A Formal Test of Assortative Matching in the Labor Market

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Abstract

We estimate a structural model of job assignment in the presence of coordination frictions due to Shimer (2005). The coordination friction model places restrictions on the joint distribution of worker and firm effects from a linear decomposition of log labor earnings. These restrictions permit estimation of the unobservable ability and productivity differences between workers and their employers as well as the way workers sort into jobs on the basis of these unobservable factors. The estimation is performed on matched employer-employee data from the LEHD program of the U.S. Census Bureau. The estimated correlation between worker and firm effects from the earnings decomposition is close to zero, a finding that is often interpreted as evidence that there is no sorting by comparative advantage in the labor market. Our estimates suggest that this finding actually results from a lack of sufficient heterogeneity in the workforce and available jobs. Workers do sort into jobs on the basis of productive differences, but the effects of sorting are not visible because of the composition of workers and employers.

1 Introduction

It is now well established that the statistical decomposition of wage rates into portions due to observable characteristics, unmeasured individual heterogeneity, unmeasured employer heterogeneity, and statistical residual attributes substantial variation to the unmeasured individual and employer components.[1] There is less agreement on the sign and magnitude of the statistical correlation between these components of heterogeneity in the observed samples of workers. Some find a small or negative correlation (Abowd et al. 1999). Others find some evidence of positive correlation (Woodcock 2008; Abowd et al. 2003).

What might explain the correlation between unmeasured individual and employer heterogeneity? Roy (1951) posited an optimal sorting of workers with heterogeneous abilities that were differentially productive in different occupations. Employer heterogeneity resulted from different occupation mixes across industries and firm sizes. Mortensen (2003) demonstrates that the within occupation decomposition into unmeasured individual and employer components is just as strong as the between occupation decomposition, which is not consistent with Roy's model.

Mortensen (2003) posits a collection of alternative explanations based on search and recruitment costs among employers and workers with heterogeneous productivity. He provides empirical analyses of the Danish labor market that are consistent with the structural models he describes. Postel-Vinay and Robin (2002) use Mortensen-style models to study the French labor market.

Our goal is to explain the observed correlations between individual and firm heterogeneity components. For this task, we adopt the model of job assignment with coordination frictions developed in Shimer (2005). In what follows, we describe only those features of the model that are essential to our analysis. Shimer's model yields a wage offer function that depends upon both the ability of the individual and the productivity of the employer. Furthermore, it predicts which jobs will be filled in equilibrium. These are the two essential features needed to construct theoretical moments of the joint distribution of individual and firm heterogeneity. The structural model and the estimated parameters help us assess whether and to what extent the correlations we observe between individual and firm heterogeneity components are due to assortative matching, mismatch, and selection effects.

The key contribution of the Shimer's model was to formally introduce the notion of coordination frictions into the job matching process. If firms and workers could condition wage offers and applications on the behavior of all other agents in the economy, it would always be possible to get a frictionless and optimal assignment of workers to employers (synonymous with jobs in this model). Shimer argues that such complete coordination of the plans of all actors is unrealistic. Workers and firms can coordinate their behavior, but only imperfectly. The model captures this idea by two refinements to worker and firm strategies. First, firms can condition their wage offers on the type, but not the identity of the worker. Second, all workers of the same type are presumed to follow the same application strategy.

[1]See Abowd et al. (1999) for France, Abowd et al. (2003) for the United States, and Mortensen (2003) for Denmark, among others.

These simple but powerful assumptions generate equilibrium unemployment, mismatch of worker and firm types, and wage dispersion that depends on both firm and worker type.

Section 2 describes the job assignment model with coordination frictions. Section 3 presents the input data from the LEHD program and how it is processed prior to estimation of the structural model. Section 4 summarizes the estimation procedure, and section 5 the results of the estimation on LEHD data for the Professional, Scientific and Technical Services and Manufacturing sectors. Section 7 concludes.

2 Theoretical Assignment Model

2.1 Job Assignment with Coordination Frictions

The economy consists of workers and employers. Each employer has one vacancy to fill by hiring one worker. Employers choose wage offers for each worker. Workers then choose where they want to apply for a job. Based on the pool of applicants, the employers hire one worker to fill the vacancy. Workers and employers only differ in productive type. There are M types of worker and N types of employers. When a worker of type m is hired into a job of type n, the output of production is $x_{m,n}$.

A decentralized equilibrium of the coordination friction economy is characterized by application strategies for all workers and wage offers from all employers to each type of worker. It turn out that this equilibrium depends only on the nature of the production technology and the number of each type of worker and of each type of employer. Out of a total measure μ of workers in the economy, we assume that μ_m of them have productive type $m \in \{1 \dots M\}$. Likewise, out of a total measure ν of employers in the economy, ν_n of them have productive type $n \in \{1 \dots N\}$. Although it will not matter in the final equilibrium, we follow Shimer in naming individual workers by (m, i) and individual employers by (n, j) where the first term is the type of the agent, and the second term is the agent's name. Thus, each worker or employer can be uniquely identified.

2.2 Equilibrium Wage Offers, Applications and Hiring

2.2.1 The Worker's Problem

Suppose each employer, (n, j), has chosen its menu of wage offers. Employers cannot offer different wages to workers of the same type. Consequently, a worker of type m must choose an application strategy based on the set of offers, $w_{m,(n,j)}$, across all (n, j). The model assumes that a worker can make only one application, so a feasible strategy for worker (m, i) is a probability distribution $p_{m,i}$ that specifies the probability with which she applies to each employer.

Since all workers of type m follow the same strategy in the coordination friction economy, and since we assume that the number of workers of each type is very large, it is easier to think in terms of the queue of applicants of type m to each employer. That is, the number of applications from type m workers to any employer is approximately Poisson with parameter $q_{m,(n,j)} = p_{m,(n,j)}\mu_m$

The equilibrium queue lengths must leave all workers of type m indifferent between applying for any two jobs. To satisfy this condition for each type of worker, all jobs for which there is a positive probability of application have the same expected income. In addition, all jobs for which the worker of type m does not apply have strictly less expected income. The expected income, $y_{m,(n,j)}$, from any job for which $q_{m,(n,j)} > 0$ is:

$$y_{m,(n,j)} = e^{-Q_{m+1,(n,j)}} \frac{1 - e^{-q_{m,(n,j)}}}{q_{m,(n,j)}} w_{m,(n,j)} \tag{1}$$

where $Q_{m+1,(n,j)} = \sum_{m'=m+1}^{M} q_{m',(n,j)}$ is the measure of workers with productive type at least $m+1$. This is the expected number of applications from workers who are strictly better than a worker of type m. Using the properties of the Poisson distribution, the expected income from (n, j) is the probability of getting the job times the wage, where the probability of getting the job is the probability that there are no applications from a better worker (the first term on the right-hand side of (1) times the probability that the worker actually ends up applying for the job (the term in the numerator) and is chosen from among all other applicants of the same type (the term in the denominator). As long as there are expected incomes, $y_{m,n}$, and queue lengths that satisfy 1, workers have no incentive to deviate.

2.2.2 The Employer's Problem

Employers choose wage offers for workers of each productive type and a hiring protocol that maximizes expected profits given the queueing behavior of workers. Shimer (2005) shows that the optimal hiring strategy is to choose the applicant of highest productive type. When pursuing the optimal strategy, the maximized expected profit of employer (n, j) is given by

$$\sum_{m=1}^{M} e^{-Q_{m+1,(n,j)}}[1 - e^{-q_{m,(n,j)}}][x_{m,n} - w_{m,(n,j)}]. \tag{2}$$

The employer's optimal wage offers maximize the difference between each worker type's productivity and wage rate times the probability that a worker of that type will apply.

2.2.3 Competitive Search Equilibrium

Shimer defines the equilibrium for this economy as follows:

Definition 1 A *Competitive Search Equilibrium* consists of wage offers, w, queue lengths, q, and expected incomes, y, chosen so that employer's profits (2) are maximized, worker's expected incomes are given by (1), and the expected number of applications from type m workers does not exceed the total measure of such workers, μ_m:

$$\mu_m = \sum_{n=1}^{N} \int_0^{\nu_n} q_{m,(n,j)} dj. \tag{3}$$

The competitive equilibrium turns out to be unique. Furthermore, the equilibrium queue lengths, $q_{m,n}$, are the same as those that would be chosen by a social planner who chooses queue lengths, or equivalently, application probabilities for workers. The social planner is restricted to queue lengths that do not condition on individual identities. The goal of the social planner is to maximize expected aggregate output. In this setting, it turns out that the optimal queue lengths will satisfy the first-order conditions:

$$\lambda_{m,n} \geq e^{-Q_{(m,n)}} x_{m,n} - \sum_{m'=1}^{m-1} e^{-Q_{m'+1,n}} (1 - e^{-q_{m',n}}) x_{m',n} \tag{4}$$

where $\lambda_{m,n}$ is the expected employment of type m workers in type n employers. The dependence on the individual's identity, i, and the employer's identity, j, have been suppressed because they are not relevant. Given these queue lengths, and the requirement that workers' expected incomes satisfy (1), the equilibrium wages must be:

$$w_{m,n} = \frac{q_{m,n} e^{-q_{m,n}}}{1 - e^{-q_{m,n}}} [x_{m,n} - \sum_{m'=1}^{m-1} e^{-Q_{m'+1,n}} (1 - e^{-q_{m',n}}) x_{m',n}]. \tag{5}$$

Equations (4) and (5) provide all the information that is required for our estimation strategy. Given assumptions about the production technology, $x_{m,n}$, and the measure of each type of employer and worker, wages and queue lengths are determined. Although the equilibrium queue lengths have no observable counterpart, we can use them to derive the expected number of workers of type m that ultimately become employed by employers of type n:

$$\lambda_{m,n} = \nu_n e^{-Q_{m+1,n}} (1 - e^{-q_{m,n}}). \tag{6}$$

2.3 Empirical Predictions and Sorting

In equilibrium, as in the frictionless model, the nature of job assignment depends on the properties of the production function. Hence, the model is still informative about the nature of assortative matching in the economy. Unlike the frictionless model, however, the coordination friction equilibrium has several properties that are more consistent with the stylized facts characterizing labor markets. These equilibrium properties are also enough to generate a structural connection between the underlying model primitives and the estimated person and employer effects from a linear decomposition of log earnings. We exploit this structural link to estimate the structure of the model.

With respect to the nature of assortative matching, Shimer derives several strong results. First, if the production function is supermodular, Shimer's Proposition 3 shows that $Q_{m,n}$ is strictly increasing in n when it is positive. In addition, a more productive job is more likely to be filled, a worker is less likely to obtain a more productive job conditional on applying for it, and a worker's wage is increasing in the employer's productivity. This result holds for production technologies that exhibit comparative advantage for higher-ability workers in higher-productivity jobs. It also holds, as is well known, for a production technology that exhibits no comparative advantage. Specifically, one in which output is multiplicative in the ability and productive types of the worker and job. A converse result holds when there is a comparative advantage for low ability workers in high-productivity jobs. In that case, $Q_{m,n}$ is decreasing in n so that the assortative matching again corresponds to comparative advantage. We estimate the structural model assuming either that the production technology exhibits no comparative advantage or that the technology exhibits comparative advantage for low ability workers in high productivity jobs. These specifications yield the minimal differentiation sufficient to capture model uncertainty as to whether matching is positively or negatively assortative.

