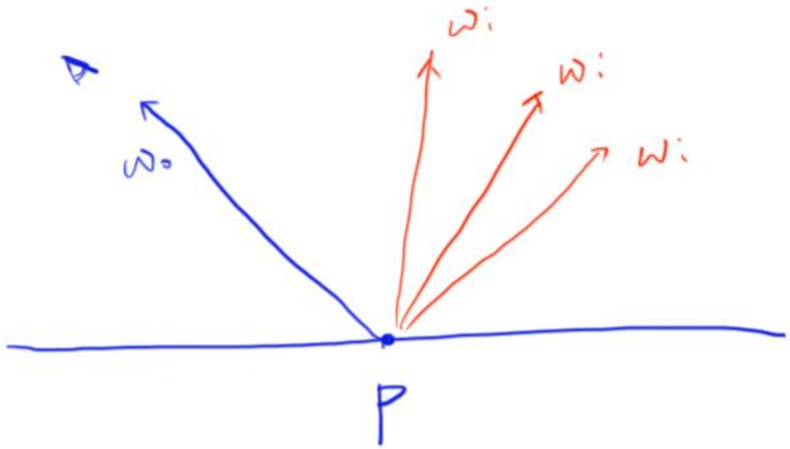
Light Transport

Rendering equation

$$L_o(p,\omega_o) = L_e(p,\omega_o) + \int_{S^2} f(p,\omega_o,\omega_i) L_i(p,\omega_i)\,|\cos(\theta)|\,d\omega_i$$

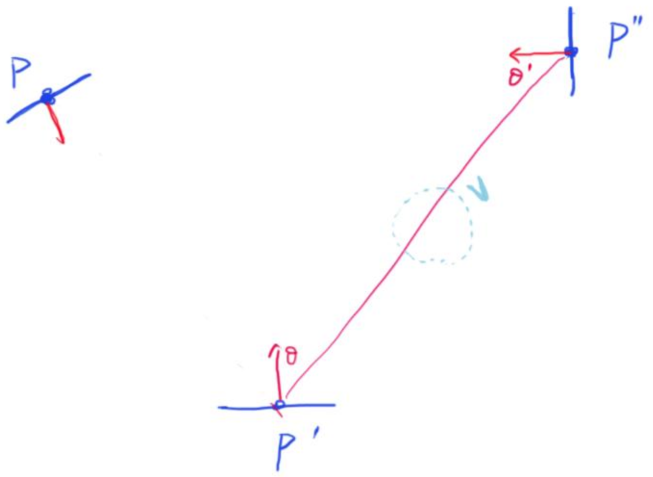


Surface form of the LTE

$$L(p' \to p) = L(p', \omega)$$

$$f(p'' \to p' \to p) = L(p', \omega_o, \omega_i)$$

$$G(p'' \leftrightarrow p') = V(p'' \leftrightarrow p')\frac{|\cos\theta||\cos\theta'|}{\|p'' - p'\|^2}$$



$$L(p' \to p) = L_e(p' \to p) + \int_A f(p'' \to p' \to p) L(p'' \to p') G(p'' \leftrightarrow p') dA(p'')$$

Integral over paths

- Expand L on the right hand side

$$L(p_1 \to p_0) = L_e(p_1 \to p_0) + \int_A L_e(p_2 \to p_1) f(p_2 \to p_1 \to p_0) G(p_2 \leftrightarrow p_1) dA(p_2)$$

$$+ \int_A \int_A L_e(p_3 \to p_2) f(p_3 \to p_2 \to p_1) G(p_3 \leftrightarrow p_2)$$

$$\times f(p_2 \to p_1 \to p_0) G(p_2 \leftrightarrow p_1) dA(p_3) dA(p_2) + \ldots$$

- For the path $\overline{p}_n = p_0, p_1, \ldots p_n$, we define

$$P(\overline{p}_n) = \underbrace{\int_A \int_A \cdots \int_A}_{n-1} L_e(p_n \to p_{n-1}) \times \left(\prod_{i=1}^{n-1} f(p_{i+1} \to p_i \to p_{i-1}) G(p_{i+1} \leftrightarrow p_i) \right) dA(p_2) \cdots dA(p_n)$$

