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Sec - SPL2

Tutorial - I

Design And Analysis of Algorithms

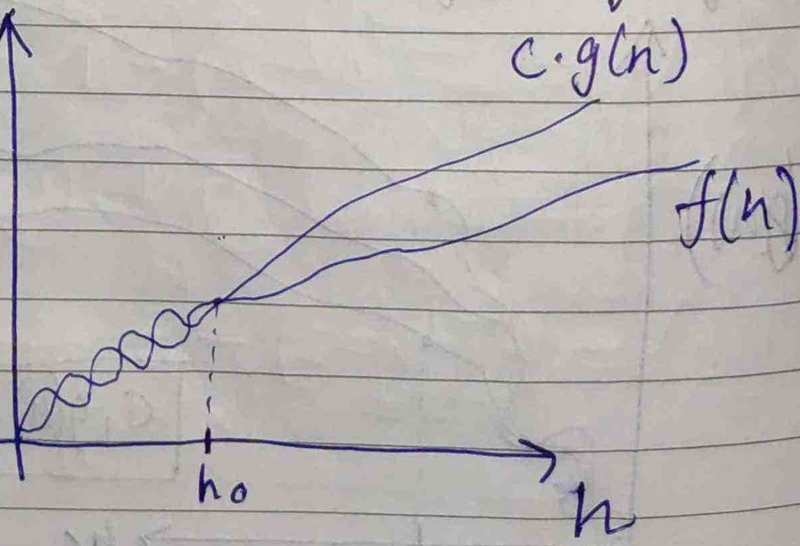
Ans 1. Asymptotic Notations are the Mathematical Notations used to describe the running time of an Algorithm when the inputs tends to a limiting value.

- The efficiency is measured with the help of asymptotic notations.

(1) <u>Big-O notation</u>

- represents upper bound of the running time of an algorithm

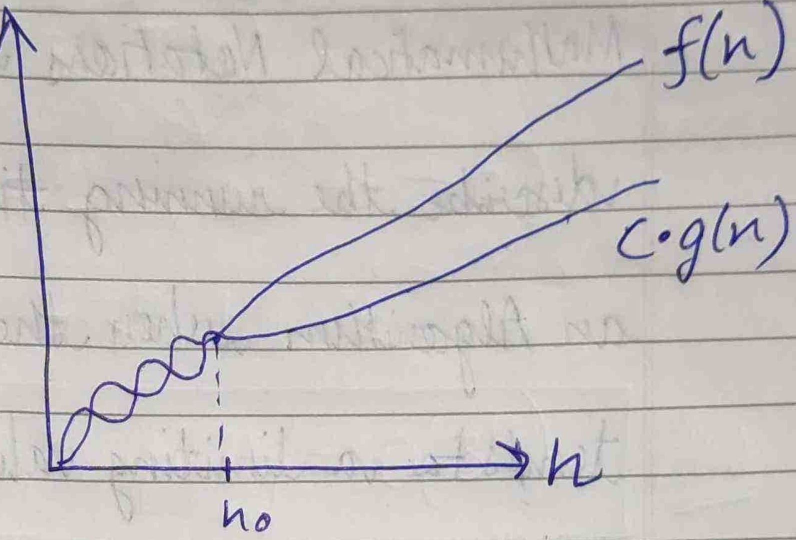
$f(n) = O(g(n))$



(2) Omega Notation (Ω)

- represents the lower bound of the running time of an algorithm.