On their own, these results do not distinguish the coordination friction economy, since the frictionless model also predicts that job assignment will be in accord with comparative advantage in production. However, the coordination friction model makes more reasonable predictions about equilibrium unemployment and vacancies, and predicts a certain amount of mismatch, which eliminates the untenable "perfect correlation" prediction for worker and employer types from comparative advantage models without the coordination friction. Since workers randomly choose the employer to which they will apply, there is, in general, a positive probability that multiple workers apply for the same job–leaving all but one unemployed. There is also a positive probability that no one will apply for any particular job, leaving it vacant. Therefore, there will be simultaneous unemployment and job vacancy for workers and jobs of all types. Furthermore, even among employed workers and occupied vacancies, there will be mismatch in the sense that the particular set of matches realized in the economy could be improved upon in the sense of maximizing total output, or in terms of aligning assignments with comparative advantage.

Finally, the coordination friction equlibrium has two properties that are essential to the estimation strategy. In equilibrium, the wage offer function violates the law of one price. Different jobs pay identically able workers different wages. That is, there is a structural employer effect in the wage. Shimer shows that the wage offer will generally be increasing in the productive type of the employer. The equivalent result does not hold for workers, however. It is possible for an employer to offer workers with low ability higher equilibrium wage than workers with higher ability. This establishes that it is possible to observe a negative correlation between person and employer effects from the empirical earnings decomposition even when there is positive assortative matching on the unobserved productivity types.

Note, however, that the model does not generally imply that log earnings can be additively decomposed into a person and employer effect. But this implication is not required by our estimation procedure. Our approach is motivated by the observation that the person and firm effects from the statistical decomposition of log earnings in a linear model will be complicated transformations of the underlying productivities and abilities along with factors determining the supply of and demand for worker skill. As we show below, Shimer's coordination friction model provides explicit formulas for these transformations that we can identify through restrictions on observable moments of the the joint distribution of the estimated person and firm effects from the earnings decomposition.

3 Data

Our estimation procedure has two stages. First, we use matched employer-employee data from the U.S. Census Bureau's LEHD Program to estimate the parameters of a statistical decomposition of log earnings into components due to individual heterogeneity, employer heterogeneity, and time-varying observables. Next, we estimate the structure of the assignment model from its predictions about the joint distribution of the person and employer effects. In the structural estimation, we use the distribution of person and employer effects stratified by deciles of the firm size distribution. This provides an additional source of variation to help identify the model in the absence of firm-level data on productivity. We estimate the model separately for the Manufacturing (NAICS 31-33) and the Professional, Scientific, and Technical Services (NAICS 541) sectors.

3.1 The Statistical Decomposition

Following Abowd et al. (1999), we decompose the annualized wage in each job as:

$$\log w_{i,t} = x_{i,t}\beta + \theta_i + \psi_{J(i,t)} + \varepsilon_{i,t} \tag{7}$$

where

- $J(i,t)$ is the dominant employer of individual i at time t;
- θ_i is the individual effect;
- $x_{i,t}\beta$ is the effect of individual labor market experience, tenure, and labor market attachment, interacted with sex; and
- $\psi_{J(i,t)}$ is the firm effect.

Abowd et al. (2002) document the identification strategy and the details of implementing the full least squares solution for estimating (7).

We estimate (7) on matched employer-employee data from the LEHD Program of the U.S. Census Bureau. The LEHD program uses administrative data from state Unemployment Insurance wage records and ES-202/QCEW establishment reports that cover approximately 98% of all nonfarm U.S. employment. Our estimation sample is based on a snapshot of the LEHD data infrastructure that includes data from 30 states collected between 1990 and 2004. A detailed description of the input data is found in Appendix B.

3.2 The Data Used in the Structural Estimation

The estimates of the assignment model are based on data, D, of the form:

$$(\hat{\theta}_i, \hat{\psi}_j, s_\ell)_\ell$$

where $\hat{\theta}$ and $\hat{\psi}$ are estimates of the true person and firm effects from (7); i and j are the identities of the worker and firm contributing to the ℓth observation; and $s_\ell = (s_1, \ldots, s_{10})$ is an indicator for the decile of the firm size distribution into which the employer falls. We assume that $\ell \in \{1, ..., N\}$ and define $\sum_{\ell=1}^{N} s_\ell = (N_1, ..., N_{10})$.

We compute the empirical first and second moments of the joint distribution of θ and ψ within employer size decile for each of the two NAICS sectors. The unit of analysis is the individual job history observation in the matched data. The deciles of the employer size distribution are computed for every year of the data, and employers are assigned to the decile that they occupy for each year. We use a bootstrap procedure to compute the covariance matrix for the estimated first and second moments. We resample the data from each sector $1,000$ times and compute the first and second moments of the joint distribution of $\hat{\theta}$ and $\hat{\psi}$ in each sample. The estimated covariance matrix is the covariance matrix of these $1,000$ estimates.

It is important to note the measures taken to protect the confidentiality of respondents providing the underlying micro-data. The U.S. data are protected by a distribution-preserving noise-infusion procedure. Each element of the microdata has been distorted in such a way that the individual observations are protected, yet the aggregate statistics are still valid for our proposed analysis. For complete details of the noise infusion procedure and the way in which it preserves analytical validity, see Abowd et al. (2009).

Figures (1),(2) and (3) provide a visual summary of data from the Professional, Scientific and Technical Services sector. Figure (1) shows that employment in this sector is concentrated in smaller firms. Figures (2) and (3) show two different kinds of evidence about the relationship between the estimated person and firm effects. The first figure plots the empirical average person and firm effects within each decile of the employer size distribution. The average firm effect is increasing in employer size, consistent with the widely documented presence of a firm size effect in the wage equation. Note that here the firm-size effect is preserved even controlling for the effects of personal heterogeneity. Abowd et al. (2003) note that the firm-size effect is indeed driven almost entirely by firm heterogeneity in wages with little of the effet caused by sorting of highly paid workers into larger firms. Here, we see that the average level of the person effect does not change much across deciles of the employer size distribution. The standard errors on these estimates are so small as to be invisible on the plot. Figure (3) shows the empirical correlation between $\hat{\theta}$ and $\hat{\psi}$ within each decile of firm size. The correlations are generally small in absolute value, ranging between about -0.2 and 0.25. Our model will fit estimated correlations that try to capture the scale of these correlations as well as the variation across size classes. Here, we observe a hump-shaped relationship between employer size decile and the correlation between $\hat{\theta}$ and $\hat{\psi}$. In terms of the model, this is feasible only if there is some considerable variation in the production technology and demand across the size classes.

Figures (4),(5) and (6) show the corresponding picture for the Manufacturing sector. The distribution of jobs is more concentrated in firms of intermediate size. As in the Professional, Scientific and Technical Services sector, we see clear evidence of a firm-size effect on earnings. Here, though, there is a more puzzling relationship between firm size and the average person effect. The relationship is clearly non-linear, but its shape defies simple interpretation. Here, the correlations are smaller, tending close to zero, with the exception of a relatively strong negative correlation between $\hat{\theta}$ and $\hat{\psi}$ for the smallest firms.

4 Estimation Procedure

4.1 Method of Moments Estimation

In this section we show that Shimer's model generates restrictions on the joint distribution of the estimated person and employer effects, $\hat{\theta}$ and $\hat{\psi}$ from the decomposition of log earnings that are sufficient to identify the free parameters of the model. We analyze the conceptual log earnings decomposition on the matched employer-employee data that would be generated in the coordination friction economy. Assuming that worker and employer types are observable, we derive formulas for the expectation of estimated person and firm effects in terms of equilibrium quantities of the model. Using these theoretical analogues to the estimated person and firm effects, we derive formulas for the expected value of the empirical second moments of $\hat{\theta}$ and $\hat{\psi}$. The full set of estimating equations uses these restrictions on the second moments of the joint distribution of $\hat{\theta}$ and $\hat{\psi}$ as well as restrictions on the distribution of realized matches across deciles of the firm size distribution.

In the exposition and the estimation, we assume that there are two types of worker and 20 types of employer. However, the argument does not depend on these assumptions; one could set an arbitrary number of worker and employer types. For both identification and computational purposes, we impose parametric restrictions on the model. Our restrictions limit the kind of production technologies under consideration as well as the manner in which productivity and the vacancy rate vary by firm size decile. These facilitate our computation of the method of moments estimator by the method of simulated annealing.

4.1.1 Theoretical Wage Decomposition

We consider two types of workers, so $M = 2$. In the coordination friction economy, for any given employer all workers of the same type are equally productive and by assumption are equally paid. The wage offer functions for the case $M = 2$ are given by:

$$
\begin{aligned}
w_{1,n} &= \frac{q_{1,n} e^{-q_{1,n}}}{1 - e^{-q_{1,n}}} x_{1,n}, \\
w_{2,n} &= \frac{q_{2,n} e^{-q_{2,n}}}{1 - e^{-q_{2,n}}} \left(x_{2,n} - \left(1 - e^{-q_{1,n}}\right) x_{1,n} \right)
\end{aligned}
$$

Recall that $\lambda_{m,n}$ is the expected number of m type workers matched to employers of type n and $w_{m,n}$ the corresponding wage. For $M = 2$ we have $\lambda_{1,n} = \nu_n e^{-q_{2,n}} (1 - e^{q_{1,n}})$ and

$\lambda_{2,n} = \nu_n(1 - e^{q_{2,n}})$.

