Formularium Wiskunde

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Inhoudsopgave

1 Algebra

1.1 Volgorde van Bewerking

Haakjes wegwerken, machtsverheffen, worteltrekken, vermenigvuldigen en delen, optellen en aftrekken.

1.2 Absolute Waarde

De absolute waarde van een getal a wordt genoteerd als $|a|$ en is altijd positief.

$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$

2 Machten en wortels

2.1 Machten met Gehele Exponenten

$\forall a \in, \forall n \in \mathbb{N}_0 : a^n = \underbrace{a.a. \dots .a}_{n\ factoren}$ $\forall a \in \mathbb{R} :\ a^1 = a$ $\forall a \in \mathbb{R}_0 :\ a^0 = 1$ $\forall a \in \mathbb{R}_0, \forall n \in \mathbb{N} :\ a^{-n} = \frac{1}{a^n}$	$\forall a, b \in \mathbb{R}_0, \forall m, n \in \mathbb{Z} : a^m \cdot a^n = a^{m+n}$ $\frac{a^m}{a^n} = a^{m-n}$ $(a^m)^n = a^{mn}$ $(a.b)^n = a^n \cdot b^n$ $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$

2.2 Vierkantswortel in $\mathbb{R}$

$\forall a \in \mathbb{R}^+, \forall b \in \mathbb{R} :$ $b = \sqrt{a} \Leftrightarrow b^2 = a \wedge (b \geq 0)$ $\forall a, b \in \mathbb{R}^+ :$ $\sqrt{a^2} = a$ $(\sqrt{a})^2 = a$ $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}.$ $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}} \wedge b \neq 0$	$\forall a \in \mathbb{R} :$ $\sqrt{a^2} = \|a\| \implies \begin{cases} \sqrt{a^2} = a & \text{als } a \geq 0, \\ \sqrt{a^2} = -a & \text{als } a \leq 0. \end{cases}$

2.3 N-de machtswortel in $\mathbb{R}$

$n\ even \Rightarrow \sqrt[n]{a^n} = \|a\| \rightarrow \begin{cases} \sqrt[n]{a^n} = a & \wedge a \geq 0 \\ \sqrt[n]{a^n} = -a & \wedge a \leq 0 \end{cases}$ $n\ oneven \Rightarrow \sqrt[n]{a^n} = a$	$\forall a, b \in \mathbb{R}_0^+, \forall m, n \in \mathbb{N}_0 :$ $\sqrt[n]{a^n} = a$ $(\sqrt[n]{a})^n = a$ $\sqrt[n]{ab} = \sqrt[n]{a}. \sqrt[n]{b}$ $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$ $\sqrt[m]{\sqrt[n]{a}} = \sqrt[m.n]{a}$

2.4 $\frac{m}{n}$-de machtswortel in $\mathbb{R}$

$\forall a \in \mathbb{R}_0^+, \forall m \in \mathbb{Z}, \forall n \in \mathbb{N}_0 : a^{\frac{m}{n}} = \sqrt[n]{a^m}$	$\forall a, b \in \mathbb{R}_0^+, \forall m, n \in \mathbb{Q}:$ $a^m.a^n = a^{m+n}$ $\frac{a^m}{a^n} = a^{m-n}$ $(a^m)^n = a^{m.n}$ $(a.b)^m = a^m.b^m$ $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

3 Veeltermen

3.1 Vierkantsvergelijking

Een vierkantsvergelijking is van de vorm : $ax^2 + bx + c = 0$, *met* $D = b^2 - 4ac$

$x \in \mathbb{R}$	$x \in \mathbb{C}$
$x_{1,2} = \frac{-b \pm \sqrt{D}}{2a}$	$x_{1,2} = \frac{-b \pm i\sqrt{-D}}{2a}$
$P = \frac{c}{a} = x_1 \cdot x_2$, $S = -\frac{b}{a} = x_1 + x_2$	
$ax^2 + bx + c = a(x - x_1)(x - x_2) = a(x^2 - Sx + P)$	

