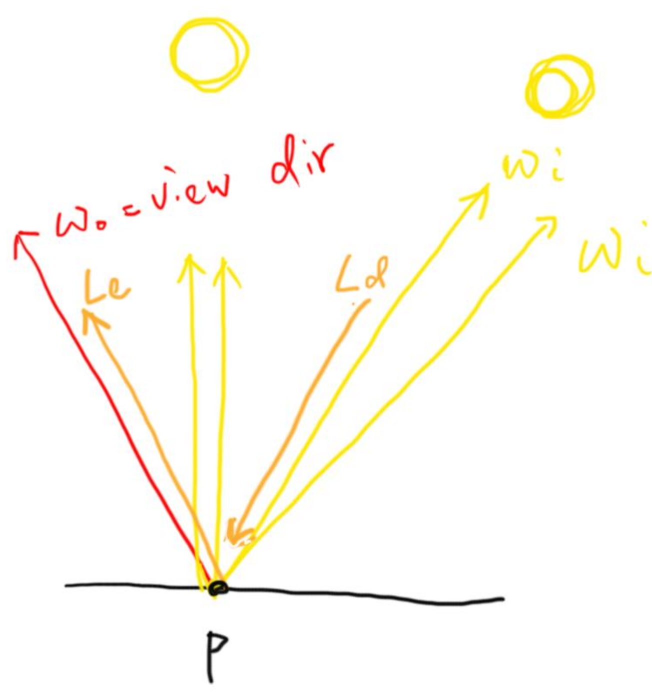
# Monte Carlo:
# Direct Lighting & Importance sampling

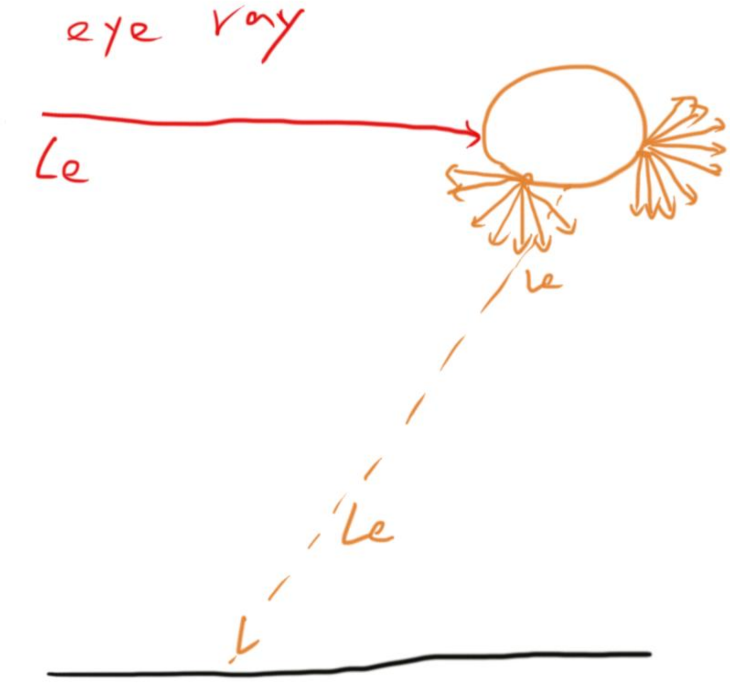
# Direct lighting

$$L_o(p,\omega_o) = L_e(p,\omega_o) + \int_{S^2} f(p,\omega_o,\omega_i) L_d(p,\omega_i)\,|\cos(\theta)|\,d\omega_i$$



# Light

- Suppose we only deal with diffuse area light



# Estimating the direct lighting integral

$$\int_{S^2} f(p,\omega_o,\omega_i)L_d(p,\omega_i)\,|\cos(\theta)|\,d\omega_i$$

$$\frac{1}{N}\sum_{j=1}^{N}\frac{f(p,\omega_o,\omega_i)L_d(p,\omega_i)\,|\cos(\theta_j)|}{p(\omega_j)}$$

