Formularium Wiskunde

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Inhoudsopgave

1 Algebra

1.1 Volgorde van Bewerking

Haakjes wegwerken, machtsverheffen, worteltrekken, vermenigvuldigen en delen, optellen en aftrekken.

1.2 Absolute Waarde

De absolute waarde van een getal a wordt genoteerd als $|a|$ en is altijd positief.

$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$

2 Machten en wortels

2.1 Machten met Gehele Exponenten

$\forall a \in, \forall n \in \mathbb{N}_0 : a^n = \underbrace{a.a. \, ... \, .a}_{n \; factoren}$ $\forall a \in \mathbb{R} : \; a^1 = a$ $\forall a \in \mathbb{R}_0 : \; a^0 = 1$ $\forall a \in \mathbb{R}_0, \forall n \in \mathbb{N} : \; a^{-n} = \frac{1}{a^n}$	$\forall a, b \in \mathbb{R}_0, \forall m, n \in \mathbb{Z} : a^m \cdot a^n = a^{m+n}$ $\frac{a^m}{a^n} = a^{m-n}$ $(a^m)^n = a^{mn}$ $(a.b)^n = a^n \cdot b^n$ $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$

2.2 Vierkantswortel in $\mathbb{R}$

$\forall a \in \mathbb{R}^+, \forall b \in \mathbb{R} :$ $b = \sqrt{a} \Leftrightarrow b^2 = a \wedge (b \geq 0)$ $\forall a, b \in \mathbb{R}^+ :$ $\sqrt{a^2} = a$ $(\sqrt{a})^2 = a$ $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}.$ $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}} \wedge b \neq 0$	$\forall a \in \mathbb{R} :$ $\sqrt{a^2} = \lvert a \rvert \implies \begin{cases} \sqrt{a^2} = a & \text{als } a \geq 0, \\ \sqrt{a^2} = -a & \text{als } a \leq 0. \end{cases}$

2.3 N-de machtswortel in $\mathbb{R}$

$n \; even \Rightarrow \sqrt[n]{a^n} = \lvert a \rvert \rightarrow \begin{cases} \sqrt[n]{a^n} = a & \wedge a \geq 0 \\ \sqrt[n]{a^n} = -a & \wedge a \leq 0 \end{cases}$ $n \; oneven \Rightarrow \sqrt[n]{a^n} = a$	$\forall a, b \in \mathbb{R}_0^+, \forall m, n \in \mathbb{N}_0 :$ $\sqrt[n]{a^n} = a$ $(\sqrt[n]{a})^n = a$ $\sqrt[n]{ab} = \sqrt[n]{a}. \sqrt[n]{b}$ $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$ $\sqrt[m]{\sqrt[n]{a}} = \sqrt[m.n]{a}$

2.4 $\frac{m}{n}$-de machtswortel in $\mathbb{R}$

$\forall a \in \mathbb{R}_0^+, \forall m \in \mathbb{Z}, \forall n \in \mathbb{N}_0 : a^{\frac{m}{n}} = \sqrt[n]{a^m}$	$\forall a, b \in \mathbb{R}_0^+, \forall m, n \in \mathbb{Q}:$ $a^m.a^n = a^{m+n}$ $\frac{a^m}{a^n} = a^{m-n}$ $(a^m)^n = a^{m.n}$ $(a.b)^m = a^m.b^m$ $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

3 Veeltermen

3.1 Vierkantsvergelijking

Een vierkantsvergelijking is van de vorm : $ax^2 + bx + c = 0$, *met* $D = b^2 - 4ac$

$x \in \mathbb{R}$	$x \in \mathbb{C}$
$x_{1,2} = \frac{-b \pm \sqrt{D}}{2a}$	$x_{1,2} = \frac{-b \pm i\sqrt{-D}}{2a}$
$P = \frac{c}{a} = x_1 \cdot x_2$, $S = -\frac{b}{a} = x_1 + x_2$	
$ax^2 + bx + c = a(x - x_1)(x - x_2) = a(x^2 - Sx + P)$	

3.2 Merkwaardige Producten en Ontbinding in Factoren

$(a \pm b)^2 = a^2 \pm 2ab + b^2$
$(a \pm b)^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3$
$(a + b)^n = a^n + C_n^1 a^{n-1}b + C_n^2 a^{n-2}b^2 + ... + C_n^{n-1} a^2 b^{n-1} + b^n \quad \wedge \quad C_n^p = \frac{n!}{(n-p)!p!}$
$a^2 - b^2 = (a + b)(a - b)$
$a^3 - b^3 = (a - b)\left(a^2 + ab + b^2\right)$
$a^n - b^n = (a - b)\left(a^{n-1} + a^{n-2}b + a^{n-3}b^2 + ... + ab^{n-2} + b^{n-1}\right)$
$a^3 + b^3 = (a + b)\left(a^2 - ab + b^2\right)$
$a^{2n+1} + b^{2n+1} = (a + b)\left(a^{2n} - a^{2n-1}b + a^{2n-2}b^2 - a^{2n-3}b^3 + ... - ab^{2n-1} + b^{2n}\right)$