To generate the theoretical analogues to the estimated person and firm effects, we perform a decomposition of log earnings into portions due to worker and employer type based on the equilibrium wage offers, w, and expected realized matches, λ, given model primitives (μ, ν, f):

$$\log w_{m,n} = \theta_m + \psi_n$$

where dependence on i and j has been suppressed because it is irrelevant. There are only two worker ability effects to estimate and N employer productivity effects. We restricted our attention to models that produce equilibrium matches so that the decomposition is identified. The worker and firm effects are the solution to the weighted least squares problem

$$\left(\hat{\theta}_1, \hat{\theta}_2, \hat{\psi}_1, \ldots, \hat{\psi}_N\right) = \arg\max \left[\sum_{m,n=1}^{2,N} \lambda_{m,n} \left(\log w_{m,n} - \theta_m - \psi_n\right)^2\right]. \tag{8}$$

The worker and employer effects are identified up to a constant Λ (see Abowd et al. (1999) and Abowd et al. (2002)). Straightforward computations (see Appendix) yield

$$\hat{\theta}_i = \frac{\sum_n \frac{\lambda_{1n}\lambda_{2n}}{\lambda_{1n}+\lambda_{2n}} \log w_{i,n}}{\sum_n \frac{\lambda_{1n}\lambda_{2n}}{\lambda_{1n}+\lambda_{2n}}} + \Lambda, \qquad i = 1, 2 \tag{9}$$

$$\hat{\psi}_n = \frac{\lambda_{1n}}{\lambda_{1n} + \lambda_{2n}}(\log w_{1,n} - \theta_1) + \frac{\lambda_{2n}}{\lambda_{1n} + \lambda_{2n}}(\log w_{2,n} - \theta_2) - \Lambda. \tag{10}$$

For consistency with the assumption used in the empirical estimation, we set $\Lambda = 0$. [2]

4.1.2 Estimating Equations

The estimating equations are based on the second moments of the $(\hat{\theta}, \hat{\psi})$ distribution resulting from solving 8, stratified by deciles of the firm size distribution s, and also from the expected fraction of filled jobs within firms of each size class, that is the first moments of s. Within each size class, k, we form a vector of second central moments as:

$$\bar{m}_k^C(D) = \bar{m}^C(\hat{\theta}, \hat{\psi}|s_k = 1) = \frac{1}{N_k} \begin{bmatrix} (\hat{\theta} - \bar{\hat{\theta}}_k)^T(\hat{\theta} - \bar{\hat{\theta}}_k) \\ (\hat{\theta} - \bar{\hat{\theta}}_k)^T(\hat{\psi} - \bar{\hat{\psi}}_k) \\ (\hat{\psi} - \bar{\hat{\psi}}_k)^T(\hat{\psi} - \bar{\hat{\psi}}_k) \end{bmatrix}$$

where $\bar{\hat{\theta}}_k$ and $\bar{\hat{\psi}}_k$ are the within size-class means for decile k. In addition, the model makes predictions on the number of jobs observed in each size class:

$$\bar{m}^S(D) = \frac{1}{N} \sum_{\ell=1}^{N} s_\ell.$$

[2] See Appendix for the general case with a continuum of firms.

The 40 empirical moments are compactly expressed as

$$\bar{m}(D) = \begin{bmatrix} \bar{m}_1^C(D) \\ \vdots \\ \bar{m}_{10}^C(D) \\ \bar{m}^S(D) \end{bmatrix}$$

The first 30 elements of $\bar{m}(D)$ are the second moments listed sequentially by size class and the last ten elements are the employment counts in each decile.[3]

We now show that the model structure is sufficient to specify a function that delivers moment restrictions corresponding to $\bar{m}(D)$ in terms of the underlying primitives of the model, (μ, ν, f):

$$E(\bar{m}(D)) = g(\mu, \nu, f).$$

We illustrate the manner in which the model determines the expectation of the empirical covariance between $\hat{\theta}$ and $\hat{\psi}$ within size-class k. The formulas for the variances are derived identically.

$$E[cov_k(\hat{\theta}, \hat{\psi})] = E\left[\frac{1}{N_k}\sum(\hat{\theta}_i\hat{\psi}_j) - \bar{\hat{\theta}}_k\bar{\hat{\psi}}_k\right]$$

$$\begin{aligned} &= \sum(\lambda_{11k}\theta_1\psi_{(k,1)} + \lambda_{12k}\theta_1\psi_{(k,2)} + \lambda_{21k}\theta_2\psi_{(k,1)} + \lambda_{22k}\theta_2\psi_{(k,2)}) \\ &\quad - \tfrac{(\lambda_{11k}+\lambda_{12k})\theta_1+(\lambda_{21k}+\lambda_{22k})\theta_2}{(\lambda_{11k}+\lambda_{12k}+\lambda_{21k}+\lambda_{22k})}\tfrac{[(\lambda_{11k}+\lambda_{21k})\psi_{(k,1)}+(\lambda_{12k}+\lambda_{22k})\psi_{(k,2)})}{(\lambda_{11k}+\lambda_{12k}+\lambda_{21k}+\lambda_{22k})}. \end{aligned} \tag{11}$$

Using formulas (9)-(10) the covariance and variances are thus functions of the expected number of each type of worker per firm $\lambda_{m,n}$ and of the wage $w_{m,n}$. We derive the theoretically predicted number of filled jobs in each size class

$$g^S(\mu, \nu, f)) = \begin{bmatrix} \frac{(\lambda_{1,(1,1)}+\lambda_{1,(1,2)}+\lambda_{2,(1,1)}+\lambda_{1,(1,2)})}{\sum_{s=1}^{10}(\lambda_{1,(s,1)}+\lambda_{1,(s,2)}+\lambda_{2,(s,1)}+\lambda_{1,(s,2)})} \\ \vdots \\ \frac{(\lambda_{1,(10,1)}+\lambda_{1,(10,2)}+\lambda_{2,(10,1)}+\lambda_{1,(10,2)})}{\sum_{s=1}^{10}(\lambda_{1,(s,1)}+\lambda_{1,(s,2)}+\lambda_{2,(s,1)}+\lambda_{1,(s,2)})} \end{bmatrix}.$$

These two variables are indeed determined in the Shimer economy.

4.2 Implementation

Our estimation procedure is based on matching the empirical second moments of the joint distribution of $\hat{\theta}$ and $\hat{\psi}$ to their theoretical counterparts. We obtain the optimal minimum distance estimator of the model parameters. Parametric restrictions on the model primitives aid in the identification of the model and also in easing the computational burden of estimation. Our inferences on the nature of assortative matching are based on estimates from several different specifications of the production technology. We use several representative

[3]For the overall economy, these counts are obviously equal in the real data; however, within each estimating sector (NAICS sector) they will not be equal.

technologies to capture the assumptions of comparative advantage for low-skilled workers in high productivity jobs (LCA) and no comparative advantage (NCA). We also fit the model under a quasi-nested combination of these production technologies.

4.2.1 Parametrization

We have 30 empirical moments from the covariance matrix for each employer size decile and an additional 10 moments from the disribution of realized jobs across size classes. With the model restricted to two latent skill groups for workers and two latent productive types for firms within each size class, the most general specification of the model primitives would have 38 parameters associated with the production technology, 1 for the distribution of workers across skill groups, and 18 for the number of vacancies for jobs of each type. To satisfy the order condition for identification, we specify parameteric restrictions that express assumptions about the relationship between firm size, vacancies, and productivity. Each of these is discussed in turn below.

There are $M = 2$ types of workers; the first parameter is α, the share of the type-1 persons in the population (α is between 0 and 1). The total number of employed workers J is known, but not the size of the labor force. We calibrate the unemployment rate at 6% (the results appear to be robust with respect to this parameter). Hence $\mu_1 = \alpha J/0.94$, $\mu_2 = (1-\alpha)J/0.94$.

We consider 10 different size classes with two latent productive types per size class for a total of 20 different types of firms. The index n of the firm is (s, j), where $s = 1, ..., 10$ indicates the size class and $j = 1, 2$ indicates the type of the firm. The type of the firm represents a variation in the production function between productive and unproductive firms. The size categories are constructed so that each size category contains the same number of employed workers, J_s. The number of jobs (vacant and filled) is not observed, but the number of employed workers by size category is observed. We model the number of jobs as

$$\begin{aligned}
\nu_{s,j} &= \pi_{s,j} \left[1 + \exp(\gamma_0 + \gamma_1 \log(\sigma_s))\right] J_s \\
\pi_{s,2} &= 1/\left[1 + \exp(\delta_0 + \delta_1 \log(\sigma_s))\right] \\
\pi_{s,1} &= 1 - \pi_{s,2}
\end{aligned}$$

where $\nu_{s,j}$ is the number of jobs in firm (s, j), $\pi_{s,j}$ is the share of type-j firms in the size category s, σ_s is the average size of the firms in size category s (σ_s is increasing in s).

The productive type of a job has two dimensions, the size-class, s, of the firm into which it is aggregated, and a latent productive type, $j \in \{1, 2\}$. The effect of size class depends only on a single parameter, ϕ, and we allow the effect of the latent type to be specified fairly generally. Since (s, j) is two-dimensional when n is one-dimensional, and because production must increase in n to apply Shimer's results, our parameterization must sort output, $x_{(m,i),(n,j)}$, in increasing order at every stage of our estimation procedure along the (s, j) dimension.

We estimate several specifications of the production technology that imply different relationships between the supply and demand for skill on the one hand, and the matching of skills to productive characteristics on the other. The LCA techology is additively separable in firm and worker types, and therefore admits comparative advantage for low-ability workers in high-productivity jobs. The NCA technology is multiplicative, and therefore admits no comparative advantage. For all of these parameterizations, we specify that the worker's skill type, m, and the employer's latent productive type, j, combine thus:

$$\begin{aligned}\bar{x}_{1,1} &= \chi_0 \\ \bar{x}_{2,1} &= \chi_0 + \chi_1 \\ \bar{x}_{1,2} &= \chi_0 + \chi_2 \\ \bar{x}_{2,2} &= \chi_0 + \chi_1 + \chi_2\end{aligned}$$

where $\bar{x}_{m,n}$ is the mean over $(m,i),(n,j)$. To assist identification, we assume that the productive capital of the employer scales linearly with the log of firm size, σ_s. Our empirical strategy is based on estimates of two different specifications for the production technology that embody different assumptions about the nature of comparative advantage in each sector:

- The LCA technology parameterizes output as $f_{LCA}(m,s,j) = \bar{x}_{m,j} + \phi \log(\sigma_s)$
- The NCA technology parameterizes output as $f_{NCA}(m,s,j) = \exp\left(f_{LCA}(m,s,j)\right)$

χ_0 is fixed exogenously for identification purposes to be 0.1. Since the production technology is required to be increasing in the productive types of both agents, we restrict $\chi_1 > 0$ and $\chi_2 > 0$. In addition to these specifications, we also estimate a quasi-nested model by considering all production functions that can be obtained as a convex combination of the two basic LCA and NCA technologies. That is, we consider a technology of the form:

$$f_{mix}(m,s,j) = \xi f_{LCA}(m,s,j) + (1-\xi) f_{NCA}(m,s,j).$$

4.2.2 Computation

We fit the parameter $\zeta = (\alpha, \chi_1, \chi_2, \phi, \delta_0, \delta_1, \gamma_0, \gamma_1, \xi)$ to the observed data by the generalized method of moments. While the estimation strategy is standard conceptually, it is complicated by the fact that there is not a general closed-form solution for the coordination friction equilibrium given an arbitrary set of model primitives. Thus, there is no general analytic closed form for the moment equations, nor for their first- and second-derivatives. We instead use a Monte Carlo approach, simulated annealing, to find the parameter estimate $\hat{\zeta}_A$ that minimizes the generalized distance between the theoretical and empirical moments. This procedure requires sampling the parameter space many times, solving the model at each draw to compute the theortical moments and thereby the distance from the empirical moments. Solving the coordination friction equilibrium for an arbitrary set of parameters is accomplished by first solving the planner's problem numerically for the equilibrium queues, and then calculating the wages from the corresponding decentralized economy. The details of these computational methods are outlined below, after first discussing the general setup for the GMM estimation.