$f(n) = \Omega(g(n))$

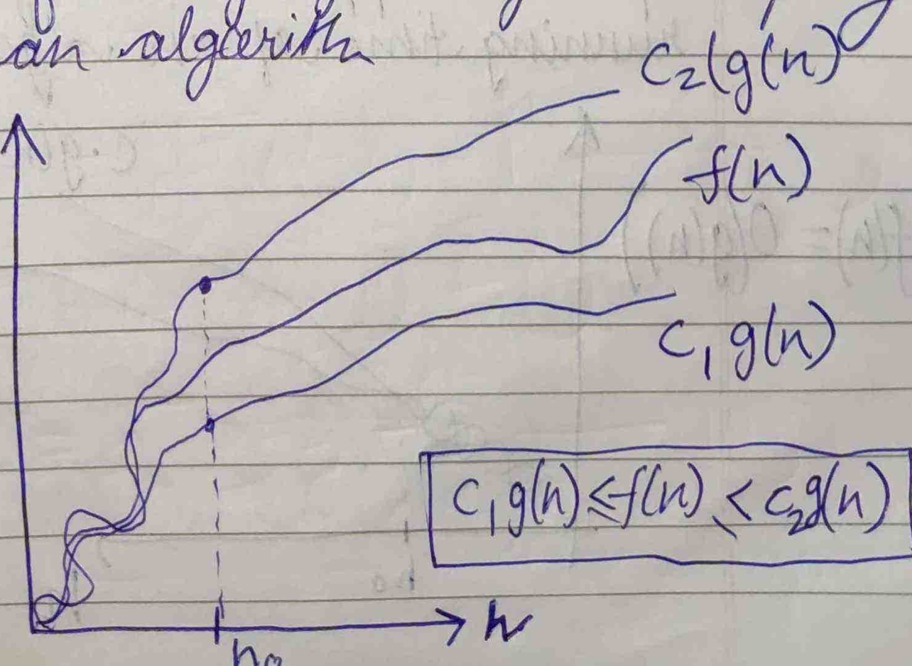


(3) Theta Notation (θ)

- encloses the function f(n) from above and below

- used for analyzing average-case complexity of an algorithm

$f(n) = \theta(g(n))$



$$\boxed{c_1 g(n) \leq f(n) < c_2 g(n)}$$

Ans2. for(i=1 to n){
i=i*2;
}

$i = 1, 2, 4, 8, 16 \ldots n$

$a=1, \ r=2$

$t_k = ar^{k-1}$

$n = (1)(2)^{k-1}$

$n = 2^{k-1}$ (taking log both sides)

$\log_2 n = \log_2 2^{k-1}$ $\because (\log_2 2^a = a)$

$\log_2 n = k-1$

$k = 1 + \log_2 n$

T.C. = $\underline{\underline{O(\log_2 n)}}$ Ans.

Ans 3. $T(n) = \begin{cases} 3T(n-1) & \text{if } n>0 \\ 1 & \text{otherwise} \end{cases}$

putting $n=0$,

$T(0) = 1 \quad (n \neq 0)$

$T(n-1) = 3T(n-2)$ —— (I)

putting (I) in $T(n)$

$T(n) = 3T(n-1)$

$= 3\left[3T(n-2)\right]$

$T(n) = 9T(n-2)$ —— (II)

$T(n-2) = 3T(n-3)$ —— (III)

putting (III) in (II)

$T(n) = 27T(n-3)$ —— (IV)

$T(n) = 3^k T(n-k)$

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put $n-k=0$

$k=n$

$T(n) = 3^n \, T(n-n)$

$T(n) = 3^n \, T(0)$

$T(0) = 1$ (as $n \ngtr 0$)

$T(n) = 3^n \cdot 1$

$\boxed{\text{Time Complexity} = O(3^n)}$ Ans

Ans 4. $T(n) = \begin{cases} 2T(n-1) - 1 & \text{if } n>0 \\ 1 & \text{else} \end{cases}$

$T(n) = 2T(n-1) - 1$ —— (I)

put $n = n-1$

$T(n-1) = 2T(n-2) - 1$ —— (II)

put $T(n-1)$ from (II) to (I)

$T(n) = 4T(n-2) - 3$ —— (III)

put $n = n-2$

$T(n-2) = 2T(n-3) - 1$ —— IV

put $T(n-2)$ from IV to III

$T(n) = 8T(n-3) - 7$ —— V

$T(n) = 2^k T(n-k) - (2^k - 1)$

put $n-k = 0$

$k = n$, $\quad (\because T(0) = 1)$

$T(n) = 2^n T(n-n) - (2^n - 1)$

$T(n) = 2^n T(0) - 2^n + 1$

$T(n) = \cancel{2^n} - \cancel{2^n} + 1$

$\boxed{T(n) = O(1)}$ Ans

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Ans 5.
```
int i=1, s=1;
while(s<=n){
    i++; s=s+i;
    printf("*");
}
```

