ME 601 Project Report

Rendezvous & Proximity Operations

Ian Ruh (iruh@wisc.edu)

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1 Overview

I'll be utilizing the material covered in class to analyze the control and relative dynamics of two spacecraft in orbit around earth. This is an area of interest due to both its current usage in vehicle docking to space stations, as well as its future applications to in-orbit debris collection and in-orbit assembly.

In order to simplify the analysis, several assumptions on the orbital parameters of the two spacecrafts, the configuration of actuators, the ideal behavior of the actuators, the control of the spacecraft attitude and knowledge of the spacecraft state will be made. These assumptions and their impact are included in the background section.

2 Background

The rendevous of two space craft, or more generally the relative positioning of two spacecraft in orbit, is a complex problem due to the dynamics involved. The first step in approaching the problem is to select a coordinate system in which the relative dynamics will be studied.

In the rest of this document, we identify two space crafts: the target and the chaser. The target is assumed to have attitude control such that it maintains a consistent orientation with respect to earth's horizon, but is otherwise inert. The chaser is the vehicle that we will focus on controlling the position of to rendezvous with the target.

We identify the Radial-Transverse-Normal (RTN) coordinate frame centered with the origin at the center of mass of the target vehicle. The three axes are aligned in the radial direction, along the vector from the origin of the center of the earth; the transverse direction, parallel to the velocity vector, and thus along the orbital path; and the normal direction, parralel to the orbit's angular momentum vector.

Within this coordinate frame the following relative dynamics are derived in [2]:

$$
\begin{aligned}
\ddot{x} - 2\dot{f}_c\dot{y} - \ddot{f}_c y - \dot{f}_c^2 x &= -\frac{\mu(r_c + x)}{\left[(r_c + x)^2 + y^2 + z^2\right]^{3/2}} + \frac{\mu}{r_c^2} + d_R \\
\ddot{y} + 2\dot{f}_c\dot{x} + \ddot{f}_c x - \dot{f}_c^2 y &= -\frac{\mu y}{\left[(r_c + x)^2 + y^2 + z^2\right]^{3/2}} + d_T \\
\ddot{z} &= -\frac{\mu z}{\left[(r_c + x)^2 + y^2 + z^2\right]^{3/2}} + d_N
\end{aligned}
\tag{1}
$$

Where f_c is the target's true anomaly, r_c is the target's radius, and $\vec{d} = [d_R, d_T, d_N]^T$ represents the relative disturbing accelerations between the target and the chaser in the target's RTN frame.

The first simplifying assumption we make is on the eccentricity of the target's orbit. If the target's orbit is near circular (the eccentricity is near 0), then $\dot{f}_c$ = mean motion = $\sqrt{\frac{\mu}{a^3}}$, where μ is the earth's gravitational parameter ($\approx 3.986 \cdot 10^{14}$ m^2/s), and a is the orbit's semi-major axis (SMA). Thus, $\dot{f}_c$ is constant and $\ddot{f}_c = 0$. This leads to the following simplified dynamics:

$$
\begin{aligned}
\ddot{x} &= -\frac{\mu(r_c + x)}{\left[(r_c + x)^2 + y^2 + z^2\right]^{3/2}} + \frac{\mu}{r_c^2} + 2n\dot{y} + n^2 x + d_R \\
\ddot{y} &= -\frac{\mu y}{\left[(r_c + x)^2 + y^2 + z^2\right]^{3/2}} + 2n\dot{x} + n^2 y + d_T \\
\ddot{z} &= -\frac{\mu z}{\left[(r_c + x)^2 + y^2 + z^2\right]^{3/2}} + d_N
\end{aligned}
\tag{2}
$$

Linearizing this around the target's position, we arrive at the following linear dynamics [1]:

$$
\begin{aligned}
\ddot{x} &= 3n^2 x + 2n\dot{y} + d_R \\
\ddot{y} &= -2n\dot{x} + d_T \\
\ddot{z} &= -n^2 z + d_N
\end{aligned}
\tag{3}
$$

2.1 Other Assumptions

The reality of controlling a spacecraft involves complex actuators including electric propulsion, mono-propellant attitude control systems, reaction wheels, control moment gyros, magnetic torquers, and others. For the purposes of this project, only the dynamics due to gravity and idealized forces in the RTN frame are considered to simplify the analysis. Therefore, we are neglecting the attitude control of the chaser and the arrangement (and therefore selection of thrusters) such that we have have a force in the RTN frame as our only control input.

3 Controllers

3.1 Infinte LQR Linearized Stabilizing Controller

I implemented an infinte LQR controller that

3.2 Infinte LQR Linearized Trajectory Tracking Controller

We deriving the expression for our error dynamics using the linearized system dynamics. We define $e(t) = x(t) - x_d(t)$, where x_d is our desired trajectory, and $v(t) = u(t) - u_d(t)$, where u_d is our desired control.

$$\begin{aligned}
\dot{e} &= \dot{x} - \dot{x}_d \\
&= f(x, u) - f(x_d, u_d) \\
&= f(x_d + e, u_d + v) - f(x_d, u_d) \\
&= (\mathbf{A}(x_d + e) + \mathbf{B}(u_d + v)) - (\mathbf{A}x_d + \mathbf{B}u_d) \\
&= \mathbf{A}e + \mathbf{B}v
\end{aligned} \tag{4}$$

For simplicity, we let $v(t) = \vec{0}$.

References

[1] Eugene M Cliff. Clohessy - Wiltshire Analysis. *N/A*, page 4, N/A.

[2] Joshua Sullivan, Sebastian Grimberg, and Simone D'Amico. Comprehensive Survey and Assessment of Spacecraft Relative Motion Dynamics Models. *Journal of Guidance, Control, and Dynamics*, 40(8):1837–1859, August 2017.