3.3 Euclidische Deling

We gaan de derdegraadsveelterm $2x^3 + 3x^2 - 4x + 5$ delen door de eerstegraadsveelterm $x + 2$ met behulp van de praktische werkwijze van lange deling.

$$
\begin{array}{r|l}
2x^3 + 3x^2 - 4x + 5 & x + 2 \\
\hline
-2x^3 - 4x^2 + 0x + 0 & 2x^2 \\
\hline
-1x^2 - 4x + 5 & \\
+1x^2 + 2x + 0 & -x \\
\hline
-2x + 5 & \\
2x + 4 & -2 \\
\hline
9 &
\end{array}
$$

We kunnen de deling als volgt uitdrukken:

$$2x^3 + 3x^2 - 4x + 5 = (x + 2)(2x^2 - x - 2) + 9$$

De rest is 25, wat een graad heeft die kleiner is dan de graad van de deler $x + 2$.

3.4 Schema van Horner

$$\frac{(3x^3 - 5x^2 + 10x - 52)}{(x - 2)}$$

	3	-5	+10	-5	
2		+6	+2	+24	
	3	+1	+12		+19

4 Complexe getallen

4.1 Rechthoekige coordinaten

Bewerking	**Formule**
Optelling/Aftrekking	$(a+j.b) \pm (c+j.d) = (a+c) \pm j(b+d)$
Vermenigvuldiging	$(a+j.b) \cdot (c+j.d) = (ac-bd) + j(ad+bc)$
Deling	$\frac{(a+j.b)}{(c+j.d)} = \frac{(a+j.b)\cdot(c-j.d)}{(c+j.d)\cdot(c-j.d)} = \left(\frac{ac+bd}{c^2+d^2}\right) + j\left(\frac{bc-ad}{c^2+d^2}\right)$
Toegevoegde van	$\overline{(a+j.b)} = (a-j.b)$ $\overline{Z_1+Z_2} = \overline{Z_1} + \overline{Z_2}, \quad \overline{Z_1 \cdot Z_2} = \overline{Z_1} \cdot \overline{Z_2}$
Inverse	$z = a+bi \implies z^{-1} = \frac{a-bi}{a^2+b^2}$
Wortel	$\sqrt{a} \ \wedge \ a<0 \implies \sqrt{a} = \pm i\sqrt{-a}$ $\sqrt{a+bi} = x+yi \iff (x+yi)^2 = a+bi$
Macht	$(a+bi)^0 = 1 \quad \forall n \in \mathbb{N}_0 :$ $(a+bi)^n = (a+bi)\cdot(a+bi)\cdots(a+bi)$
Machten of i	$i^1 = i, \quad i^2 = -1, \quad i^3 = -i, \quad i^4 = 1$

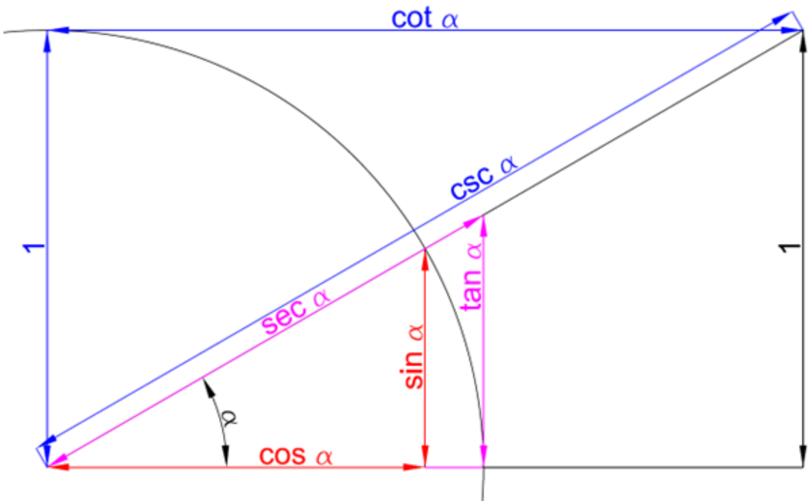
4.2 Poolcoördinaten

$$z = a + i.b = r\left(\cos(\varphi) + i.\sin(\varphi)\right) = r\angle\varphi, \quad \tan(\varphi) = \frac{b}{a}, \quad r = \sqrt{a^2+b^2}$$

