While Working on my Notes

Kunihiko Kodaira

Nothing is more unreadable to me than books (and papers) on mathematics. Reading a book on mathematics with hundreds of pages cover to cover is an arduous task. When you open such a book, you first encounter some definitions and axioms, followed by theorems and proofs. Since mathematics becomes plain and easy once the insight is there, you make an effort to get to it just by reading the theorems - and try to produce the proofs on your own. Most likely, your thinking doesn't get you very far, so you have no choice but to read the proof in your book, but you can't make sense of it by looking at it once or twice. That is why you copy the proof to your notebook. But this time, the parts of the proof you dislike leap out. You ask yourself if there could be another proof, and ideally find one immediately, but if you don't, it takes quite some time until you let go. After you have spent an entire month getting finally through one chapter this way, you have forgotten the beginning, which you now have to revise. This time, the arrangement of the material in the chapter starts to bother you, and makes you think something like "Wouldn't it be better to prove theorem 7 before theorem 3," and you rewrite the entire chapter. Now you feel confident that you have finally understood chapter 1, but at the same time troubled, since this all cost you so much time. Time alone can make it next to impossible to get through hundreds of pages in a book to the last chapter. If anybody knows a faster method of reading mathematical texts, I would like them to teach me.

Somebody might say that perhaps it is better to read straight to the end without thinking about anything non-essential. This is certainly true and I can read books from fields of mathematics other than my own faster and more smoothly (although I rarely read mathematical texts unrelated to my field). I may have read them, but it is acutely dubious whether there is any insight. At what stage can one say that one has read and understood a written piece of mathematics? Is having verified the proof step by step and having agreed that there is no error enough? If I read some book on mathematics from an unfamiliar field, I notice that the theorems that I don't understand puzzle me even after having verified their proofs. The proof may be correct, but the overall picture is fuzzy and foggy. When I understand a theorem from my own field on the other hand, I comprehend it perfectly even after forgetting the proof, as perfectly as the fact that $2+2=4$: The understanding behind $2+2=4$ comes from an instinctive grasp of the mathematical truth behind $2+2=4$, and not from a proof. In a similar vein, understanding any theorem seems to mean having a sense of the underlying mathematical truth that it provides. I think that following a proof step by step serves more as one good method for absorbing a theorem intuitively than for verifying that the arguments in the proof are correct. (The fact that the proof of a famous theorem is correct should be so clear that it doesn't have to be checked by everybody.) This is why in order to understand a theorem well, reading the proof just once won't suffice, but it is more beneficial to read it again and again, copy it to your notebook, and try to apply it to various problems. Writing the proof out is not something you do for memorizing it, but for taking your time to look in detail at what builds up to the mathematical idea behind the theorem. Once you have gained a complete understanding of the theorem in this way, there is no objection at all to forgetting the entire proof (except if you haven't graduated from university, in which case you better have it memorized for the exam). If you happen to need the forgotten proof later and review it, it may then even seem like an unnatural tag on the theorem which itself is as clear as $2+2=4$.

Mathematics is a highly technical subject. Acquiring anything that people call technical needs extensive and repetitious practice. For example, anybody who wants to become a pianist has no choice but to practice every day for hours from childhood on. Mathematics also has a similar aspect to this and I think you need to spend many hours every day doing repetitious exercises in order to master it. This is how your intuitive grasp for mathematical truth develops. Reading books on mathematics from a field unrelated to your own and not understanding theorems even after having verified the proofs is a sign that your intuition for that area isn't yet mature.

Suugaku Seminaa, August 1980. Translation: Florian Sprung.