- Then

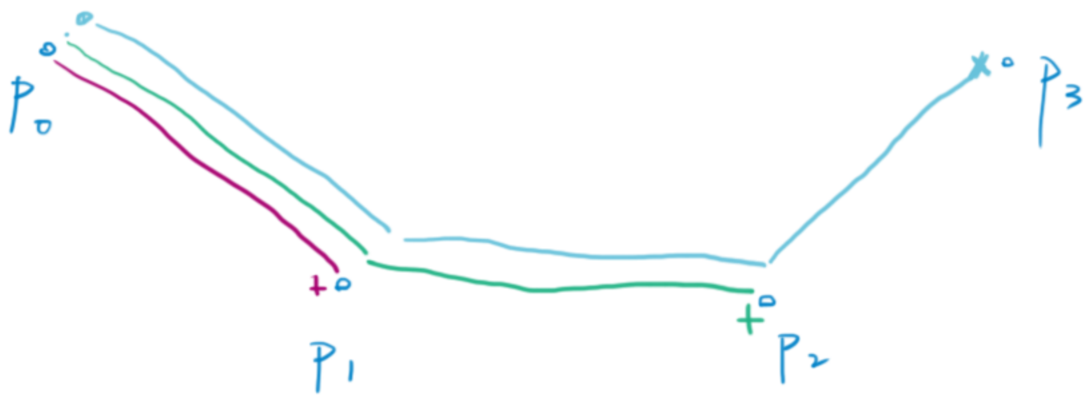
$$L(p_1 \to p_0) = \sum_{n=1}^{\infty} P(\overline{p}_n)$$

Integral over paths

$$L(p_1 \to p_0) = L_e(p_1 \to p_0) + \int_A L_e(p_2 \to p_1) f(p_2 \to p_1 \to p_0) G(p_2 \leftrightarrow p_1) dA(p_2)$$

$$+ \int_A \int_A L_e(p_3 \to p_2) f(p_3 \to p_2 \to p_1) G(p_3 \leftrightarrow p_2)$$

$$\times f(p_2 \to p_1 \to p_0) G(p_2 \leftrightarrow p_1) dA(p_3) dA(p_2) + \ldots$$



Integral over paths

- Define $T(\overline{p}_n) = \prod_{i=1}^{n-1} f(p_{i+1} \to p_i \to p_{i-1}) G(p_{i+1} \leftrightarrow p_i)$ as the throughput as the path

$$P(\overline{p}_n) = \underbrace{\int_A \int_A \cdots \int_A}_{n-1} L_e(p_n \to p_{n-1}) T(\overline{p}_n) dA(p_2) \cdots dA(p_n)$$

Termination

- For unbiased render
- Russian Roulette

$$F' = \begin{cases} \dfrac{F - qc}{1 - q} & \xi > q \\ c & \text{otherwise} \end{cases}$$

$$E[F'] = (1-q)\left(\frac{E[F] - qc}{1-q}\right) + qc = E[F]$$

- For example

$$p(\overline{p}_1) + p(\overline{p}_2) + p(\overline{p}_3) + \frac{1}{1-q}\sum_{i=4}^{n} p(\overline{p}_4)$$

$$\frac{1}{1-q_1}(p(\overline{p}_1) + \frac{1}{1-q_2}(p(\overline{p}_2) + \frac{1}{1-q_3}(p(\overline{p}_3) + \cdots,$$

Path sampling

- Uniform area sampling
- Form j objects, sample the i-the object, the pdf is $p_i = \frac{A_i}{\sum_j A_j}$
- Then uniform sample the surface of the object i, pdf is $\frac{A_i}{\sum_j A_j} \frac{1}{A_i}$
- Then for each sample point $\mathrm{p_i}$

$$p_A(\mathrm{p_i}) = \frac{1}{\sum_j A_j},$$

$$\text{MonteCarlo estimator is } \mathrm{L_e}(p_n \to p_{n-1}) \prod_{i=1}^{n-1} \frac{f(p_{i+1} \to p_i \to p_{i-1}) G(p_{i+1} \leftrightarrow p_i)}{p_A(\mathrm{p_i})}$$