3.2 Merkwaardige Producten en Ontbinding in Factoren

$(a \pm b)^2 = a^2 \pm 2ab + b^2$
$(a \pm b)^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3$
$(a + b)^n = a^n + C_n^1 a^{n-1} b + C_n^2 a^{n-2} b^2 + ... + C_n^{n-1} a^2 b^{n-1} + b^n \quad \wedge \quad C_n^p = \frac{n!}{(n-p)!p!}$
$a^2 - b^2 = (a + b)(a - b)$
$a^3 - b^3 = (a - b)\left(a^2 + ab + b^2\right)$
$a^n - b^n = (a - b)\left(a^{n-1} + a^{n-2}b + a^{n-3}b^2 + ... + ab^{n-2} + b^{n-1}\right)$
$a^3 + b^3 = (a + b)\left(a^2 - ab + b^2\right)$
$a^{2n+1} + b^{2n+1} = (a + b)\left(a^{2n} - a^{2n-1}b + a^{2n-2}b^2 - a^{2n-3}b^3 + ... - ab^{2n-1} + b^{2n}\right)$

3.3 Euclidische Deling

We gaan de derdegraadsveelterm $2x^3 + 3x^2 - 4x + 5$ delen door de eerstegraadsveelterm $x + 2$ met behulp van de praktische werkwijze van lange deling.

$2x^3 + 3x^2 - 4x + 5$	$x + 2$
$-2x^3 - 4x^2 + 0x + 0$	$2x^2$
$-1x^2 - 4x + 5$	
$+1x^2 + 2x + 0$	$-x$
$-2x + 5$	
$2x + 4$	-2
9	

We kunnen de deling als volgt uitdrukken:

$$2x^3 + 3x^2 - 4x + 5 = (x + 2)(2x^2 - x - 2) + 9$$

De rest is 25, wat een graad heeft die kleiner is dan de graad van de deler $x + 2$.

3.4 Schema van Horner

$$\frac{(3x^3 - 5x^2 + 10x - 52)}{(x - 2)}$$

	3	-5	+10	-5
2		+6	+2	+24
	3	+1	+12	+19

4 Complexe getallen

4.1 Rechthoekige coordinaten

Bewerking	**Formule**
Optelling/Aftrekking	$(a+j.b) \pm (c+j.d) = (a+c) \pm j(b+d)$
Vermenigvuldiging	$(a+j.b) \cdot (c+j.d) = (ac-bd) + j(ad+bc)$
Deling	$\frac{(a+j.b)}{(c+j.d)} = \frac{(a+j.b)\cdot(c-j.d)}{(c+j.d)\cdot(c-j.d)} = \left(\frac{ac+bd}{c^2+d^2}\right) + j\left(\frac{bc-ad}{c^2+d^2}\right)$
Toegevoegde van	$\overline{(a+j.b)} = (a-j.b)$ $\overline{Z_1+Z_2} = \overline{Z_1} + \overline{Z_2}, \quad \overline{Z_1 \cdot Z_2} = \overline{Z_1} \cdot \overline{Z_2}$
Inverse	$z = a+bi \implies z^{-1} = \frac{a-bi}{a^2+b^2}$
Wortel	$\sqrt{a} \wedge a<0 \implies \sqrt{a} = \pm i\sqrt{-a}$ $\sqrt{a+bi} = x+yi \iff (x+yi)^2 = a+bi$
Macht	$(a+bi)^0 = 1 \quad \forall n \in \mathbb{N}_0 :$ $(a+bi)^n = (a+bi)\cdot(a+bi)\cdots(a+bi)$
Machten of i	$i^1 = i, \quad i^2 = -1, \quad i^3 = -i, \quad i^4 = 1$