Model estimation is complicated by the absence of a closed form expression for the model equilibrium in terms of the parameters. It is not possible to explicitly compute the partial derivatives of the moment equations with respect to the model parameters. We therefore turn to simulation methods to find the GMM estimator. The moment estimator, $\hat{\zeta}_A$ minimizes

$$\hat{\zeta}_A = \arg\min Q_A(\zeta) = \arg\min(\bar{m}(D) - f(\zeta))^T A\,(\bar{m}(D) - f(\zeta))$$

Simulated annealing is an optimization technique that can be thought of as a variant of greedy search that allows for 'uphill' moves. The uphill moves prevent the algorithm from getting stuck in a local minimum. The probability of such a move decreases over the course of the algorithm. During early iterations, the algorithm can easily bounce out of local minima, leading to a more complete exploration of the parameter space.

Given the unusual nature of the problem, it is important to check that the results are robust. In this spirit, we employ two methods to solve for the minimum distance estimator: simulated annealing and a more standard non-linear solver from the `KNITRO` package. Simulated annealing has the advantage that it does not require the stepwise evaluation of first- or second derivatives of the objective function. It also has a theoretical guarantee of convergence to the global minimum. `KNITRO` is a local solver that can be applied to global minimization by using a grid of starting values. If `KNITRO` converges to a better solution than simulated annealing, or can improve the solution found through simulated annealing, then we have not tuned the simulated annealing algorithm properly.

Simulated annealing follows an iterative structure starting from an initial guess, ζ_0 using the algorithm:

1. Given ζ_{t-1}, propose candidate ζ_t
2. Compute $(\eta(\zeta_t), \mu(\zeta_t), x(\zeta_t))$
3. Given $(\eta(\zeta_t), \mu(\zeta_t), x(\zeta_t))$, solve for equilibrium $(q(\zeta_t), w(\zeta_t))$
4. Use $(q(\zeta_t), w(\zeta_t))$ to compute theoretical $(\theta(\zeta_t), \psi(\zeta_t), \lambda(\zeta_t))$
5. Compute $Q(\zeta_t)$
6. Check $Q(\zeta_t)$ against stopping conditions.

The simulated annealing algorithm gives specific details on the process by which ζ_t is proposed and the stopping conditions. When the algorithm is properly specified, the sequence $(\zeta_t)_t$ will converge to the true distance-minimizing parameter. We use the Adaptive Simulated Annealing (ASA) algorithm described in Ingber (1993). Simulated annealing is known to be effective on other estimation problems (Goffe et al. 1994), and the ASA variant of simulated annealing has been successfully applied in a few economic settings. In spite of theoretical assurances of convergence tuning the algorithm to the problem is essential.

At each iteration in the algorithm, we must solve the decentralized equilibrium of the coordination friction economy. We exploit the theoretical result from Shimer's paper that the equilibrium queue lengths are identical in the decentralized economy and in a centralized economy with a planner that faces coordination constraints equivalent to those faced by the workers and employers. At each iteration in the minimization, it is sufficient to solve the planner's problem by choosing the vector of applicant queues that maximizes the economy's expected output. Again, because there is no closed form for these queues in terms of the model primitives, or the parameters, we have to solve this problem by numerical methods. This is a constrained nonlinear optimization in 40 decision variables. We have used the built-in Matlab function `fmincon` as well as `KNITRO` to solve this problem at each iteration with comparable results.

Once the simulated annealing procedure converges to an estimate, $\hat{\zeta}_A$, we use standard results to estimate its covariance matrix as:

$$(F^T AF)^{-1} F^T ASAF (F^T AF)^{-1}$$

where $F = F(\zeta) = \frac{\partial f(\zeta)}{\partial \zeta}$. $F(\hat{\zeta}_A)$ is computed using automatic differentiation at the parameter estimate. We compute the sample variance of $\bar{m}$, denoted S, using a bootstrap procedure. From the full data, we compute $\bar{m}$ on 1,000 bootstrap samples and compute the empirical covariance matrix for the elements of m. We continue to impose the restriction that there is no correlation between the sample moments across size classes, except for the estimates of $\bar{m}^S(D)$.

4.2.3 Identification

The parametric restrictions on the relationship between employer size and the number of vacancies and the production technology yield an order condition necessary for identification. It remains to be shown that model satisfies the sufficient condition for identification:

$$E(\bar{m}(D)) - f(\zeta) = 0 \Leftrightarrow \zeta = \zeta_0$$

where ζ_0 is the true parameter of the data generating process. We take note of a theoretical result which states that as long as $F(\zeta) = \frac{\partial f(\zeta)}{\partial \zeta}$ has constant rank in a neighborhood of ζ_0 and full-column rank at ζ_0, then ζ_0 is locally identified (Ruud 2003). Noting that the smallest singular value of $F(\zeta_0)$ is the 2-norm distance of F from the space of rank-deficient matrices (Golub and Van Loan, 2008), it is sufficient to check this value is non-zero, since this combined with the continuous differentiability of F in ζ ensures that there is a neighborhood of ζ_0 for which F always has full column rank.

5 Results

We have obtained results from estimating the model under the LCA technology, NCA technology, and a nested technology in which the output from every match is a convex combination of the LCA and NCA technologies for that match given the same parameter values. In the nested model, the parameter ξ is the weight on the LCA technology. Table 2 shows estimates obtained from estimating the LCA, NCA, and nested models on data for the Professional, Scientific and Technical Services sector and the Manufacturing sector from the LEHD sample. We present results only for the general specification that mixes the two types of production technology. The estimates of the LCA and NCA technologies are equivalent to restrictions on the value of ξ. Where appropriate, we comment on the validity of those assumed restrictions.

5.1 The Professional, Scientific and Technical Services Sector

5.1.1 Model Primitives

The estimated parameters reveal the type of workers, the type of jobs, and the production technology in the sector. Our best estimate of the distribution of workers available for work in the sector indicates that 86% of the workforce is of high ability, a reasonable result given the nature of the sector. These workers sort themselves into jobs across deciles that are distinguished in terms of productivity. Figure 7 shows the split between high and low productivity vacancies across firm size deciles. We find that the concentration of high ability workers is not matched by the distribution of vacancies. The majority of vacancies in this sector are of low productivity.

The estimated production technology is illustrated in figure 8. Output is measured in implied units that have been scaled by our assumption that $\chi_0 = 0.1$. There is very little productive heterogeneity by employer size. However, there is a productive advantage associated with the latent employer type, but most of the output differentials within this sector are associated with matches of high ability workers into high productivity employers. Output in those matches is roughly four times greater than the output in other matches. Combining the estimated production technology with the estimated distribution of vacancies, we note that this sector is composed of a large low-productivity segment of jobs in which it does not matter much what kind of worker is employed. These are complemented by a small high productivity segment in which there is a substantial advantage to employing high ability workers.

5.1.2 Characteristics of the Estimated Equilibrium

The estimated equilibrium wages and job assignments in the Professional Services sector reflect a small degree of positive assortative matching of workers to jobs. The equilibrium wages, illustrated in figure 9, reflect the peculiarities of the coordination friction equilibrium. Note that even though there is a very high production advantage to high ability workers in high ability jobs, the low ability workers actually have a higher equilibrium wage offer from high ability employers. In terms of the model, this occurs because there are so few high productivity jobs and so many high ability workers that competition limits the wages that high ability workers receive. At the same time, the high productivity employers limit their risk of vacancy by making sure even low ability workers are willing to apply for those jobs. The high wage offers received by low ability workers is compensation for the unemployment risk they face in applying for those jobs. The expected income from applying for such a job has to balance the expected income from applying for a low productivity job, which can almost always be obtained due to the large number of low productivity vacancies relative to the number of job-seekers.

The positive assortative matching result follows from the fact that the estimated production technology is primarily one with no comparative advantage. How the assortativity of matching manifests itself in this sector is revealed in figures 10 and 11. The first figure shows the probability that a low ability worker applies to high productivity and low productivity employers. The second shows the same probabilities for high ability workers. For both ability types, the application probabilities sum to one. Most of the mass is concentrated on the low productivity jobs since there are many more of them, and the shape is goverened by the way vacancies are distributed across the size classes. Note that the probability of application to low productivity jobs is considerably higher for low ability workers. In other words, better workers are more likely to apply for better jobs, and, of course, to receive them conditional on applying for them. These application strategies result in the expected distribution of realized matches shown in figures 12 and 13.

The low productivity vacancies are occupied by a mixture of high and low ability workers, as shown in figure 12. We also see that there is a large number of unoccupied vacancies in equilibrium. By contrast, high productivity jobs, which are a small fraction of the total number of jobs, are almost entirely occupied in equilibrium by high ability workers. Based on these estimates of the number of occupied vacancies, we estimate that 88% of the employed workforce is of high productivity. In addition, fully 85% of the filled vacancies are of low productivity.

The estimates give an overall impression of this sector as being split between a large, low productivity segment and a small high productivity segment. In the low productivity segment, workers' ability differentials are largely irrelevant to production. Furthermore, there are sufficiently many vacancies that the coordination friction is not binding. Workers can be relatively certain of obtaining a job conditional on applying for it. By contrast, the high productivity segment has very few vacancies and pays much higher wages. Thus, many workers queue for, but fail to receive, these jobs. The relatively mild correlation we observe between wage components hides the extent of sorting both because there are relatively few high productivity vacancies and low ability workers. In addition, the low ability workers actually earn higher wages when they are employed in high productivity jobs.

5.2 The Manufacturing Sector

5.2.1 Model Primitives

The estimated parameters in the manufacturing sector are listed in the second column of table 2. We estimate that 92% of the workforce participating in this sector is of low productivity. This sector is characterized by a dominance of low productivity vacancies. Figure 14 shows the distribution of vacancies between high and low productivity jobs across deciles of the employer size distribution. Here though, the estimates suggest that most of the high productivity vacancies are concentrated in smaller firms, rather than being evenly distributed by size. The mixing parameter on the production technology, ξ is essentially equal to 1, providing strong evidence that there is a comparative advantage for low ability workers in high productivity jobs. The estimated output indicates that both worker and employer types have a relevant impact on output.

5.2.2 Characteristics of the Estimated Equilibrium

The estimated equilibrium wages (figure 16) show that high ability workers receive the same wage in most jobs that they actually occupy in equilibrium. By contrast, low ability workers receive a wage premium when they obtain high productivity jobs. The application behaviors of both types of workers reflect the incentives created by these wages and the nature of comparative advantage that is ultimately reflected in the equilibrium assignment of workers to jobs. Low ability workers are more likely to apply to high productivity jobs than are high ability workers. As a result, we see in figure 20 that a large portion of the high productivity vacancies are filled by low ability workers.