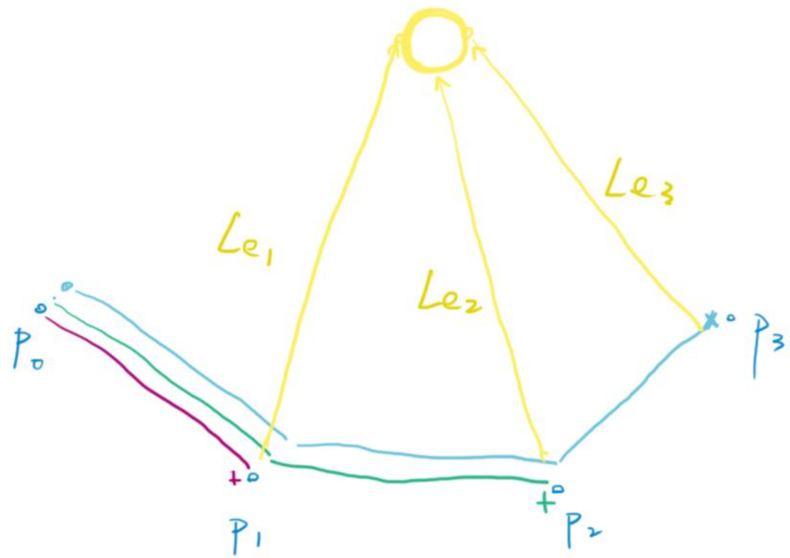
Path sampling

- Path tracing
- Incremental path construction
  - At each vertex, the BSDF is sampled to generate a new direction and find the closest intersection.
  - from last vertex, sample the surface of the light source

because $p_A = p_\omega \frac{|\cos\theta_i|}{\|p_i - p_{i+1}\|^2}$ the Monte Carlo estimator is

$$\frac{L_e(p_i \to p_{i-1}) f(p_i \to p_{i-1} \to p_{i-2}) |\cos\theta_{i-1}|}{P_A(p_i)} \left(\prod_{j=1}^{i-2} \frac{f(p_{j+1} \to p_j \to p_{j-1}) |\cos\theta_j|}{p_\omega(p_{j+1} - p_j)}\right)$$

