. You can really can see the “change in the marginal effect of the other IV’s”
- But you don’t know the statistical significance of these differences.
- And you may run out of observations.

# Interaction terms provide another way to test robustness



- And can also unearth false negatives in your analysis
- "Does x affect y? It depends. . . " A significant interaction terms shows this conditional effect,
- And also creates an additional variable to analyze.

# Another recap on t-tests from the lab



- We reconnected with the idea of t-tests,
- Do religious respondents demonstrate less political knowledge than non-religiou respondents?
- We can learn something from the responses without a full blown regression.
- We can sim[ply compare the means of the two subroups on our measure of political knowledge.

# The formula is this:



$$t = \frac{\overline{x_1} - \overline{x_2}}{\sqrt{\frac{S_1{}^2}{N_1} + \frac{S_2{}^2}{N_2}}}$$

## Notice the similarities to the Z-score calculation

- t equals: the differences in the observed means, divided by:
- the square root of the first sd over sample size, + the second sd over its sample size
- The denominator is also the standard error.
- How do you determine the sample size? Roughly, use the smaller of the two N's. Formulas that want the "degrees of freedom" just take this sample size minus one. If it's big enough, the t-score won't differ from the z-score.

# So what's the big idea here?



- Did these means come from the same source distribution? If so, they might be different just because of randomness.
- But if they are sufficiently different, given the sample size, we reject that story.
- "The same source distribution could not have generated both of these samples. One must have been generated from a different source."

# Wednesday: We will run a logit regression from the UCLA website instructions,



- plus review the last homework and assign the final one.