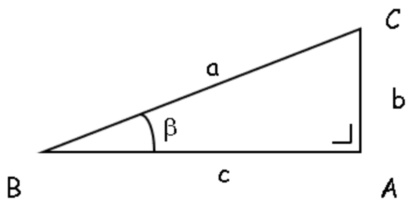
Bewerking	**Formule**
Vermenigvuldiging	$z_1 \cdot z_2 = r_1 \cdot r_2 \angle \varphi_1 + \varphi_2$
Deling	$\frac{z_1}{z_2} = \frac{r_1\angle\varphi_1}{r_2\angle\varphi_2} = \frac{r_1}{r_2}\angle\varphi_1 - \varphi_2$
Inverse	$z^{-1} = \frac{1}{r}\angle-\varphi$
Macht	$z^n = r^n\left[\cos(n\cdot\varphi) + i\sin(n\cdot\varphi)\right] \quad n \in \mathbb{N}$
Wortel	$\sqrt{r(\cos\varphi + i\sin\varphi)} = \pm\sqrt{r}\left(\cos\frac{\varphi}{2} + i\sin\frac{\varphi}{2}\right)$
$\sqrt[n]{r\left(\cos\varphi + i\sin\varphi\right)} = \sqrt[n]{r}\left(\cos\frac{\varphi+k\cdot 2\pi}{n} + i\sin\frac{\varphi+k\cdot 2\pi}{n}\right) \quad \wedge \quad k = 0, 1, \cdots, n-1$	

5 Goniometrie

5.1 De Goniometrische Cirkel



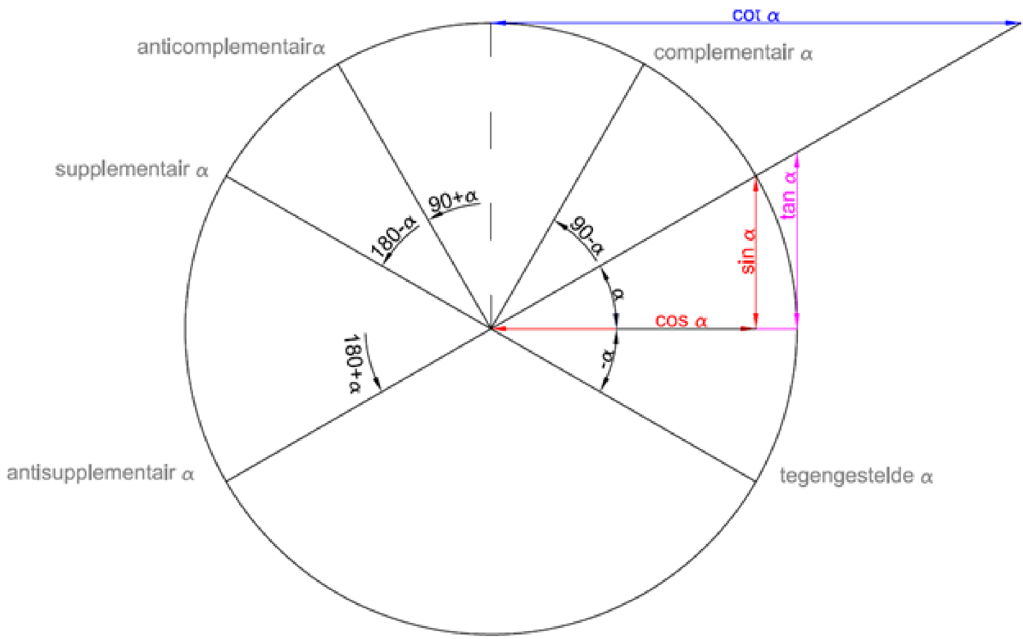
5.2 formules uit de goniometrie



$\begin{array}{ccc} \csc\beta & sec\beta & \cot\beta \\ \leftarrow & \leftarrow & \leftarrow \\ os & as & oa \\ \rightarrow & \rightarrow & \rightarrow \\ \sin\beta & \cos\beta & \tan\beta \end{array}$	waarin: $\begin{cases} o: & \text{overstaande rechthoekszijde} \\ s: & \text{schuine zijde (hypotenusa)} \\ a: & \text{aanliggende rechthoekszijde} \end{cases}$

$\sin\beta = \frac{b}{a}$	$\cos\beta = \frac{c}{a}$	$\tan\beta = \frac{b}{c}$
$\csc\beta = \frac{a}{b}$	$\sec\beta = \frac{a}{c}$	$\cot\beta = \frac{c}{b}$
$\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$	$\cot\alpha = \frac{\cos\alpha}{\sin\alpha}$	$\cot\alpha = \frac{1}{\tan\alpha}$
$\sec\alpha = \frac{1}{\cos\alpha}$	$\csc\alpha = \frac{1}{\sin\alpha}$	