Implementation



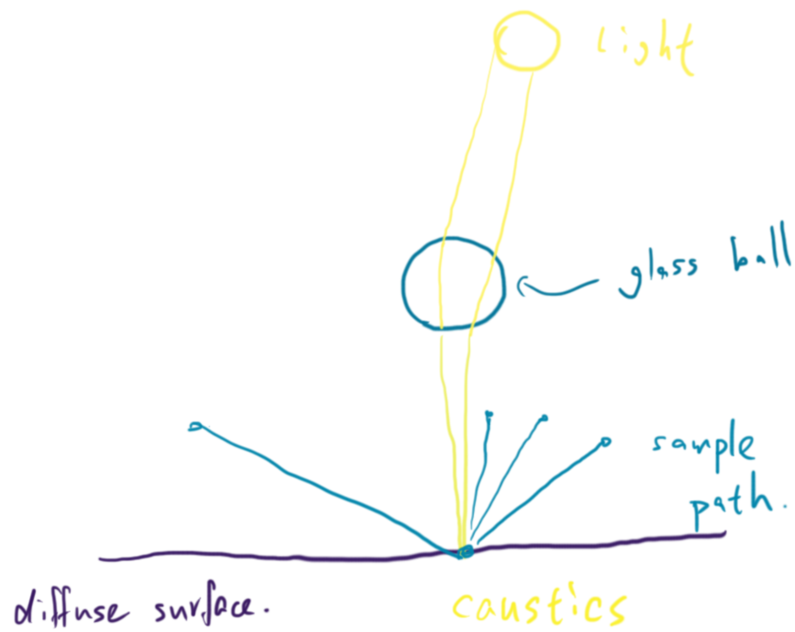
$T_1 = f(p_1)\cos\theta_1$

$T_2 = f(p_2)\cos\theta_2 T_1$

$T_3 = f(p_3)\cos\theta_3 T_2$

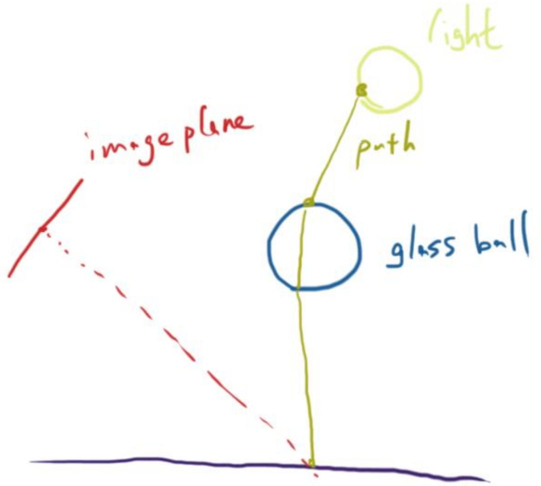
$$L = T_1 L_{e1} + T_2 L_{e2} + T_3 L_{e3}$$

Difficult case for path tracing



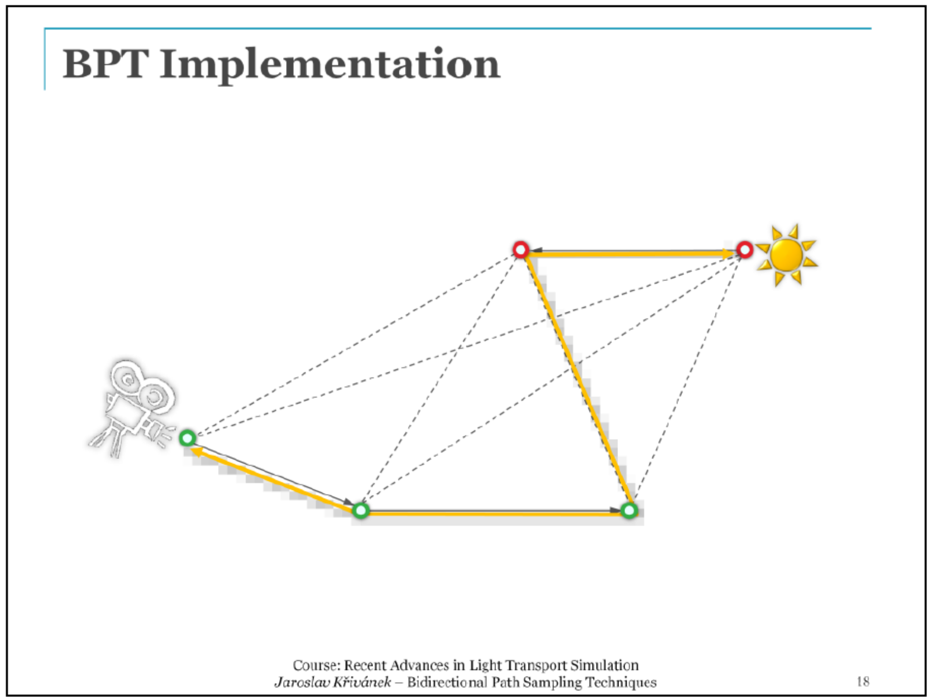
Light tracing

- Reverse of the path tracing
  - Incremental path construction from light
  - Other variants: photon map, instant radiosity (virtual point lights)



Combining path sampling techniques

- Bidirectional path tracing



- For more detail, see SIGGRAPH 2013 course
  - **“Recent Advances in Light Transport Simulation: Theory & Practice”**