When we consider the estimated equilibrium job matches, we find that 88% of the workforce employed in this segment is of low ability, and 98% of the occupied job vacancies are of low productivity. Therefore, there is very little variation, especially for larger firms in employer type. Hence, the small correlations between estimated wage components are an artefact of the very small amount of heterogeneity in employer types in the sector. Nevertheless, the model does a rather good job of capturing the empirical correlation between $\hat{\theta}$ and $\hat{\psi}$. Figure 21 shows the estimated and actual correlations. The model captures both the overall location of these correlations as well as some of the variation across deciles of the employer size distribution.

6 Conclusion

We implemented a formal test of Shimer's model of job assignment with coordination frictions. The full structural model was estimated using the method of moments. The model predicts the equilibrium number of job matches as well as the empirical moments of the joint distribution of estimated person and employer effects from a linear decomposition of log labor earnings. Our estimates of this model for two major sectors of the U.S. economy provide evidence that there is matching in the labor market in terms of comparative advantage, but the empirical relevance of matching is attenuated because there is insufficient heterogeneity of workers and employers for assignment to influence the distribution of earnings.

This paper provides some of the first estimates of a structural job assignment model with transferrable surplus using matched employer-employee data. These estimates allow a structural interpretation of the observed correlations between person and employer effects from the log wage decomposition. While we have focused in this paper on estimating the assignment model with coordination friction, our empirical strategy is general. Any model that delivers a wage offer function which depends on both worker and employer type and which predicts the way in which workers match to jobs in equilibrium could be estimated using this strategy.

Estimates from both sectors, Professional Services and Manufacturing, have similar characteristics. The underlying production technology should generate comparative advantage. In the case of Professional Services, there is positive assortative matching, while in Manufacturing the estimated production technology has a comparative advantage for low ability workers in high productivity employers. In both cases, the influence of sorting by comparative advantage is limited because there are very few of the workers who should be sorting into low productivity jobs. Likewise, there is a very limited number of high productivity vacancies. In other words, each sector has a small, high productivity segment that generates unemployment based on queueing.

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Appendices

A An Economy with Heterogeneous Workers and Firms

In this Appendix we derive worker and firm effects in an economy with only two types of workers and a continuum of firms differing by size and production technology. Notation in this appendix differs slightly from the notation in the main paper in order to accommodate the more general specification.

A.1 The Economy

Let the economy consist of two types of workers. This is not as limiting as it sounds, for it may only represent heterogeneity inside a particular class of workers since we consider the effects net of observables. The economy is divided in a finite number $\bar{k}$ of industries. The workers are hired by a continuum of firms, indexed by the share x of type 1 workers each firm hires, the size t of the firm, and the industry k. The share x is unobserved by the econometrician. In industry k, there is a mass (density) $\mu_k(x,t)$ of firms of type (k,x,t). We omit from what follows the k index except when it is necessary.

The economy as a whole is represented by the space $\Omega = \{\Omega_1, \ldots, \Omega_{\bar{k}}\}$ with $\Omega_k = [0,1] \times \mathbb{R}^+$, the space defining each industry. There is a quantity (or number) $J = \int_\Omega d\mu$ of firms, $J_k = \int_{\Omega_k} d\mu_k$ in each industry. Firm (x,t) hires xt of type 1 workers, and $(1-x)t$ of type 2. We write $\int_\Omega xt\, d\mu(x,t) = M$ and $\int_\Omega (1-x)t\, d\mu(x,t) = N$ so that there are $M+N$ workers in the economy. Type 1 workers earn $w(x,t,k)$ in firm (x,t) in industry k while type 2 earn $u(x,t,k)$; the wage only depends on the industry, the share of type 1 workers, and the size of the firm.

A.2 The Economy as Estimated by Person and Firm Effects

As in Abowd et al. (1999) (hereafter, AKM99), decompose the individual wage, $\ln w_{i,j(i,t),t}$, after conditioning for observables, in each period as the sum of an individual effect, θ_i, a firm effect, $\psi_{j(i,t)}$, and a residual, $\varepsilon_{i,j(i),t}$

$$\ln w_{i,j(i,t),t} = \theta_i + \psi_{j(i,t)} + \varepsilon_{i,j(i),t}$$

We consider here only the limiting (and purely theoretical) case where the number of time periods goes to infinity and the workers visit ergodically all firms and all industries. This frees us from the identification problem discussed in detail in the above paper. Moreover, the effects estimated in AKM99 would converge in probability to the theoretical values we show below.

In this case, even though the type of the worker is not directly known by the econometrician, observing the workers jump from job to job provides enough information to be able to reduce the above ordinary least squares (OLS) problem to the following minimization problem. Find W, U (worker effects) and $\psi(k,x,t)$ (firm effect in industry k for a firm of size t hiring a proportion x of type 1 workers) that solve

$$\min_{W,U,\psi} \int_\Omega \left[xt\,(w(x,t) - W - \psi(x,t))^2 + (1-x)t\,(u(x,t) - U - \psi(x,t))^2\right] d\mu(x,t). \tag{12}$$

This problem is exactly equivalent to minimizing the sum of squared residuals in ordinary least squares. The solutions to this problem are unique up to a constant. We show that:

Theorem 2 Under integrability assumptions on w and u, the solutions to the OLS program (12) are of the type

$$W = \frac{\int x(1-x)tw(x,t)}{\int x(1-x)t} + \Lambda, \tag{13}$$

$$U = \frac{\int x(1-x)tu(x,t)}{\int x(1-x)t} + \Lambda, \tag{14}$$

$$\psi(x,t) = x(w(x,t) - W) + (1-x)(u(x,t) - U) - \Lambda \tag{15}$$

where the integrals are all taken on Ω with respect to the measure μ, and Λ is an arbitrary constant (we will set $\Lambda = 0$ in the rest of the paper).

Proof. Set $U = 0$ for the time being, as the solutions are only identified up to a constant. Let us write $J(W,\psi) = \int_\Omega \rho(W,\psi)(x,t)\,d\mu(x,t)$. J is defined on $\mathbb{R} \times L^2(\Omega,\mu)$, and may take the value $+\infty$. It is clear that ρ is convex in its two arguments (as sum of two squares); hence J is also convex, and can be show to be strictly convex if $w(x,t) - u(x,t)$ is not a constant. Hence it has a unique minimum under this assumption, characterized by the first order constraints

$$\int_\Omega xt(w(x,t) - W - \psi(x,t))\,d\mu(x,t) = 0 \tag{16}$$

$$\int_\Omega xt(w(x,t) - W - \psi(x,t)) + (1-x)t(u(x,t) - \psi(x,t)) = 0 \tag{17}$$

Hence $W = \int xt(w-\psi)\,d\mu / \int xt\,dB$ and $\psi = x(w - W) + (1-x)u$. Replacing ψ by this last value in equation 16 and simplifying with the fact that $\int x^2 t = M - \int x(1-x)t$ and $\int x^2 tw = \int xtw - \int x(1-x)w$ proves the theorem. ■

Remark 3 (Some special cases) If all firms hire the same number of both types of workers, the distribution μ_k for all k is $\delta_{1/2,2}$, a Dirac distribution of firms of equal size and same number of type 1 workers. This leads to $W = \sum_k w_k/\bar{k}$, $U = \sum_k u_k/\bar{k}$, $\psi_k = \frac{1}{2}(w_k - \bar{w} + u_k - \bar{u})$, where w_k is the wage in industry k.
The case where the distribution μ has all mass in t at $t = 2$ and $w(x,t) = u(x,t) + \delta$ leads to $W = U + \bar{\delta}$ and $\psi(x,t) = u(x) - U$.

We define the firm-employee effects covariances $\mathrm{cov}(\hat{\theta}, \hat{\psi})$ as

$$\frac{\int (xtW + (1-x)tU)\psi(x,t)\, d\mu}{\int t\, d\mu} - \frac{\int (xtW + (1-x)tU)\, d\mu}{\int t\, d\mu} \frac{\int \psi(x,t)\, d\mu}{\int t\, d\mu} \tag{18}$$

For total covariance the integrals are taken over Ω, for within-sector covariance over Ω_k for all k, and the between-sector covariance is the discrete covariance between the means (i.e. integrals) on the Ω_k. Total covariance is as usual the sum of between-sector and within-sector covariances.

B Data Appendix

B.1 The LEHD Data

We use data from the Longitudinal Employer-Household Dynamics Program (LEHD), universe data for four large American states with information from 1990-2004. See Abowd et al. (2004) and Abowd et al. (2009) for more detailed discussions of the underlying data.

Unemployment Insurance:

The individual data were derived from the universe of unemployment insurance (UI) quarterly wage records from four of the following seven states: California, Florida, Illinois, Maryland, Minnesota, North Carolina, and Texas.[4] The BLS Handbook of Methods (1997) describes UI coverage as "broad and basically comparable from state to state," and claims over 96 percent of total wage and salary civilian nonfarm jobs were covered in 1994. The Federal Unemployment Tax Act (FUTA), first enacted in 1938, lays the ground rules for the kinds of employment which must be covered in state unemployment insurance laws. While technically mandating coverage of all employers with one or more employees in a calendar year, FUTA allows for numerous exceptions to covered employment (Stevens 2002). These include workers at small agricultural co-operatives, employees of the federal government, and certain employees of state governments, most notably elected officials, members of the judiciary, and emergency workers. According to the Handbook, UI wage records measure "gross wages and salaries, bonuses, stock options, tips, and other gratuities, and the value of meals and lodging, where supplied." They do not include OASDI, health insurance, workers compensation, unemployment insurance, and private pension and welfare funds. Individuals are uniquely identified and followed for all quarters in which their employers had reporting requirements in the UI system. Thus, cross-state mobility can be observed for individuals moving between any of the seven states for which we have data. Although coverage dates vary, all states provide between seven and thirteen years of data. Table 1 in Abowd et al. (2003) (ALM, hereafter) details the starting dates and number of individuals appearing in each of these states up to the year 2000. In our analysis, by combining them into a single

[4]We cannot disclose which states are included in the analysis as a condition imposed by the Census Bureau on the public use data. We can, however, release our analysis moment matrices to any user who requests them as they have been designated as public-use data.

"pooled" file, we have information on approximately 75 million workers, accounting for over 40% of the U.S. workforce.

Creation of Variables:

Using Census Bureau and other LEHD data bases, sex, race, date of birth, and education are combined with the individual earnings data.[5] When a variable was created with an exact link to another database, the actual value from that data source is used. When a variable was created with a statistical link to another database, the value of the variable is imputed 10 times, thereby providing information on the precision of the statistical links. Upon each individual's first appearance in the data, labor force experience as potential labor force experience (age - education - 6) is calculated. In subsequent periods, experience is measured as the sum of observed experience and initial (potential) experience. The UI wage records connect individuals to every employer from which they received wages in any quarter of a given calendar year. Therefore, individual employment histories are constructed using the same personal identifier used in the individual data. Employers are identified by their state unemployment insurance account number (SEIN). While large employers undoubtedly operate in multiple states, their SEINs are unfortunately state specific, meaning they cannot be connected. In addition, while we match workers to their employers, it is not possible to connect those employed in firms with multiple establishments to specific places of work. This problem is not overly pervasive, as over 70% of employment occurs in firms with only a single establishment. There are approximately 4.7 million firms in this version of the data.