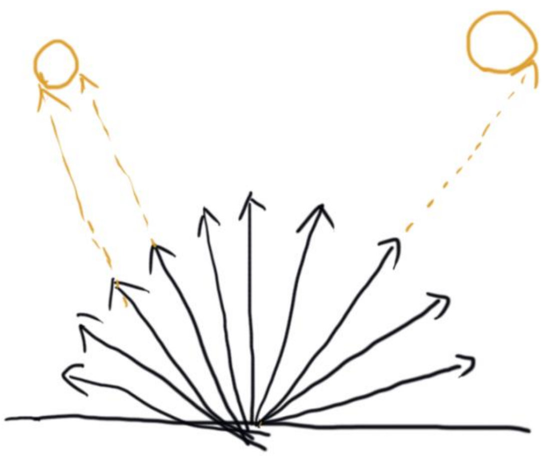
# Sampling strategy

- Uniform sample
- Sample each lights
- Sample one light at a time

# Uniform sample

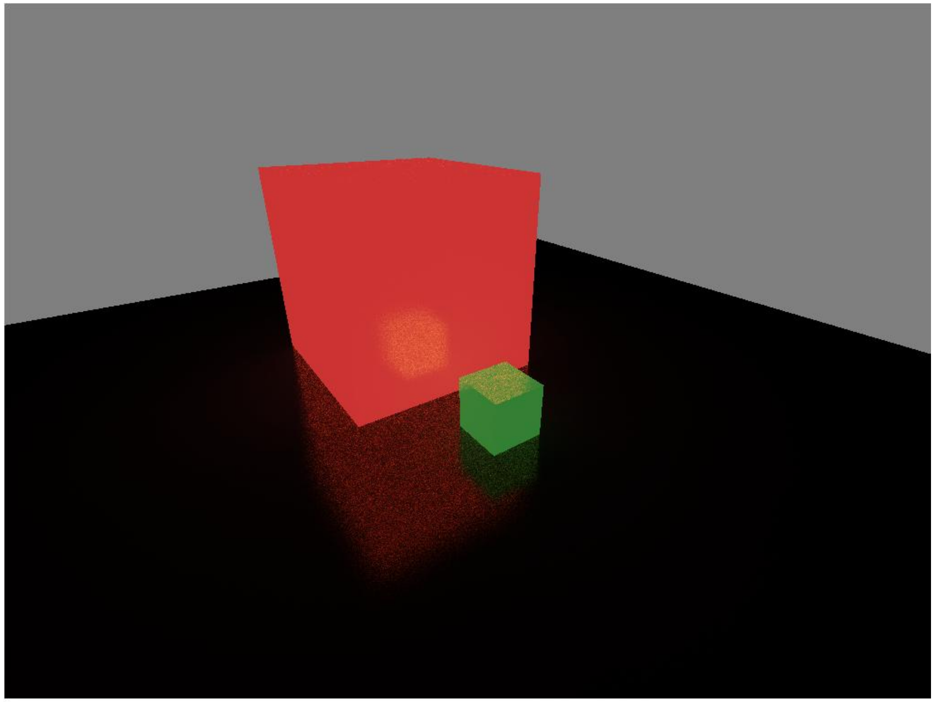
$$\int_{S^2} f(p,\omega_o,\omega_i) L_d(p,\omega_i) \,|\cos(\theta)|\, d\omega_i$$

$$\frac{1}{N}\sum_{j=1}^{N} \frac{f(p,\omega_o,\omega_i) L_d(p,\omega_i) \,|\cos(\theta_j)|}{p(\omega_j)=2\pi}$$



