

Let's consider

$$\begin{cases} u_t = u_{xx} + v_x + v \left(\pi^2 - \frac{1}{1+\pi^2} - \lambda \right) & t > 0, x \in (0, 1) \\ 0 = v_{xx} - v + u & t > 0, x \in (0, 1) \\ u_x(t, 0) = u_x(t, 1) = 0, \quad v_x(t, 0) = v_x(t, 1) = 0 & t > 0 \\ u(0, x) = \cos(\pi x) & x \in (0, 1). \end{cases}$$

The exact solution should be

$$\begin{aligned} u(t, x) &= e^{-\lambda t} \cos(\pi x), \\ v(t, x) &= \frac{1}{1 + \pi^2} e^{-\lambda t} \cos(\pi x). \end{aligned}$$