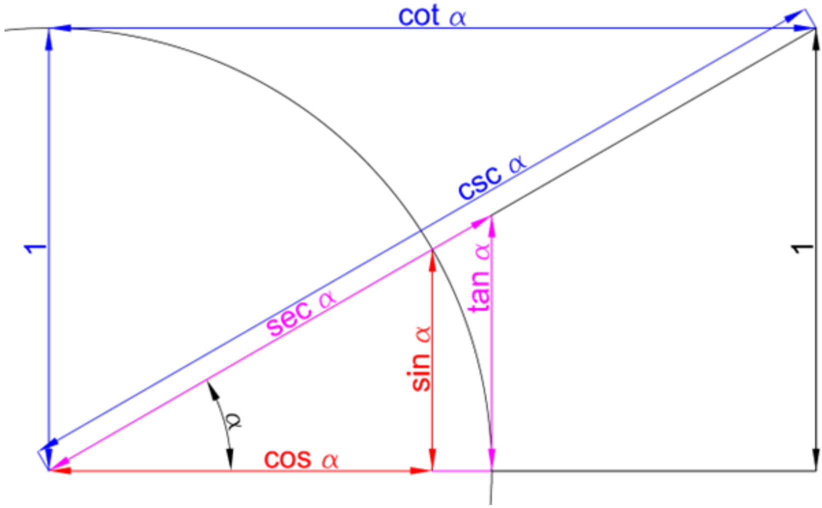
4.2 Poolcoördinaten

$$z = a + i.b = r\left(\cos(\varphi) + i.\sin(\varphi)\right) = r\angle\varphi, \quad \tan(\varphi) = \frac{b}{a}, \quad r = \sqrt{a^2+b^2}$$

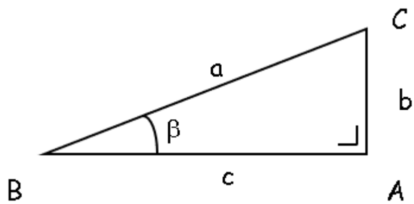
Bewerking	**Formule**
Vermenigvuldiging	$z_1 \cdot z_2 = r_1 \cdot r_2 \angle \varphi_1 + \varphi_2$
Deling	$\frac{z_1}{z_2} = \frac{r_1\angle\varphi_1}{r_2\angle\varphi_2} = \frac{r_1}{r_2}\angle\varphi_1 - \varphi_2$
Inverse	$z^{-1} = \frac{1}{r}\angle - \varphi$
Macht	$z^n = r^n\left[\cos(n\cdot\varphi) + i\sin(n\cdot\varphi)\right] \quad n \in \mathbb{N}$
Wortel	$\sqrt{r(\cos\varphi + i\sin\varphi)} = \pm\sqrt{r}\left(\cos\frac{\varphi}{2} + i\sin\frac{\varphi}{2}\right)$
$\sqrt[n]{r\left(\cos\varphi + i\sin\varphi\right)} = \sqrt[n]{r}\left(\cos\frac{\varphi+k\cdot 2\pi}{n} + i\sin\frac{\varphi+k\cdot 2\pi}{n}\right) \quad \wedge \quad k = 0, 1, \cdots, n-1$	