$\boxed{\sin^2\alpha + \cos^2\alpha = 1}$ $\boxed{\tan^2\alpha + 1 = \sec^2\alpha}$ $\boxed{1 + \cot^2\alpha = \csc^2\alpha}$



gelijkehoeken	supplementairehoeken	complementairehoeken
$\sin(\alpha + k2\pi) = \sin\alpha$	$\sin(\pi - \alpha) = \sin\alpha$	$\sin\left(\frac{\pi}{2} - \alpha\right) = \cos\alpha$
$\cos(\alpha + k2\pi) = \cos\alpha$	$\cos(\pi - \alpha) = -\cos\alpha$	$\cos\left(\frac{\pi}{2} - \alpha\right) = \sin\alpha$
$\tan(\alpha + k2\pi) = \tan\alpha$	$\tan(\pi - \alpha) = -\tan\alpha$	$\tan\left(\frac{\pi}{2} - \alpha\right) = \cot\alpha$
$\cot(\alpha + k2\pi) = \cot\alpha$	$\cot(\pi - \alpha) = -\cot\alpha$	$\cot\left(\frac{\pi}{2} - \alpha\right) = \tan\alpha$
$\sec(\alpha + k2\pi) = \sec\alpha$	$\sec(\pi - \alpha) = -\sec\alpha$	$\sec\left(\frac{\pi}{2} - \alpha\right) = \csc\alpha$
$\csc(\alpha + k2\pi) = \csc\alpha$	$\csc(\pi - \alpha) = \csc\alpha$	$\csc\left(\frac{\pi}{2} - \alpha\right) = \sec\alpha$

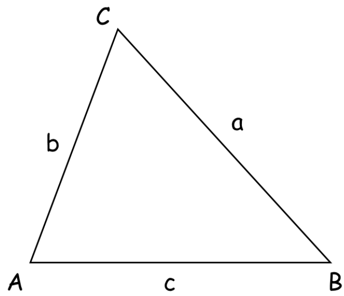
tegengesteldehoeken	antisupplementairehoeken	anticomplementairehoeken
$\sin(-\alpha) = -\sin\alpha$	$\sin(\pi + \alpha) = -\sin\alpha$	$\sin\left(\frac{\pi}{2} + \alpha\right) = \cos\alpha$
$\cos(-\alpha) = \cos\alpha$	$\cos(\pi + \alpha) = -\cos\alpha$	$\cos\left(\frac{\pi}{2} + \alpha\right) = -\sin\alpha$
$\tan(-\alpha) = -\tan\alpha$	$\tan(\pi + \alpha) = \tan\alpha$	$\tan\left(\frac{\pi}{2} + \alpha\right) = -\cot\alpha$
$\cot(-\alpha) = -\cot\alpha$	$\cot(\pi + \alpha) = \cot\alpha$	$\cot\left(\frac{\pi}{2} + \alpha\right) = -\tan\alpha$
$\sec(-\alpha) = \sec\alpha$	$\sec(\pi + \alpha) = -\sec\alpha$	$\sec\left(\frac{\pi}{2} + \alpha\right) = -\csc\alpha$
$\csc(-\alpha) = -\csc\alpha$	$\csc(\pi + \alpha) = -\csc\alpha$	$\csc\left(\frac{\pi}{2} + \alpha\right) = \sec\alpha$

Desinusregel :

$$\frac{a}{\sin \hat{A}} = \frac{b}{\sin \hat{B}} = \frac{c}{\sin \hat{C}}$$

Decosinusregel :

$$\begin{cases} a^2 = b^2 + c^2 - 2bc\cos \hat{A} \\ b^2 = c^2 + a^2 - 2ca\cos \hat{B} \\ c^2 = a^2 + b^2 - 2ab\cos \hat{C} \end{cases}$$



$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$	$\sin 2\alpha = 2\sin\alpha\cos\alpha$
$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$	$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$ $= 1 - 2\sin^2\alpha \quad (*)$ $= 2\cos^2\alpha - 1 \quad (**)$
$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$	$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha}$

$\sin^2\alpha = \frac{1-\cos 2\alpha}{2} \quad (*)$ $\cos^2\alpha = \frac{1+\cos 2\alpha}{2} \quad (**)$ $\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}$ $\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$	$\sin 2\alpha = \frac{2\tan\alpha}{1+\tan^2\alpha}$ $\cos 2\alpha = \frac{1-\tan^2\alpha}{1+\tan^2\alpha}$ $\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha}$	$\sin\alpha = \frac{2t}{1+t^2} \quad \wedge \quad \tan\frac{\alpha}{2} = t$ $\cos\alpha = \frac{1-t^2}{1+t^2}$ $\tan\alpha = \frac{2t}{1-t^2}$

5.3 Omgekeerde formules van Simpson

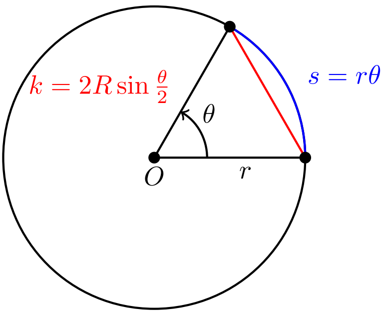
$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$	$\sin(\alpha+\beta) + \sin(\alpha-\beta) = 2\sin\alpha\cos\beta$ $\sin(\alpha+\beta) - \sin(\alpha-\beta) = 2\cos\alpha\sin\beta$
$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$	$\cos(\alpha+\beta) + \cos(\alpha-\beta) = 2\cos\alpha\cos\beta$ $\cos(\alpha+\beta) - \cos(\alpha-\beta) = -2\sin\alpha\sin\beta$