# Uniform sample

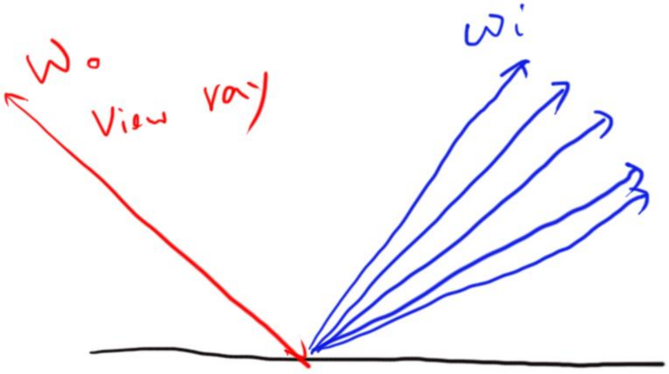
- 1024 samples per pixel



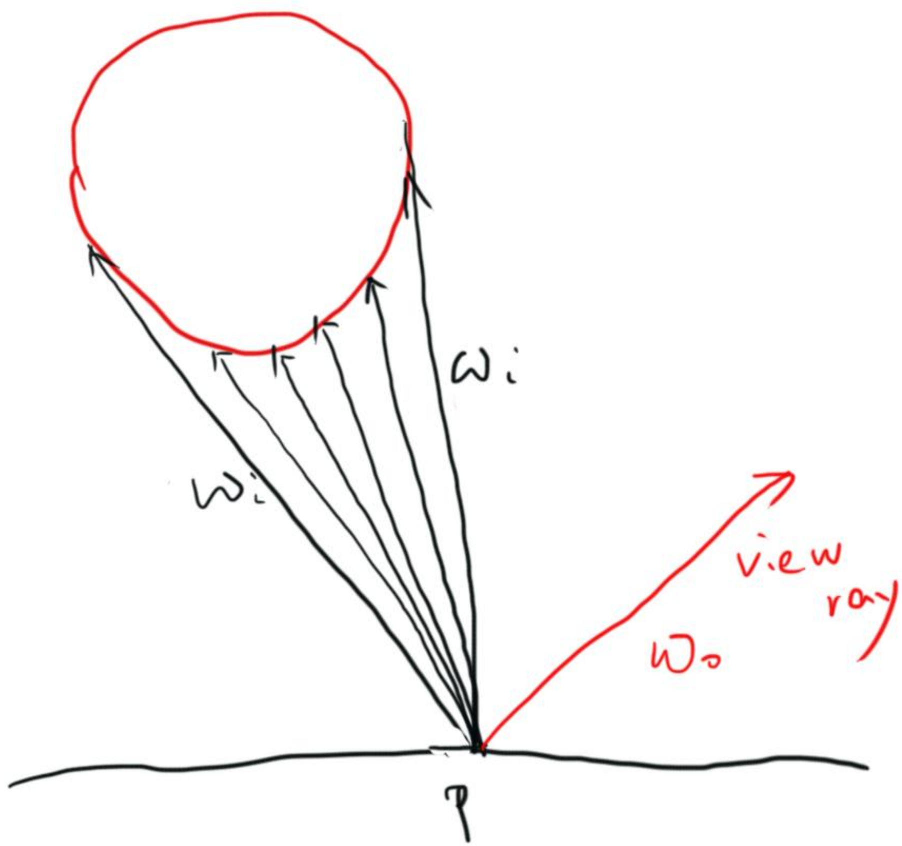
# Importance sample

- Ideally, We want $p(\omega_j)$ match $f(p,\omega_o,\omega_i)L_d(p,\omega_i)|\cos(\theta_j)|$
- Practically, we can draw samples according to f or L