Earnings:

For every year an individual appears in a state, a "dominant" employer — the employer for whom the sum of quarterly earnings is the highest—is identified in order to better approximate the individual's full-time, full-year annual wage rate using the following steps. First, define full quarter employment in quarter t as having an employment history with positive earnings for quarters $t-1, t$, and $t+1$. Continuous employment during quarter t means having an employment history with positive earnings for either $t-1$ and t or t and $t+1$. Employment spells that are neither full quarter nor continuous are designated discontinuous. If the individual was full quarter employed for at least one quarter at the dominant employer, the annualized wage is computed as 4 times average full quarter earnings at that employer (total full quarter earnings divided by the number of full quarters worked). This accounts for 84% of the person-year-state observations in our eventual analysis sample. Otherwise, if the individual was continuously employed for at least one quarter at the dominant employer, the annualized wage is average earnings in all continuous quarters of employment at the dominant employer multiplied by 8 (i.e., 4 quarters divided by an expected employment duration during the continuous quarters of 0.5). This accounts for 11% of all observations. For the remaining 5%, annualized wages are average earnings in each quarter multiplied by 12 (i.e., 4 quarters divided by an expected employment duration during discontinuous quarters of 0.33).

Annual Hours of Work and Full-time Status:

[5] Sex, race, and date of birth are based on an exact match to administrative data. Education is based on a statistical match.

We restrict our sample to individuals who worked full-time for their dominant employer. Full-time status is taken to mean that the individual worked at least 35 hours per week. The average number of hours per week for a worker is given by dividing the number of hours worked over the year with the dominant employer by the number of weeks worked. The number of weeks worked is approximated based on the observed number of full, continuous, and discontinuous quarters with one's dominant employer. Specifically, we compute weeks worked as 13 times the number of full quarters + 6.5 times the number of continuous quarters + 4.33 times the number of discontinuous quarters. Annual hours of work are observed for a subset of individuals in the sample via a database link to other LEHD and Census sources. For those missing annual hours of work, a value is statistically assigned using a Bayesian multiple imputation procedure. Bjelland (2007) provides complete details of the imputation of annual hours of work.

Our analysis sample is restricted to individuals aged 18-70, employed full-time at their dominant employer. Table C presents sample means for several earnings, demographic, industry, and labor force attachment variables for the period 1990-2000. The final analysis sample contains 278 million person-year-state observations for the aforementioned 75 million individuals and 4.7 million firms. In comparison to the base LEHD file, this analysis file has considerably higher wages and earnings, and is slightly more educated, male, white, and experienced.

C Tables and Figures

Table 1: Summary of the LEHD Data

N	277,527,039
Log Raw Earnings ($2000)	9.64
Education	13.48
Male	0.57
Age	37.82
White	0.62
Annual Hours	1952.46
Theta (Person-Effect)	0.07
Psi (Firm-Effect)	0.12
Employer Size	3543.69
Native	0.84

Note: Earnings data originate from Unemployment Insurance wage records of four states in the LEHD database between 1990-2004. Demographic characteristics were added through database links to other Census Bureau and LEHD sources. Education and annual hours of work have been imputed using multivariate multiple imputation procedures. The sample is restricted to full-time workers, aged 18-70, employed in their dominant jobs. *Source:*Authors' calculations.

Table 2: Estimates of the Model Parameters

Parameter	Prof. Svcs.	Mfg.
α	0.139	0.924
	(0.0001)	(0.0003)
χ_1	2.133	0.767
	(0.0007)	(0.0009)
χ_2	3.829	3.106
	(0.0116)	(0.0030)
ϕ	0.027	0.0282
	(0.0004)	(0.0001)
δ_0	2.322	0.358
	(0.0063)	(0.0047)
δ_1	0.022	1.034
	(0.0009)	(0.0021)
γ_0	1.681	-0.276
	(0.0507)	(0.0321)
γ_1	-2.216	-1.616
	(0.0376)	(0.0102)
ξ	0.105	0.999
	(0.0048)	(0.0000)
Residual	0.0900	0.0214

GMM parameter estimates for the nested model that mixes the two technologies according to the parameter ξ. See text for details. Standard errors in parentheses.

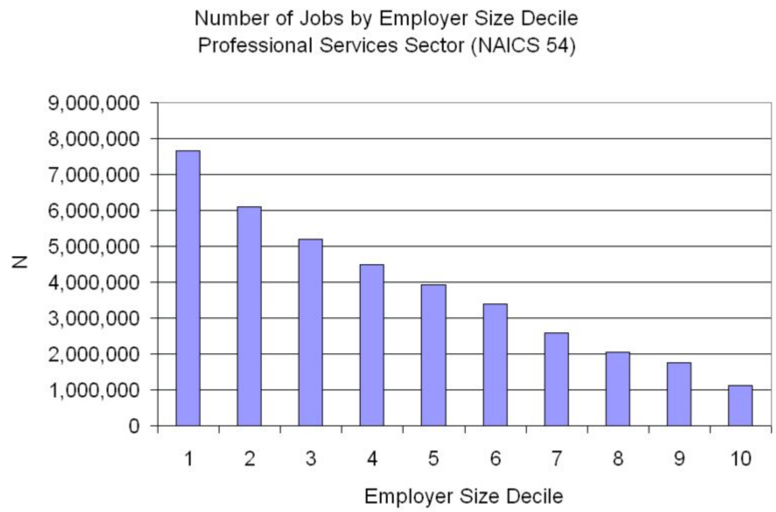

Figure 1: Average $\hat{\theta}$ and $\hat{\psi}$ by employer size class: Professional, Scientific and Technical Services Sector (NAICS 541)

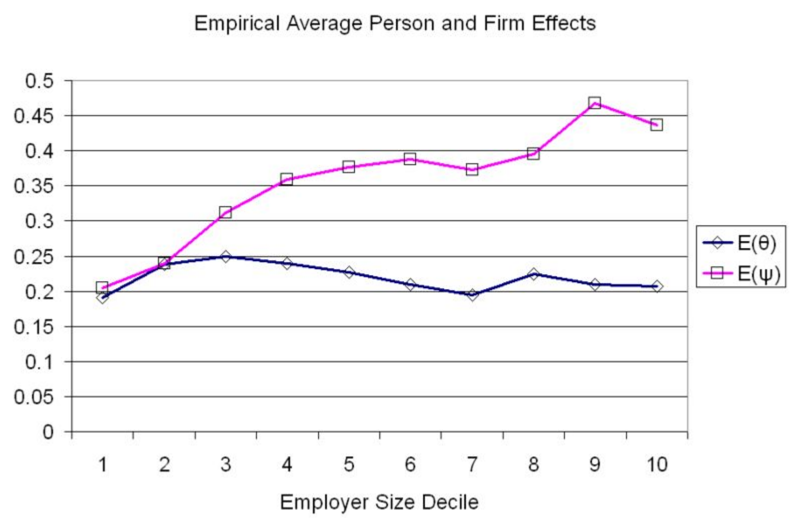

Figure 2: Average $\hat{\theta}$ and $\hat{\psi}$ by employer size class: Professional, Scientific and Technical Services Sector (NAICS 541)

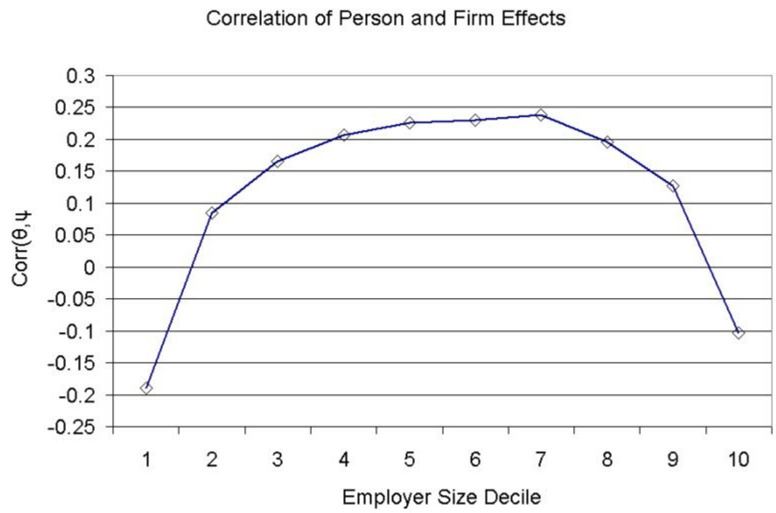

Figure 3: Correlation of $\hat{\theta}$ and $\hat{\psi}$ by employer size class: Professional, Scientific and Technical Services Sector (NAICS 541)

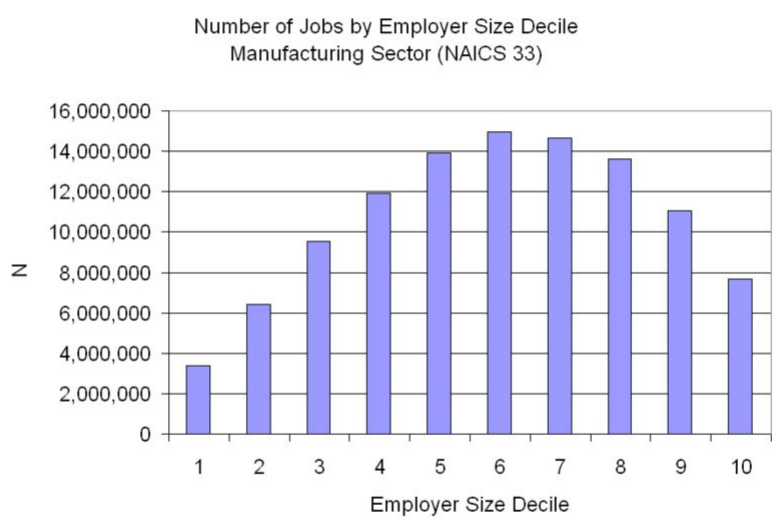

Figure 4: Average $\hat{\theta}$ and $\hat{\psi}$ by employer size class: Manufacturing Sector (NAICS 31-33)