5.4 Formules van Simpson

$\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$ $\sin\alpha - \sin\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$	$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$ $\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$

5.5 Belangrijke goniometrische waarden

Angle	$0°$	$30°$	$45°$	$60°$	$90°$	$180°$	$270°$	$360°$
α	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\sin\alpha$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
$\cos\alpha$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0	1
$\tan\alpha$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	/	0	/	0

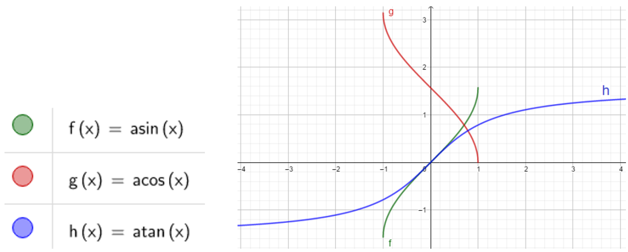


5.6 Cyclometrische formules

$$y = \text{Bgsin}x \Leftrightarrow \left(x = \sin y \quad \wedge y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right],\ x \in [-1,1]\right)$$

$$y = \text{Bgcos}x \Leftrightarrow (x = \cos y \quad \wedge y \in [0,\pi],\ x \in [-1,1])$$

$$y = \text{Bgtan}\ x \Leftrightarrow \left(x = \tan y \quad \wedge y \in \left]-\frac{\pi}{2}, \frac{\pi}{2}\right[,\ x \in \mathbb{R}\right)$$



$$\sin(\text{Bgsin}x) = x \wedge \forall x \in [-1,1]$$

$$\cos(\text{Bgcos}x) = x \wedge \forall x \in [-1,1]$$

$$tg\,(\text{Bgtan}) = x \wedge \forall x \in \mathbb{R}$$

$$cotg\,(\text{Bgcot}x) = x \wedge \forall x \in \mathbb{R}$$

$$\cos(\text{Bgsin}x) = \sqrt{1-x^2} \wedge \forall x \in [-1,1]$$

$$\sin(\text{Bgcos}x) = \sqrt{1-x^2} \wedge \forall x \in [-1,1]$$

$$Bgsin\,(-x) = -\text{Bgsin}x \wedge \forall x \in [-1,1]$$

$$\cos(\text{Bgtan}x) = \frac{1}{\sqrt{1+x^2}} \forall x \in \mathbb{R}$$

$$\sin(\text{Bgtan}x) = \frac{x}{\sqrt{1+x^2}} \forall x \in \mathbb{R}$$

$$\text{Bgsin}x + \text{Bgcos}x = \frac{\pi}{2} \wedge \forall x \in [-1,1]$$

$$\text{Bgcot}x + \text{Bgtan}x = \frac{\pi}{2} \wedge \forall x \in \mathbb{R}$$

$$\text{Bgtan}(-x) = -\text{Bgtan}x \wedge \forall x \in \mathbb{R}$$

$$\text{Bgcot}(-x) = -\text{Bgcot}x \wedge \forall x \in \mathbb{R}$$

$$\text{Bgcos}(-x) = \pi - \text{Bgcos}x \forall x \in [-1,1]$$

6 Meetkunde

Afstand 2 punten	$\lvert P_1(x_1,y_1),P_2(x_2,y_2)\rvert = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $\lvert P_1(x_1,y_1,z_1),P_2(x_2,y_2,z_2)\rvert =$ $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$
Midden v/e lijnstuk	$co(M) = (\frac{(x_1+x_2)}{2}, \frac{(y_1+y_2)}{2})$
Zwaartepunt v/e driehoek	$co(Z) = (\frac{(x_1+x_2+x_3)}{3}, \frac{(y_1+y_2+y_3)}{3})$

Vergelijking v/e rechte dr punt met rico m	$y - y_1 = m(x - x_1)$
Vergelijking v/e rechte dr punt met rico m	$y - y_1 = \frac{y_2-y_1}{x_2-x_1}(x - x_1)$
Vergelijking v/e rechte dr snijpunt met x-as (r,0) en y-as (0,s)	$\frac{x}{r} + \frac{y}{s} = 1$
Hoek tussen twee rechten a,b met rico m1,m2	$\cos(\widehat{ab}) = \frac{\lvert 1+m_1m_2\rvert}{\sqrt{1+{m_1}^2}\sqrt{1+{m_2}^2}}$
Afstand tussen rechte a-¿ux+vy+w=0 en P(x1,y1)	$d(P,a) = \frac{\lvert ux_1+vy_1+w\rvert}{\sqrt{u^2+v^2}}$