# Importance sample BRDF

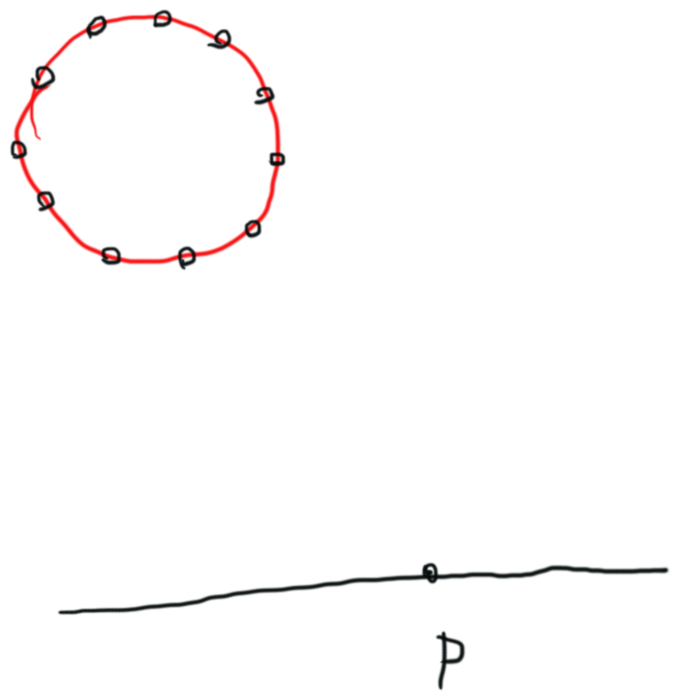


# Importance sample light



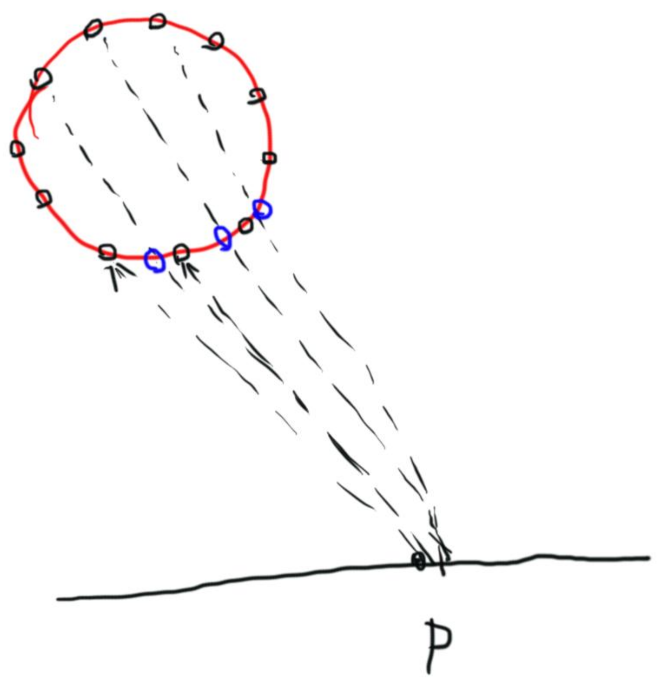
# Importance sample light

- Step 1: Uniform sample the shape of light



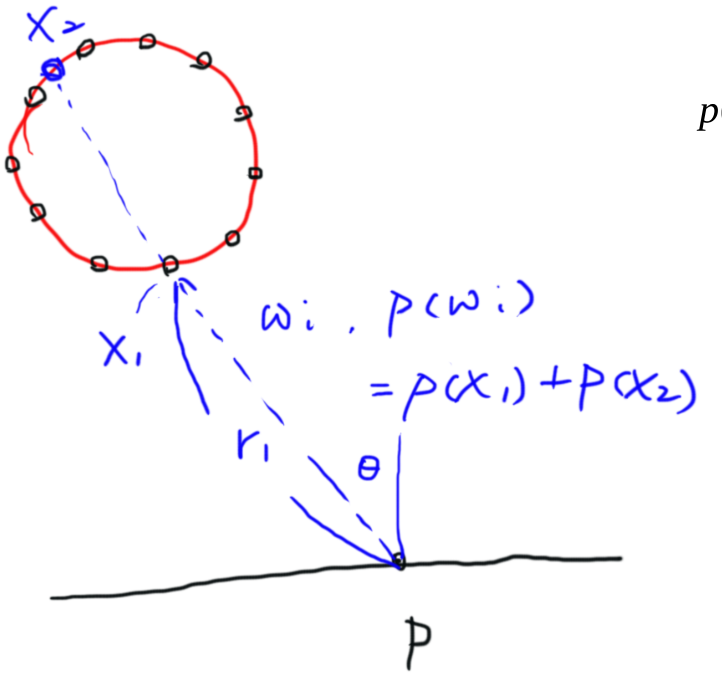
# Importance sample light

- Step 2: ray trace to get the nearest points on light



# Importance sample light

- Step 3: convert the pdf from light surface to position angle




$$p(X_1) = \frac{r_1^2}{|\cos(\theta_1)| \cdot Area(\text{light surface})}$$

# Multiple importance sampling

- Sample BRDF works poorly on diffuse surface

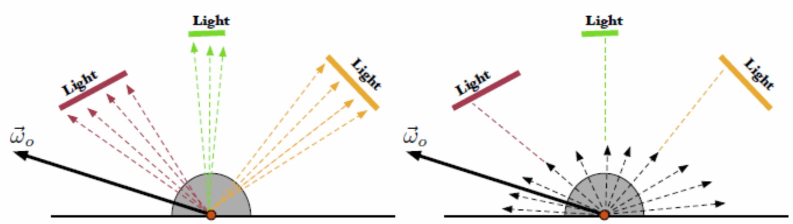



Figure 16: Diffuse BRDF and area lights. When we have a diffuse BRDF and multiple area lights of different sizes, two obvious sampling density choices are lighting (left) and BRDF (right). Note that BRDF sampling produces many samples that will be wasted, because there is no light emission in those directions. Sampling according to only the lighting produces lower variance.

