Let's consider

$$\begin{cases} u_t = u_{xx} - \frac{1}{1+\pi^2} u_x + v_x + v & t > 0,\ x \in (0,1) \\ 0 = v_{xx} - v + u & t > 0,\ x \in (0,1) \\ u_x(t,0) = u_x(t,1) = 0, \quad v_x(t,0) = v_x(t,1) = 0 & t > 0 \\ u(0,x) = \cos(\pi x) & x \in (0,1). \end{cases}$$

The exact solution should be

$$\begin{aligned} u(t,x) &= e^{-\lambda t} \cos(\pi x), \\ v(t,x) &= \frac{1}{1+\pi^2} e^{-\lambda t} \cos(\pi x), \end{aligned}$$

with $\lambda = \pi^2 - \frac{1}{1+\pi^2}$.