6.1 De cirkel

Cartesiaanse vergelijking	$(x-x_1)^2 + (y-y_1)^2 = r^2$
Algemene vergelijking	$x^2+y^2+2ax+2by+c=0 \quad \wedge \quad a^2+b^2-c \geq 0$
Parameter vergelijking	$\begin{cases} x = x_M + r.\cos t \\ y = y_M + r.\sin t \end{cases} \qquad met\ t \in [0,2\pi[$

6.2 De parabool

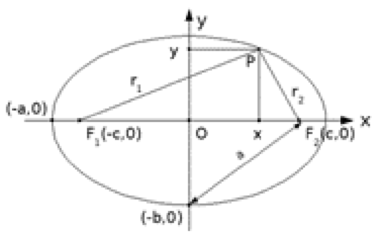
Top vergelijking	$y^2 = 2px$
Parameter vergelijking	$x = 2p\lambda^2 \qquad met\ \lambda \in \mathbb{R}$ $y = 2p\lambda$

6.3 De ellips

$Cartesiaansevgl.: \frac{x^2}{a^2} + \frac{y}{b^2}^2 = 1$

$Parametervgl.:$

$$\begin{cases} x = a.\cos t \\ y = b.\sin t \end{cases} \qquad met\ t \in [0,2\pi[$$

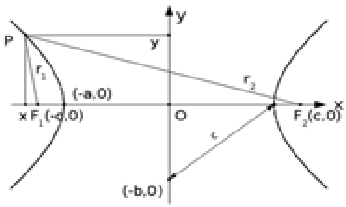


6.4 De hyperbool

$$Cartesiaansevgl.: \frac{x^2}{a^2} - \frac{y}{b^2}^2 = 1$$

$$Parametervgl.:$$

$$\begin{cases} x = a.\sec t \\ y = b.\tan t \end{cases} \quad met\ t \in \left]\frac{-\pi}{2}, \frac{3\pi}{2}\right[\setminus \left\{\frac{\pi}{2}\right\}$$



6.5 Oppervlakte Formules

Vorm	**Formule**	**Variabelen**
Vierkant	$A = s^2$	s: zijlengte
Rechthoek	$A = l.w$	l: lengte, w: breedte
Driehoek	$A = \frac{1}{2}b.h$	b: basis, h: hoogte
Cirkel	$A = \pi r^2$	r: straal
Parallellogram	$A = b.h$	b: basis, h: hoogte
Trapezium	$A = \frac{1}{2}(b_1 + b_2).h$	b_1, b_2: bases, h: hoogte
Ellips	$A = \pi a.b$	a, b: halve grote en halve kleine as
Regelmatig Veelhoek	$A = \frac{1}{2}P.a$	P: omtrek, a: apothema

6.6 Volume Formules

Vorm	**Formule**	**Variabelen**
Kubus	$V = s^3$	s: zijlengte
Rechthoekig Prisma	$V = l \times w \times h$	l: lengte, w: breedte, h: hoogte
Bol	$V = \frac{4}{3}\pi r^3$	r: straal
Cilinder	$V = \pi r^2 h$	r: straal, h: hoogte
Kegel	$V = \frac{1}{3}\pi r^2 h$	r: straal, h: hoogte
Piramide	$V = \frac{1}{3}B \times h$	B: basisoppervlakte, h: hoogte
Ellipsoïde	$V = \frac{4}{3}\pi abc$	a, b, c: halve hoofdaslengtes
Prisma	$V = B \times h$	B: basisoppervlakte, h: hoogte

6.7 Basis reële functies

Functie	**Definitie**
Identiteitsfunctie	$f(x) = x$
Constante functie	$f(x) = c,\ c \in \mathbb{R}$
Lineaire functie	$f(x) = mx + b,\ m, b \in \mathbb{R}$
Kwadratische functie	$f(x) = ax^2 + bx + c,\ a, b, c \in \mathbb{R},\ a \neq 0$
Cubische functie	$f(x) = ax^3 + bx^2 + cx + d,\ a, b, c, d \in \mathbb{R},\ a \neq 0$
Polynoomfunctie	$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0,\ a_i \in \mathbb{R}$ $a_n \neq 0$
Rationale functie	$f(x) = \frac{P(x)}{Q(x)},\ P(x), Q(x)$ zijn polynomen, $Q(x) \neq 0$
Exponentiële functie	$f(x) = a^x,\ a > 0,\ a \neq 1$
Logaritmische functie	$f(x) = \log_a(x),\ a > 0,\ a \neq 1,\ x > 0$
Absolute-waarde functie	$f(x) = \lvert x \rvert = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$
Goniometrische functies	$f(x) = \sin(x)$ $f(x) = \cos(x)$ $f(x) = \tan(x)\ x \neq \frac{\pi}{2} + k\pi,\ k \in \mathbb{Z}$
Inverse goniometrische functies	$f(x) = \arcsin(x),\ x \in [-1, 1]$ $f(x) = \arccos(x),\ x \in [-1, 1]$ $f(x) = \arctan(x),\ x \in \mathbb{R}$
Hyperbolische functies	$f(x) = \sinh(x) = \frac{e^x - e^{-x}}{2}$ $f(x) = \cosh(x) = \frac{e^x + e^{-x}}{2}$ $f(x) = \tanh(x) = \frac{\sinh(x)}{\cosh(x)},\ x \in \mathbb{R}$
Stukjesfunctie	$f(x) = \begin{cases} x^2, & x < 0 \\ x + 1, & x \geq 0 \end{cases}$