- Sample Light works poorly on glossy surface

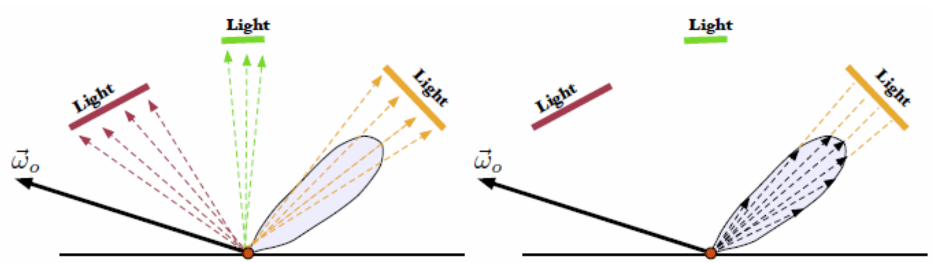



Figure 17: Glossy (specular) BRDF and area lights. Lighting (left) or BRDF (right) can be used as an importance sampling density. Note that light sampling produces many wasted samples because there will be no reflection in those directions. Sampling according to the BRDF provides better results.

Pictures from“Importance sampling for production rendering”

# Multiple importance sampling

- Mix two distributions

if we want to estimate $\int f(x)g(x)dx$

we can draw samples seperately from f and g

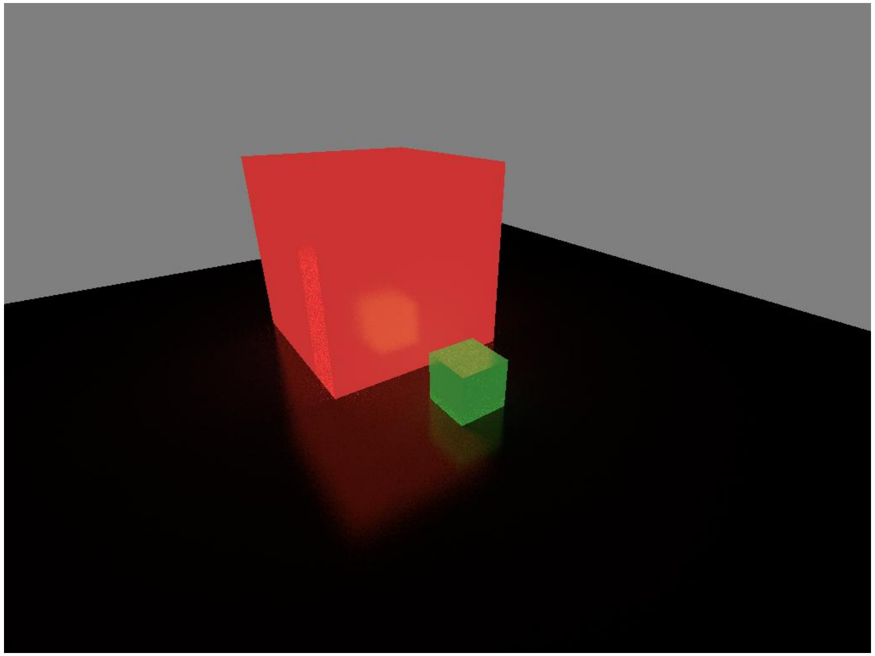
and the estimator is

$$\frac{1}{n_f}\sum_{i=1}^{n_f}\frac{f(X_i)g(X_i)w_f(X_i)}{p_f(X_i)}+\frac{1}{n_g}\sum_{j=1}^{n_g}\frac{f(X_j)g(X_j)w_g(X_j)}{p_g(X_j)}$$

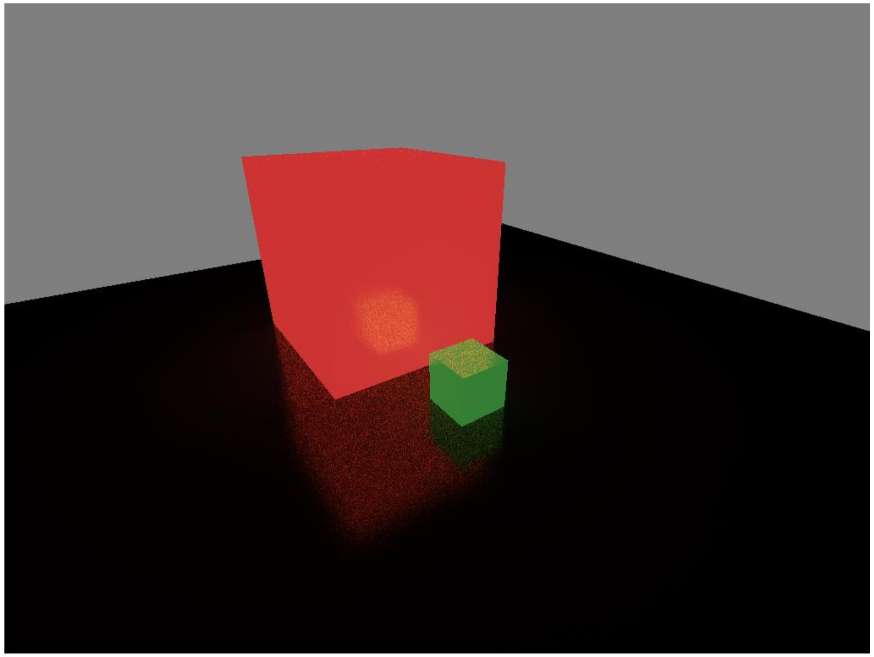
$$w_f=\frac{n_f p_f(X_i)}{n_f p_f(X_i)+n_g p_g(X_i)}$$