5 Goniometrie

5.1 De Goniometrische Cirkel



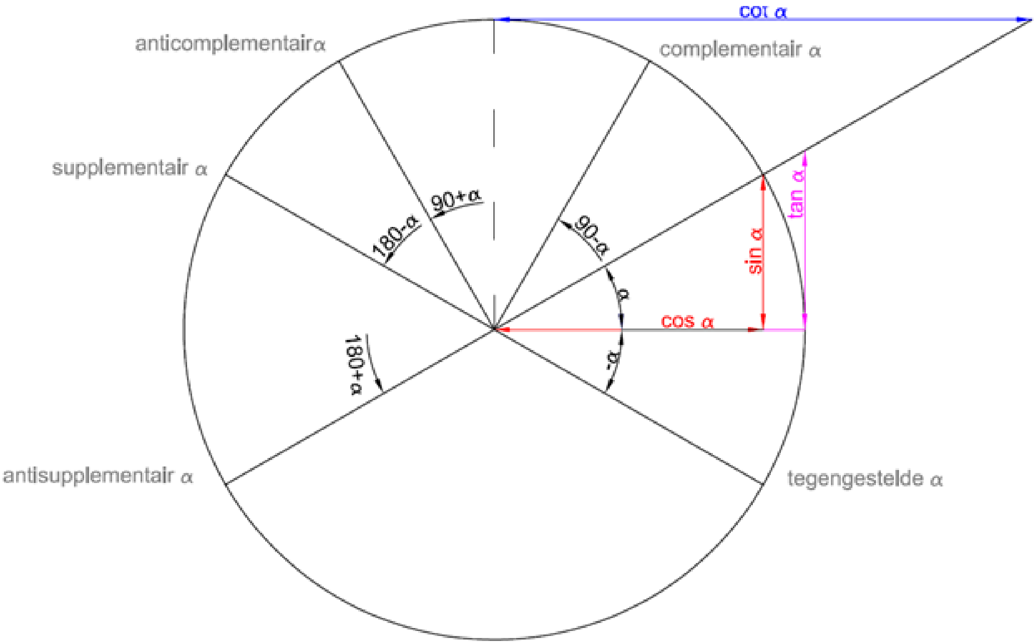
5.2 formules uit de goniometrie



$$\boxed{\begin{array}{ccc} \csc\beta & sec\beta & \cot\beta \\ \leftarrow & \leftarrow & \leftarrow \\ os & as & oa \\ \rightarrow & \rightarrow & \rightarrow \\ \sin\beta & \cos\beta & \tan\beta \end{array}} \boxed{\quad \text{waarin: } \begin{cases} o: & \text{overstaande rechthoekszijde} \\ s: & \text{schuine zijde (hypotenusa)} \\ a: & \text{aanliggende rechthoekszijde} \end{cases}}$$

$\sin\beta = \frac{b}{a}$	$\cos\beta = \frac{c}{a}$	$\tan\beta = \frac{b}{c}$
$\csc\beta = \frac{a}{b}$	$\sec\beta = \frac{a}{c}$	$\cot\beta = \frac{c}{b}$
$\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$	$\cot\alpha = \frac{\cos\alpha}{\sin\alpha}$	$\cot\alpha = \frac{1}{\tan\alpha}$
$\sec\alpha = \frac{1}{\cos\alpha}$	$\csc\alpha = \frac{1}{\sin\alpha}$	

$\boxed{\sin^2\alpha + \cos^2\alpha = 1}$ $\boxed{\tan^2\alpha + 1 = \sec^2\alpha}$ $\boxed{1 + \cot^2\alpha = \csc^2\alpha}$



gelijkehoeken	supplementairehoeken	complementairehoeken
$\sin(\alpha + k2\pi) = \sin\alpha$	$\sin(\pi - \alpha) = \sin\alpha$	$\sin\left(\frac{\pi}{2} - \alpha\right) = \cos\alpha$
$\cos(\alpha + k2\pi) = \cos\alpha$	$\cos(\pi - \alpha) = -\cos\alpha$	$\cos\left(\frac{\pi}{2} - \alpha\right) = \sin\alpha$
$\tan(\alpha + k2\pi) = \tan\alpha$	$\tan(\pi - \alpha) = -\tan\alpha$	$\tan\left(\frac{\pi}{2} - \alpha\right) = \cot\alpha$
$\cot(\alpha + k2\pi) = \cot\alpha$	$\cot(\pi - \alpha) = -\cot\alpha$	$\cot\left(\frac{\pi}{2} - \alpha\right) = \tan\alpha$
$\sec(\alpha + k2\pi) = \sec\alpha$	$\sec(\pi - \alpha) = -\sec\alpha$	$\sec\left(\frac{\pi}{2} - \alpha\right) = \csc\alpha$
$\csc(\alpha + k2\pi) = \csc\alpha$	$\csc(\pi - \alpha) = \csc\alpha$	$\csc\left(\frac{\pi}{2} - \alpha\right) = \sec\alpha$