7 Analyse

7.1 Limieten van rijen)

$\lim\limits_{n\to\pm\infty}\left(a_m n^m + a_{m-1}n^{m-1} + ... + a_1 n + a_0\right) = \lim\limits_{n\to\pm\infty} a_m n^m$
$\lim\limits_{n\to\pm\infty}\frac{\left(a_m n^m + a_{m-1}n^{m-1} + ... + a_1 n + a_0\right)}{\left(b_q n^p + b_{q-1}n^{p-1} + ... + b_1 n + b_0\right)} = \lim\limits_{n\to\pm\infty}\frac{a_m n^m}{b_q n^p}$

7.2 Limieten van functies)

$\lim\limits_{x\to a}\left[f\left(x\right) \pm g\left(x\right)\right] = \lim\limits_{x\to a} f\left(x\right) \pm \lim\limits_{x\to a} g\left(x\right)$
$\lim\limits_{x\to a}\left[f\left(x\right).g\left(x\right)\right] = \lim\limits_{x\to a} f\left(x\right).\lim\limits_{x\to a} g\left(x\right)$
$\lim\limits_{x\to a}\left[f\left(x\right)\right]^n = \left[\lim\limits_{x\to a} f\left(x\right)\right]^n \qquad (n \in {}_0)$
$\lim\limits_{x\to a}\sqrt[n]{f\left(x\right)} = \sqrt[n]{\lim\limits_{x\to a} f\left(x\right)}$
$\lim\limits_{x\to a}\frac{f(x)}{g(x)} = \frac{\lim\limits_{x\to a} f(x)}{\lim\limits_{x\to a} g(x)}$ $\lim\limits_{x\to\pm\infty}\left(a_n x^n + a_{n-1}x^{n-1} + ... + a_1 x + a_0\right) = \lim\limits_{x\to\pm\infty} a_n x^n$
$\lim\limits_{x\to\pm\infty}\frac{a_n x^n + a_{n-1}x^{n-1} + ... + a_1 x + a_0}{b_m x^m + b_{m-1}x^{m-1} + ... + b_1 x + b_0} = \lim\limits_{x\to\pm\infty}\frac{a_n x^n}{b_m x^m}$

$\lim\limits_{x\to a}\sin(x) = \sin(a)$ $\lim\limits_{x\to a}\cos(x) = \cos(a)$	$\lim\limits_{x\to 0}\frac{\sin x}{x} = 1$ $\lim\limits_{x\to 0}\frac{\tan x}{x} = 1$

7.3 Afgeleiden - differentialen

$Dc = 0$	
$D\,(c.f) = c.Df$	$dc = 0$
$D\,(f \pm g) = Df \pm Dg$	$dx^n = nx^{n-1}dx$
$D\,(f.g) = fDg + gDf$	$dx^{-1} = -1.x^{-2}dx$
$D\left(\frac{f}{g}\right) = \frac{gDf - fDg}{g^2}$	$d\sin x = \cos x dx$
$Dx^n = nx^{n-1}$	$d\cos x = -\sin x dx$
$Dx^{-1} = -1.x^{-2}$	$d\tan x = \sec^2 x dx = \frac{1}{\cos^2 x}dx$
$D\sin x = \cos x$	$d\cot x = -csc^2 x dx = \frac{-1}{\sin^2 x}dx$
$D\cos x = -\sin x$	$d\text{Bgsin}x = \frac{dx}{\sqrt{1-x^2}}$
$D\tan x = \sec^2 x = \frac{1}{\cos^2 x}$	$d\text{Bgcos}x = \frac{-dx}{\sqrt{1-x^2}}$
$D\cot x = -csc^2 x = \frac{-1}{\sin^2 x}$	$d\text{Bgtan}x = \frac{dx}{1+x^2}$
$D\text{Bgsin}x = \frac{1}{\sqrt{1-x^2}}$	$dshx = chxdx$
$D\text{Bgcos}x = \frac{-1}{\sqrt{1-x^2}}$	$dchx = shxdx$
$D\text{Bgtan}x = \frac{1}{1+x^2}$	$dthx = \frac{dx}{ch^2 x}$
$D\text{sh}x = \text{ch}x$	$d^a\log x = \frac{dx}{x\ln a}$
$D\text{ch}x = \text{sh}x$	$d\ln\|x\| = \frac{dx}{x}$
$D\text{th}x = \frac{1}{ch^2 x}$	$da^x = a^x \ln a dx$
$De^x = e^x$	$de^x = e^x dx$
$Da^x = a^x \ln a$	$d\ln\left\|x + \sqrt{x^2+k}\right\| = \frac{dx}{\sqrt{x^2+k}}$
$D\ln x = \frac{1}{x} \quad D\ln\|x\| = \frac{1}{x}$	$d\,(f+g) = df + dg$
$D^a\log x = \frac{1}{x\ln a}$	$d\,(f.g) = fdg + gdf$
$D\ln\left\|x + \sqrt{x^2+k}\right\| = \frac{1}{\sqrt{x^2+k}}$	$d\left(\frac{f}{g}\right) = \frac{gdf - fdg}{g^2}$
$Du^v = vu^{v-1}Du + u^v \ln u Dv$	