$$w_g=\frac{n_g p_g(X_j)}{n_f p_f(X_j)+n_g p_g(Xj_i)}$$

# Multiple importance sampling

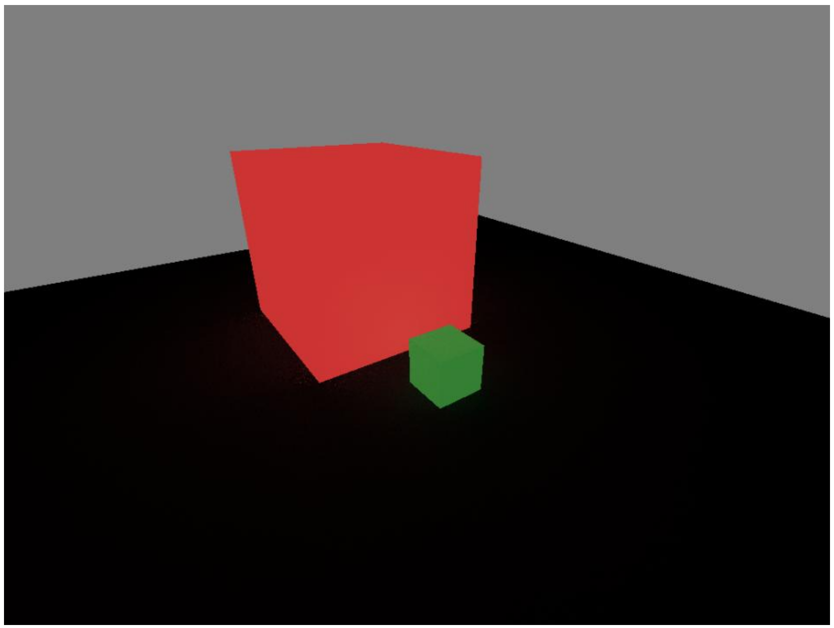


Multiple importance sampling
32 light samples + 32 BRDF samples

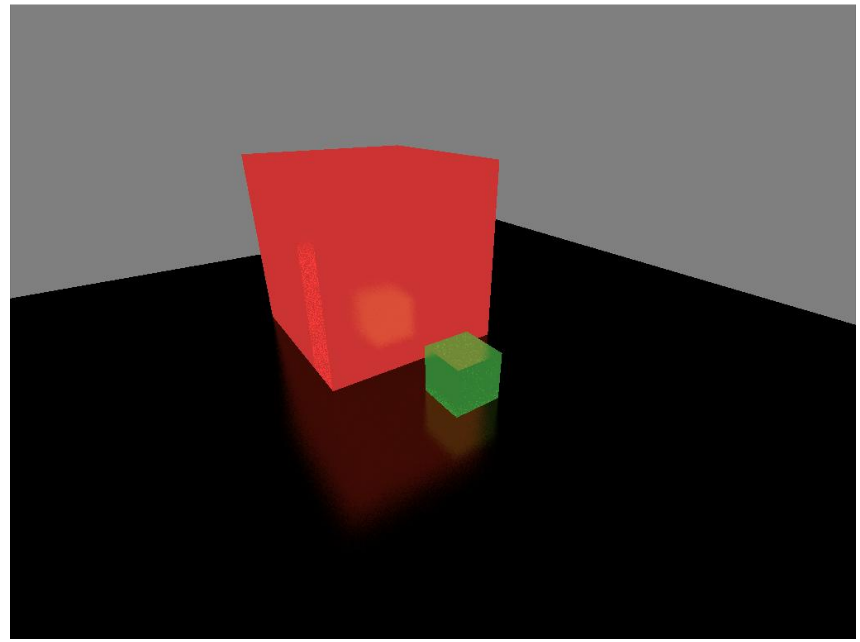


Uniform sampling
1024 samples

# Multiple importance sampling



Light samples contribution



BRDF samples contribution