Let the loop run for k times,

After 1st iteration: $s = s + 1$

After 2nd iteration: $s = s + 1 + 2$

It goes on for k times, as long as 's' is less than equal to 'n'

$1 + 2 + \text{-----} + k <= n$

$\Rightarrow \left(k \left(\frac{1+k}{2} \right) \right) <= n$

$\Rightarrow \left(\frac{k^2 + k}{2} \right) <= n$

$\Rightarrow O(k^2) <= n$

$\Rightarrow \boxed{k = O(\sqrt{n})}$ Time Complexity.

Ans 6.

```
void function (int n){
    int i, count = 0;
    for(i=1; i*i<=n; i++){
        count++;
    }
}
```

$= i*i <= n$

$= i^2 <= n$ (taking root both sides)

$= i <= \sqrt{n}$

$$\sum_{i=1}^{\sqrt{n}} 1 = 1+1+1 - - - - +1 \ (\sqrt{n} \text{ times})$$

$\boxed{T.C. = O(\sqrt{n})}$ Ans

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Ans7.
```
void function(int n){
    int i, j, k, count = 0;
    for(i=n/2; i<=n; i++){
        for(j=1; j<=n; j=j*2){
            for(k=1; k<=n; k=k*2){
                count++;
            }
        }
    }
```

i	j	k
$n/2$	$\log_2 n$	$\log_2 n * \log_2 n$
$n/2+1$	$\log_2 n$	$\log_2 n * \log_2 n$
$n/2+2$	$\log_2 n$	
$n/2+3$		
n	$\log_2 n$	$\log_2 n * \log_2 n$

$$= O\left(\frac{n}{2} * \cancel{\log_2 n} * (\log_2 n * \log_2 n)\right)$$

$$= O(n(\log_2 n)^2) \text{ Ans}$$

Ans 8.

```
function (int n){
    if (n==1) return;        → O(1)
    for(i=1 to n){           → O(n)
        for(j=1 to n){       → O(n)
            printf("*");
        }
    } function (n-3);        → T(n-3)
}
```

$\Rightarrow T(n) = T(n-3) + n^2$

put $n = n-3$

$T(n-3) = T(n-6) + (n-3)^2$

put $T(n-3)$ in $T(n)$

$T(n) = T(n-6) + (n-3)^2 + n^2$

$T(n) = T(n-3k) + (n-$

Ans 9. void function(int n) {

for(i=1 to n) {

for(j=1; j<=n; j+=i) {

printf("*");

}

}

}

i	j
1	{1, 2, 3, 4 ---- n}
2	{1, 3, 5, 7, 9, 11 ---}
3	{1, 4, 7, 10, 13, ----}
4	{1, 5, 9, 13 ------}
|	|
|	|
|	|
|	|
|	|
|	|
n-2	{1, n-1}
n-1	{1, n}
n	{1}

$$\Rightarrow O\left(\sum_{i=1}^{n} 1 + \sum_{i=1\,(i=i+2)}^{n} 1 + \sum_{i=1\,(i=i+3)}^{n} 1 + \sum_{i=1\,(i=i+4)}^{n} 1 + \sum_{i=1\,(i=i+5)}^{n} 1 - - - + - - + - - + \\ - - - - - - + \sum_{i=1\,(i=i+n)}^{n} 1\right)$$

$$\cancel{\Rightarrow O\left((n) + \left(\frac{n+1}{2}\right) + (n^2) +\right.}$$

$$\Rightarrow O\left(n + \frac{n}{2} + \frac{n}{3} + \frac{n}{4} - - - + 1\right)$$

$$\Rightarrow O\left(n\left(1 + \frac{1}{2} + \frac{1}{3} + - - + \frac{1}{n}\right)\right)$$

$$\Rightarrow \boxed{O(n \log n)}$$ Ans.