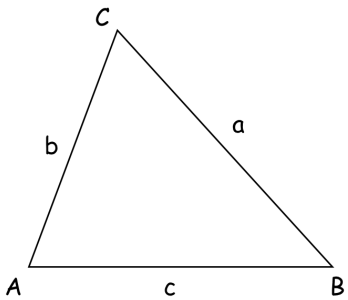
tegengesteldehoeken	antisupplementairehoeken	anticomplementairehoeken
$\sin(-\alpha) = -\sin\alpha$	$\sin(\pi + \alpha) = -\sin\alpha$	$\sin\left(\frac{\pi}{2} + \alpha\right) = \cos\alpha$
$\cos(-\alpha) = \cos\alpha$	$\cos(\pi + \alpha) = -\cos\alpha$	$\cos\left(\frac{\pi}{2} + \alpha\right) = -\sin\alpha$
$\tan(-\alpha) = -\tan\alpha$	$\tan(\pi + \alpha) = \tan\alpha$	$\tan\left(\frac{\pi}{2} + \alpha\right) = -\cot\alpha$
$\cot(-\alpha) = -\cot\alpha$	$\cot(\pi + \alpha) = \cot\alpha$	$\cot\left(\frac{\pi}{2} + \alpha\right) = -\tan\alpha$
$\sec(-\alpha) = \sec\alpha$	$\sec(\pi + \alpha) = -\sec\alpha$	$\sec\left(\frac{\pi}{2} + \alpha\right) = -\csc\alpha$
$\csc(-\alpha) = -\csc\alpha$	$\csc(\pi + \alpha) = -\csc\alpha$	$\csc\left(\frac{\pi}{2} + \alpha\right) = \sec\alpha$

- **De sinusregel:**

$$\frac{a}{\sin \hat{A}} = \frac{b}{\sin \hat{B}} = \frac{c}{\sin \hat{C}}$$

- **De cosinusregel:**

$$a^2 = b^2 + c^2 - 2bc\cos \hat{A}$$
$$b^2 = c^2 + a^2 - 2ca\cos \hat{B}$$
$$c^2 = a^2 + b^2 - 2ab\cos \hat{C}$$



$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$	$\sin 2\alpha = 2\sin\alpha\cos\alpha$
$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$	$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$ $= 1 - 2\sin^2\alpha \quad (*)$ $= 2\cos^2\alpha - 1 \quad (**)$
$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$	$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha}$

$\sin^2\alpha = \frac{1-\cos 2\alpha}{2} \quad (*)$ $\cos^2\alpha = \frac{1+\cos 2\alpha}{2} \quad (**)$ $\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}$ $\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$	$\sin 2\alpha = \frac{2\tan\alpha}{1+\tan^2\alpha}$ $\cos 2\alpha = \frac{1-\tan^2\alpha}{1+\tan^2\alpha}$ $\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha}$	$\sin\alpha = \frac{2t}{1+t^2} \quad \wedge \quad \tan\frac{\alpha}{2} = t$ $\cos\alpha = \frac{1-t^2}{1+t^2}$ $\tan\alpha = \frac{2t}{1-t^2}$

5.3 Omgekeerde formules van Simpson

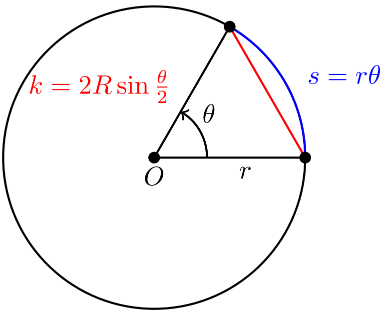
$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$	$\sin(\alpha+\beta) + \sin(\alpha-\beta) = 2\sin\alpha\cos\beta$ $\sin(\alpha+\beta) - \sin(\alpha-\beta) = 2\cos\alpha\sin\beta$
$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$	$\cos(\alpha+\beta) + \cos(\alpha-\beta) = 2\cos\alpha\cos\beta$ $\cos(\alpha+\beta) - \cos(\alpha-\beta) = -2\sin\alpha\sin\beta$

5.4 Formules van Simpson

$\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$	$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$
$\sin\alpha - \sin\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$	$\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$

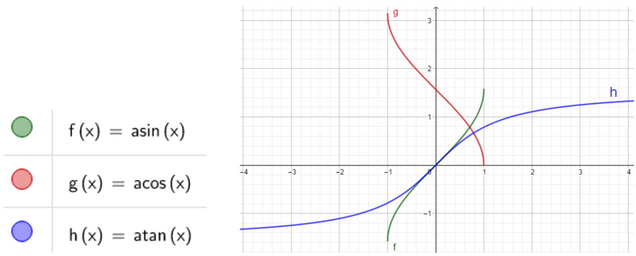
5.5 Belangrijke goniometrische waarden

Angle	0°	30°	45°	60°	90°	180°	270°	360°
α	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\sin\alpha$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
$\cos\alpha$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0	1
$\tan\alpha$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	/	0	/	0