# Multiple importance sampling

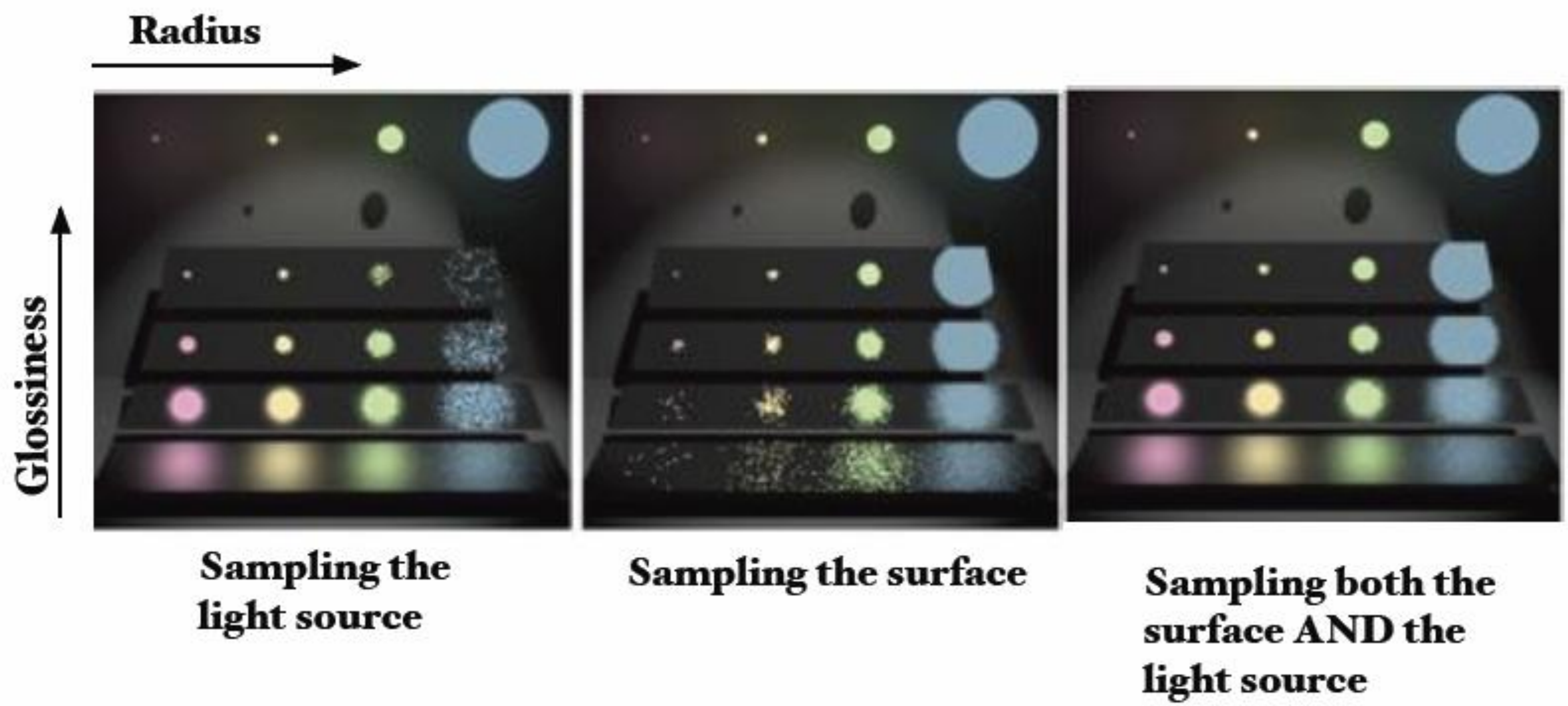



Figure 19: Combining many sampling strategies using Multiple Importance Sampling (MIS) produces superior results to using a single sampling density. Image from Veach and Guibas [VG95].

Pictures from"Importance sampling for production rendering"

# Special case

- “Delta” light
  - Point light
  - Directional light