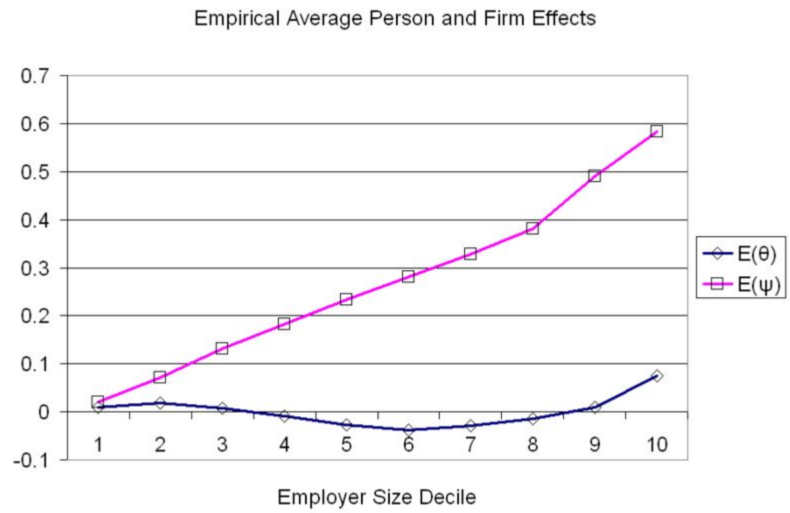

Figure 5: Average $\hat{\theta}$ and $\hat{\psi}$ by employer size class: Manufacturing Sector (NAICS 31-33)

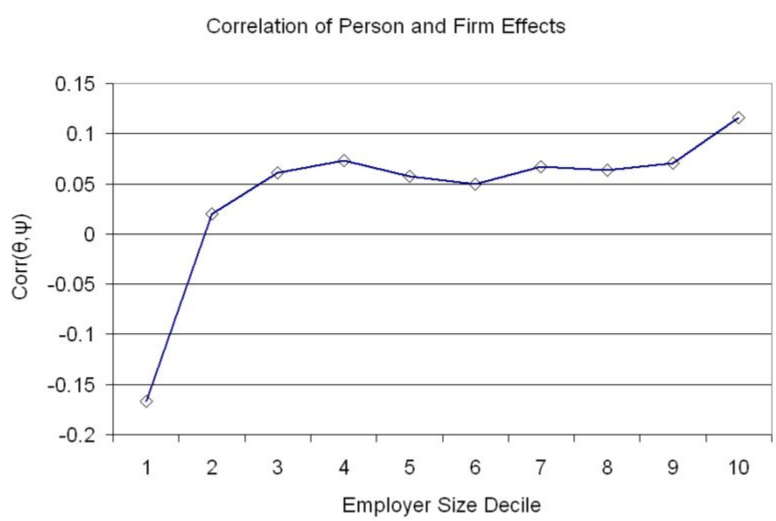

Figure 6: Correlation of $\hat{\theta}$ and $\hat{\psi}$ by employer size class: Manufacturing Sector (NAICS 31-33)

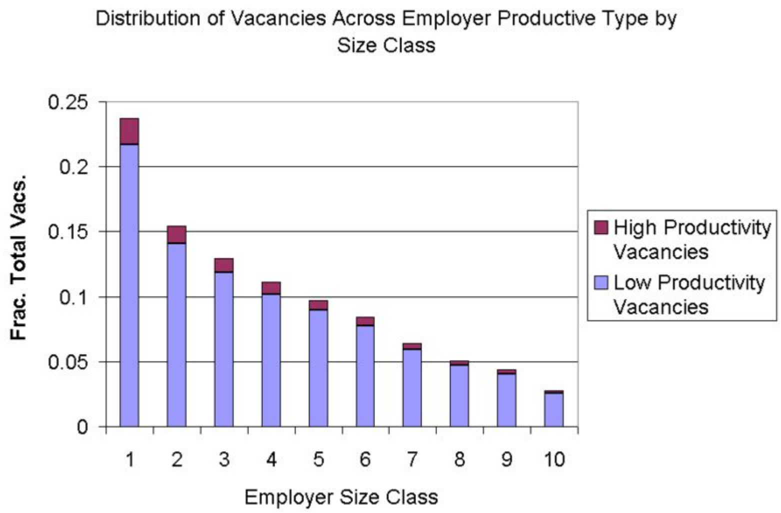

Figure 7: Estimated Distribution of Vacancies by Employer Size Decile: Professional, Scientific and Technical Services Sector (NAICS 541)

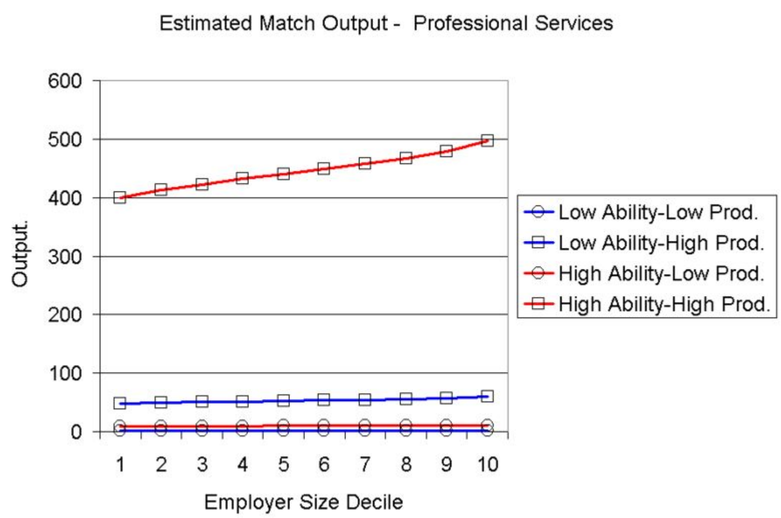

Figure 8: Estimated Output: Professional, Scientific and Technical Services Sector (NAICS 541)

Figure 9: Estimated Equilibrium Wage Offers: Professional, Scientific and Technical Services Sector (NAICS 541)

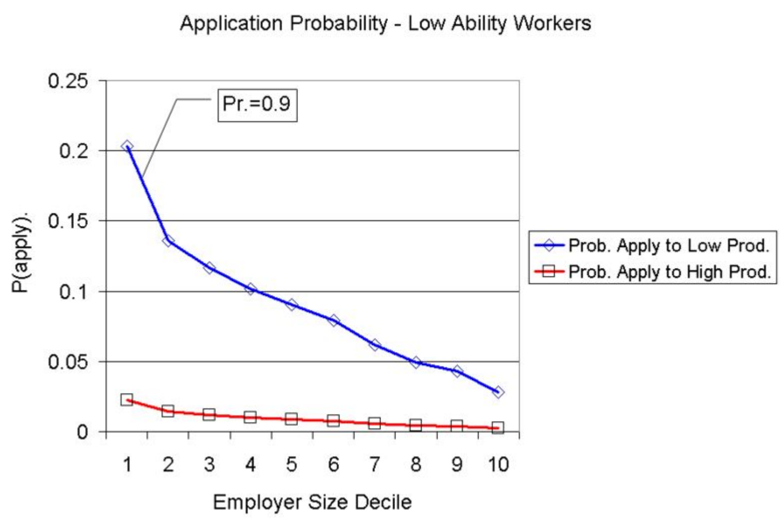

Figure 10: Estimated Equilibrium Application Probabilities for Low Ability Workers: Professional, Scientific and Technical Services Sector (NAICS 541)

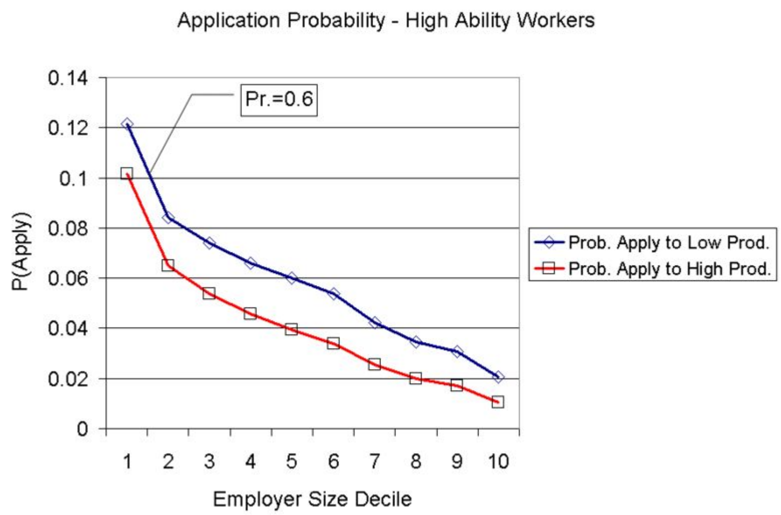

Figure 11: Estimated Equilibrium Application Probabilities for High Ability Workers: Professional, Scientific and Technical Services Sector (NAICS 541)

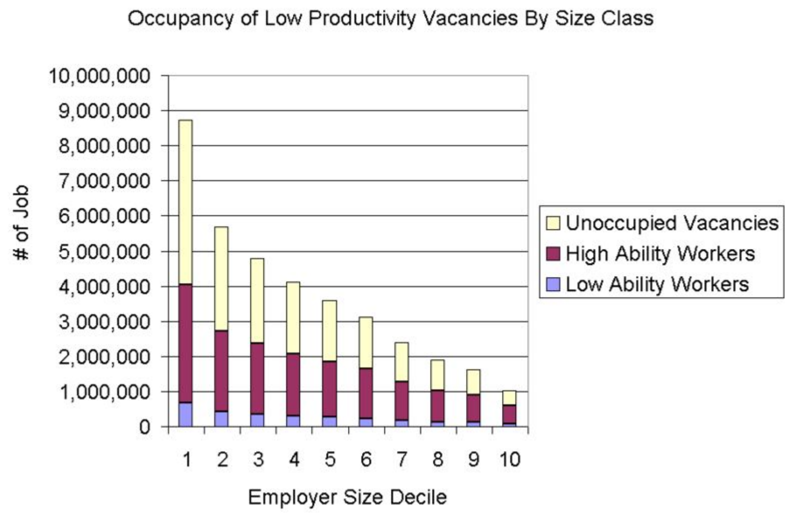

Figure 12: Estimated Equilibrium Occupancy of Low Productivity Jobs: Professional, Scientific and Technical Services Sector (NAICS 541)

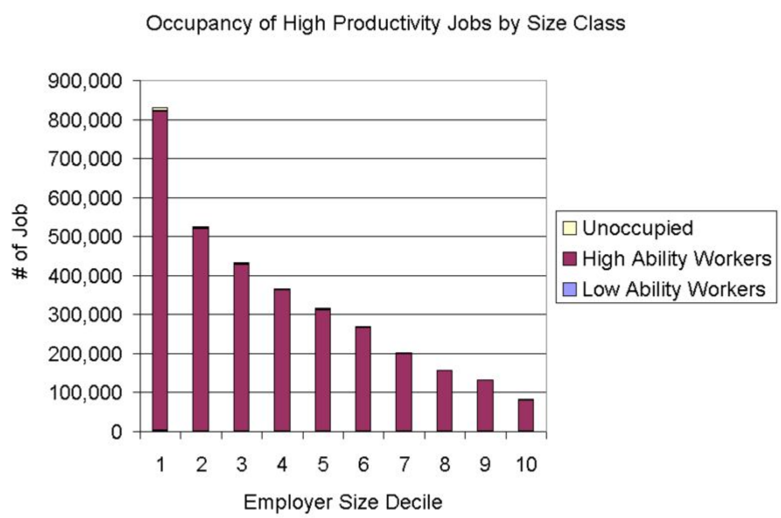

Figure 13: Estimated Equilibrium Occupancy of High Productivity Jobs: Professional, Scientific and Technical Services Sector (NAICS 541)