5.6 Cyclometrische formules

$$y = \text{Bgsin}x \Leftrightarrow \left(x = \sin y \;\; \wedge y \in \left[-\tfrac{\pi}{2}, \tfrac{\pi}{2}\right], \; x \in [-1,1]\right)$$
$$y = \text{Bgcos}x \Leftrightarrow \left(x = \cos y \;\; \wedge y \in [0,\pi], \; x \in [-1,1]\right)$$
$$y = \text{Bgtan}\, x \Leftrightarrow \left(x = \tan y \;\; \wedge y \in \left]-\tfrac{\pi}{2}, \tfrac{\pi}{2}\right[, \; x \in\right)$$



$$\sin(\text{Bgsin}x) = x \forall x \in [-1,1] \qquad \cos(\text{Bgtan}x) = \frac{1}{\sqrt{1+x^2}} \forall x \in$$
$$\cos(\text{Bgcos}x) = x \forall x \in [-1,1] \qquad \sin(\text{Bgtan}x) = \frac{x}{\sqrt{1+x^2}} \forall x \in$$
$$tg(\text{Bgtan}) = x \forall x \in \qquad \text{Bgsin}x + \text{Bgcos}x = \frac{\pi}{2} \forall x \in [-1,1]$$
$$cotg(\text{Bgcot}x) = x \forall x \in \qquad \text{Bgcot}x + \text{Bgtan}x = \frac{\pi}{2} \forall x \in$$
$$\cos(\text{Bgsin}x) = \sqrt{1-x^2} \forall x \in [-1,1] \qquad \text{Bgtan}(-x) = -\text{Bgtan}x \forall x \in$$
$$\sin(\text{Bgcos}x) = \sqrt{1-x^2} \forall x \in [-1,1] \qquad \text{Bgcot}(-x) = -\text{Bgcot}x \forall x \in$$
$$Bgsin(-x) = -\text{Bgsin}x \forall x \in [-1,1] \qquad \text{Bgcos}(-x) = \pi - \text{Bgcos}x \forall x \in [-1,1]$$

6 Diversen

6.1 Wiskundige Symbolen (ISO 31/XI)

$x \in A$	is een element van de verzameling
$x \notin A$	is geen element van de verzameling
$\{x_1, x_2, \ldots, x_n\}$	de verzameling door opsomming
$\{x \in A \mid p(x)\}$	de verzameling waar de elementen voldoen aan de eigenschap $p(x)$
$\emptyset$	de lege verzameling
$\mathbb{N}$	de natuurlijke getallen $(0, 1, 2, \ldots)$
$\mathbb{Z}$	de gehele getallen $(\ldots, -2, -1, 0, 1, 2, \ldots)$
$\mathbb{Q}$	de rationale getallen (breuken van $\mathbb{Z}$)
$\mathbb{R}$	de reële getallen
$\mathbb{C}$	de complexe getallen
$B \subseteq A$	B behoort tot A (kan er mee samenvallen)
$B \subset A$	B behoort strikt tot A
$A \cup B$	samenvoeging van A en B (unie)
$A \cap B$	doorsnede van A en B (de gemeenschappelijke elementen)
$A \setminus B$	A verschilt B, wat tot A behoort en niet tot B
$\mathcal{C}_U A$	het complement van A in het universum U
(a, b)	het geordend paar
$(a_1, a_2, \ldots, a_n)$	een geordend n-tal
$A \times B$	de productverzameling van A en B
$\#$	rangnummer of aantal

6.2 Logische symbolen

$p \wedge q$	conjunctie, de beweringen p en q zijn geldig
$p \vee q$	disjunctie, de bewering p of q is geldig
$\neg p$	negatie, de bewering p is niet geldig
$p \Rightarrow q$	implicatie, als p dan q
$p \Leftrightarrow q$	equivalentie, de beweringen p en q zijn gelijkwaardig
$\forall x$	universele kwantor, voor alle elementen geldt
$\exists x$	existentiële kwantor, er zijn elementen die voldoen aan