7.4 Afgeleiden - fundamentele integralen

Bg = arc

Afgeleiden	**Integraal**
$D[c] = 0$	$\int dx = x + C$
$D[x^n] = nx^{n-1}$	$\int x^n dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$
$D[\sin x] = \cos x$	$\int \cos x \, dx = \sin x + C$
$D[\cos x] = -\sin x$	$\int \sin x \, dx = -\cos x + C$
$D[\tan x] = \sec^2 x = \frac{1}{\cos^2 x}$	$\int \frac{1}{\cos^2 x} \, dx = \tan x + C$
$D[\cot x] = -\csc^2 x = \frac{-1}{\sin^2 x}$	$\int \frac{1}{\sin^2 x} \, dx = -\cot x + C$
$D[\arcsin x] = \frac{1}{\sqrt{1-x^2}}$	$\int \frac{dx}{\sqrt{1-x^2}} = \arcsin x + C$
$D[\arccos x] = \frac{-1}{\sqrt{1-x^2}}$	$\int \frac{dx}{\sqrt{1-x^2}} = -\arccos x + C$
$D[\arctan x] = \frac{1}{1+x^2}$	$\int \frac{dx}{1+x^2} = \arctan x + C$
$D[e^x] = e^x$	$\int e^x dx = e^x + C$
$D[a^x] = a^x \ln a$	$\int a^x dx = \frac{a^x}{\ln a} + C$
$D[\ln x] = \frac{1}{x}$	$\int \frac{dx}{x} = \ln \lvert x \rvert + C$
$D\left[\ln \left\lvert x + \sqrt{x^2 + k} \right\rvert\right] = \frac{1}{\sqrt{x^2+k}}$	$\int \frac{dx}{\sqrt{x^2+k}} = \ln \left\lvert x + \sqrt{x^2 + k} \right\rvert + C$
$D^a \log x = \frac{1}{x \ln a}$	*

8 Diversen

8.1 Wiskundige Symbolen (ISO 31/XI)

$x \in A$	is een element van de verzameling
$x \notin A$	is geen element van de verzameling
$\{x_1, x_2, \ldots, x_n\}$	de verzameling door opsomming
$\{x \in A \mid p(x)\}$	de verzameling waar de elementen voldoen aan de eigenschap $p(x)$
$\emptyset$	de lege verzameling
$\mathbb{N}$	de natuurlijke getallen $(0, 1, 2, \ldots)$
$\mathbb{Z}$	de gehele getallen $(\ldots, -2, -1, 0, 1, 2, \ldots)$
$\mathbb{Q}$	de rationale getallen (breuken van $\mathbb{Z}$)
$\mathbb{R}$	de reële getallen
$\mathbb{C}$	de complexe getallen
$B \subseteq A$	B behoort tot A (kan er mee samenvallen)
$B \subset A$	B behoort strikt tot A
$A \cup B$	samenvoeging van A en B (unie)
$A \cap B$	doorsnede van A en B (de gemeenschappelijke elementen)
$A \setminus B$	A verschilt B, wat tot A behoort en niet tot B
$\mathcal{C}_U A$	het complement van A in het universum U
(a, b)	het geordend paar
$(a_1, a_2, \ldots, a_n)$	een geordend n-tal
$A \times B$	de productverzameling van A en B
$\#$	rangnummer of aantal

8.2 Logische symbolen

$p \wedge q$	conjunctie, de beweringen p en q zijn geldig
$p \vee q$	disjunctie, de bewering p of q is geldig
$\neg p$	negatie, de bewering p is niet geldig
$p \Rightarrow q$	implicatie, als p dan q
$p \Leftrightarrow q$	equivalentie, de beweringen p en q zijn gelijkwaardig
$\forall x$	universele kwantor, voor alle elementen geldt
$\exists x$	existentiële kwantor, er zijn elementen die voldoen aan