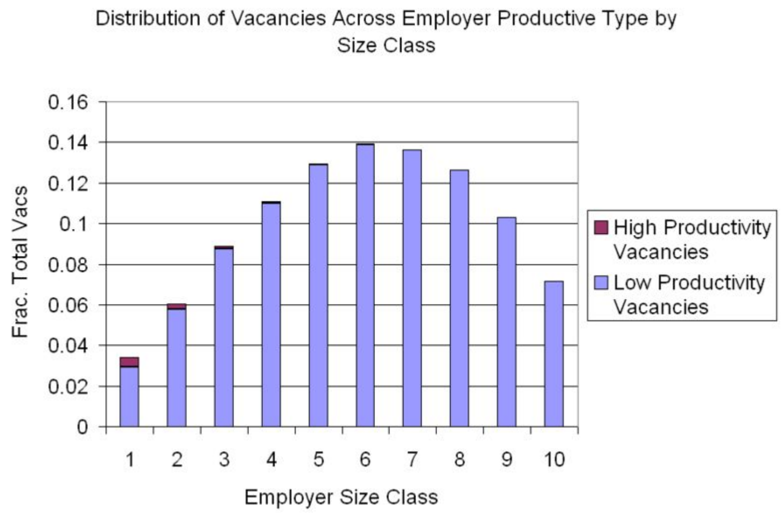

Figure 14: Estimated Distribution of Vacancies by Employer Size Decile: Manufacturing Sector (NAICS 31-33)

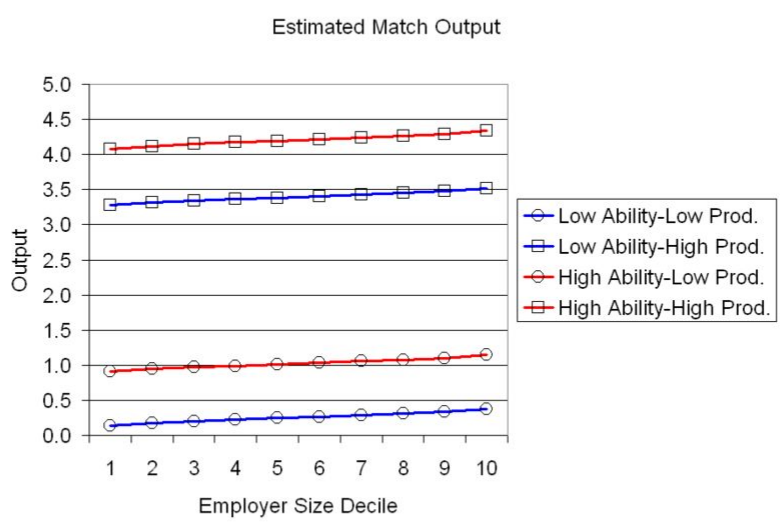

Figure 15: Estimated Output: Manufacturing Sector (NAICS 31-33)

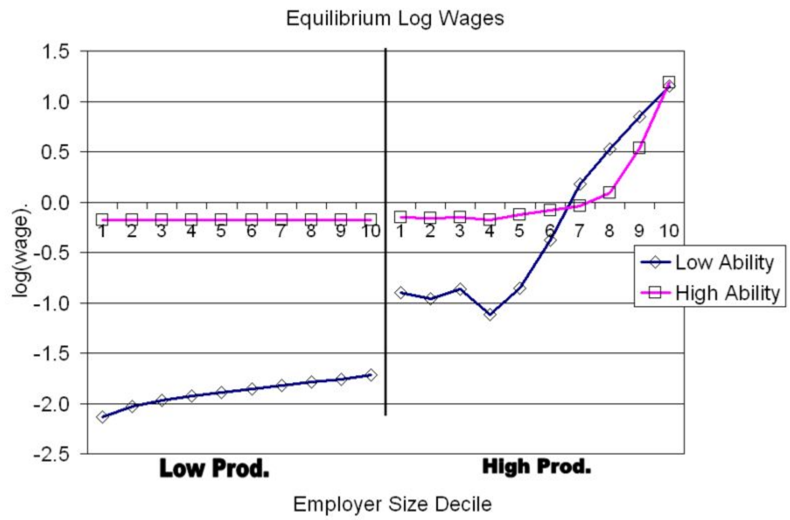

Figure 16: Estimated Equilibrium Wage Offers: Manufacturing Sector (NAICS 31-33)

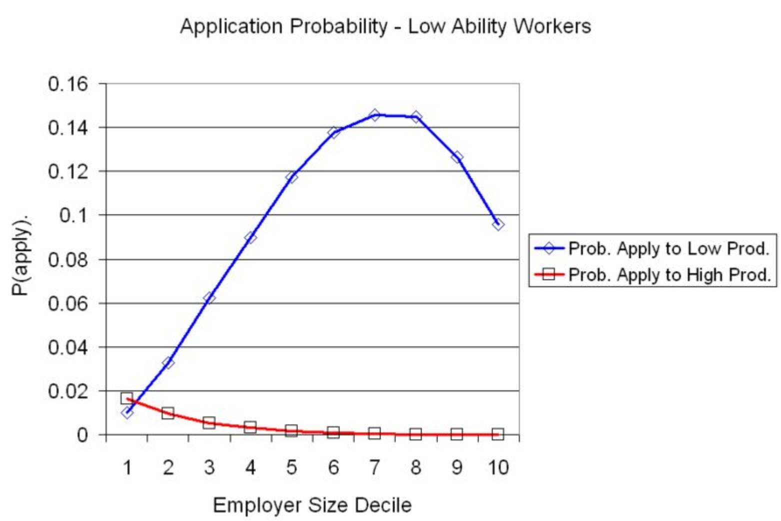

Figure 17: Estimated Equilibrium Application Probabilities for Low Ability Workers: Manufacturing Sector (NAICS 31-33)

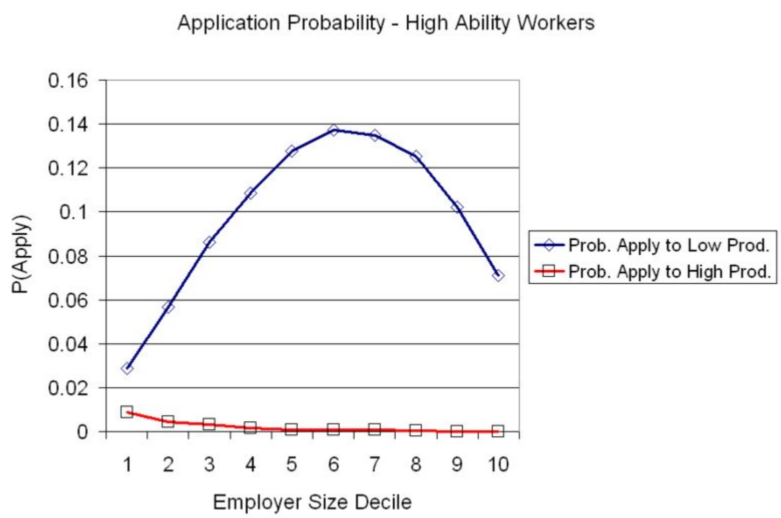

Figure 18: Estimated Equilibrium Application Probabilities for High Ability Workers: Manufacturing Sector (NAICS 31-33)

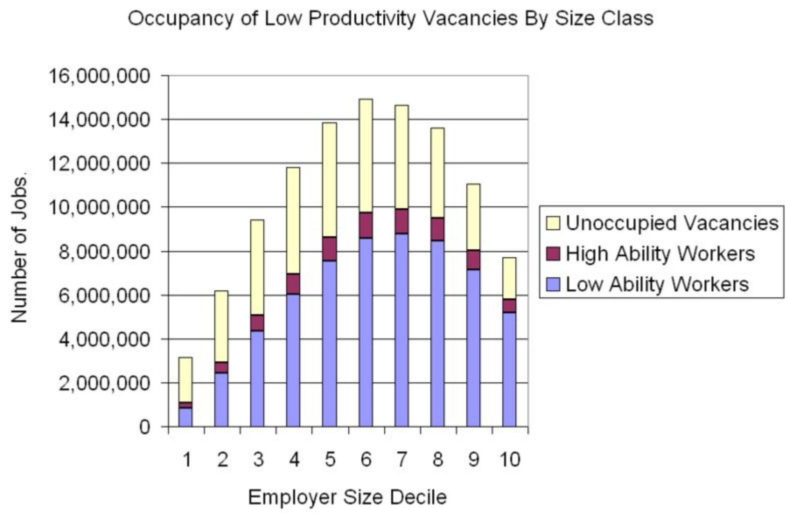

Figure 19: Estimated Equilibrium Occupancy of Low Productivity Jobs: Manufacturing Sector (NAICS 31-33)

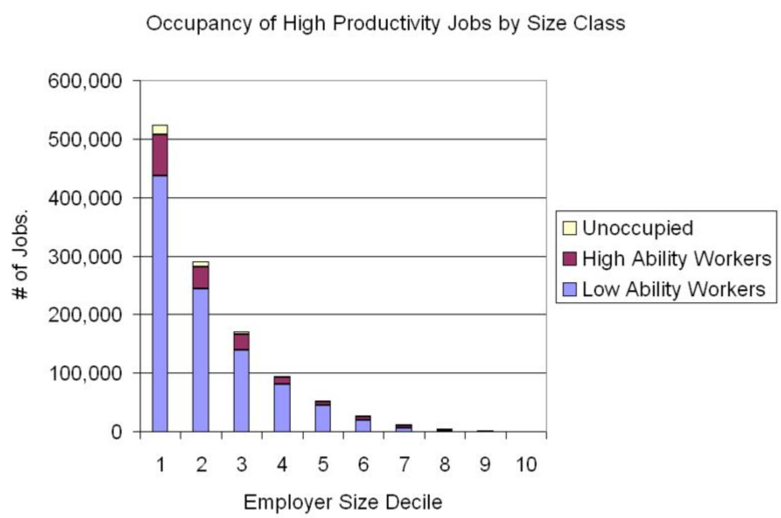

Figure 20: Estimated Equilibrium Occupancy of High Productivity Jobs: Manufacturing Sector (NAICS 31-33)

Figure 21: Estimated Equilibrium Occupancy of High Productivity Jobs: Manufacturing Sector (NAICS 31-33)