# Formularium Wiskunde

Ian Claesen

8 december 2024

## Inhoudsopgave

# 1 Algebra

## 1.1 Volgorde van Bewerking

Haakjes wegwerken, machtsverheffen, worteltrekken, vermenigvuldigen en delen, optellen en aftrekken.

## 1.2 Absolute Waarde

De absolute waarde van een getal $a$ wordt genoteerd als $|a|$ en is altijd positief.

$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$

# 2 Machten en wortels

## 2.1 Machten met Gehele Exponenten

| | |
|---|---|
| $\forall a \in, \forall n \in \mathbb{N}_0 : a^n = \underbrace{a.a.\ ...\ .a}_{n\ factoren}$<br>$\forall a \in \mathbb{R} :\ a^1 =\ a$<br>$\forall a \in \mathbb{R}_0 :\ a^0 =\ 1$<br>$\forall a \in \mathbb{R}_0, \forall n \in \mathbb{N} :\ a^{-n} =\ \frac{1}{a^n}$ | $\forall a, b \in \mathbb{R}_0, \forall m, n \in \mathbb{Z} : a^m \cdot a^n = a^{m+n}$<br>$\frac{a^m}{a^n} = a^{m-n}$<br>$(a^m)^n = a^{mn}$<br>$(a.b)^n = a^n \cdot b^n$<br>$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$<br>$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$ |

## 2.2 Vierkantswortel in $\mathbb{R}$

| | |
|---|---|
| $\forall a \in \mathbb{R}^+, \forall b \in \mathbb{R} :$<br>$b = \sqrt{a} \Leftrightarrow b^2 = a \wedge (b \geq 0)$<br>$\forall a, b \in \mathbb{R}^+ :$<br>$\sqrt{a^2} = a$<br>$\left(\sqrt{a}\right)^2 = a$<br>$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}.$<br>$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}} \wedge b \neq 0$ | $\forall a \in \mathbb{R} :$<br>$\sqrt{a^2} = \|a\| \implies \begin{cases} \sqrt{a^2} = a & \text{als } a \geq 0, \\ \sqrt{a^2} = -a & \text{als } a \leq 0. \end{cases}$ |

## 2.3 N-de machtswortel in $\mathbb{R}$

| | |
|---|---|
| $n\ even \Rightarrow \sqrt[n]{a^n} = \|a\| \rightarrow \begin{cases} \sqrt[n]{a^n} = a & \wedge\, a \geq 0 \\ \sqrt[n]{a^n} = -a & \wedge\, a \leq 0 \end{cases}$<br>$n\ oneven \Rightarrow \sqrt[n]{a^n} = a$ | $\forall a, b \in \mathbb{R}_0^+, \forall m, n \in \mathbb{N}_0 :$<br>$\sqrt[n]{a^n} = a$<br>$\left(\sqrt[n]{a}\right)^n = a$<br>$\sqrt[n]{ab} = \sqrt[n]{a}. \sqrt[n]{b}$<br>$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$<br>$\sqrt[m]{\sqrt[n]{a}} = \sqrt[m.n]{a}$ |

### 2.4 $\frac{m}{n}$-de machtswortel in $\mathbb{R}$

| | |
|---|---|
| $\forall a \in \mathbb{R}_0^+, \forall m \in \mathbb{Z}, \forall n \in \mathbb{N}_0 : a^{\frac{m}{n}} = \sqrt[n]{a^m}$ | $\forall a, b \in \mathbb{R}_0^+, \forall m, n \in \mathbb{Q}:$<br>$a^m.a^n = a^{m+n}$<br>$\frac{a^m}{a^n} = a^{m-n}$<br>$(a^m)^n = a^{m.n}$<br>$(a.b)^m = a^m.b^m$<br>$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$ |

# 3 Veeltermen

## 3.1 Vierkantsvergelijking

*Een vierkantsvergelijking is van de vorm :* $ax^2 + bx + c = 0$ , *met* $D = b^2 - 4ac$

| $x \in \mathbb{R}$ | $x \in \mathbb{C}$ |
|---|---|
| $x_{1,2} = \frac{-b \pm \sqrt{D}}{2a}$ | $x_{1,2} = \frac{-b \pm i\sqrt{-D}}{2a}$ |
| $P = \frac{c}{a} = x_1 \cdot x_2$ , $S = -\frac{b}{a} = x_1 + x_2$ | |
| $ax^2 + bx + c = a(x - x_1)(x - x_2) = a(x^2 - Sx + P)$ | |

## 3.2 Merkwaardige Producten en Ontbinding in Factoren

| |
|---|
| $(a \pm b)^2 = a^2 \pm 2ab + b^2$ |
| $(a \pm b)^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3$ |
| $(a + b)^n = a^n + C_n^1 a^{n-1} b + C_n^2 a^{n-2} b^2 + ... + C_n^{n-1} a^2 b^{n-1} + b^n \quad \wedge \quad C_n^p = \frac{n!}{(n-p)!p!}$ |
| $a^2 - b^2 = (a + b)(a - b)$ |
| $a^3 - b^3 = (a - b)\left(a^2 + ab + b^2\right)$ |
| $a^n - b^n = (a - b)\left(a^{n-1} + a^{n-2}b + a^{n-3}b^2 + ... + ab^{n-2} + b^{n-1}\right)$ |
| $a^3 + b^3 = (a + b)\left(a^2 - ab + b^2\right)$ |
| $a^{2n+1} + b^{2n+1} = (a + b)\left(a^{2n} - a^{2n-1}b + a^{2n-2}b^2 - a^{2n-3}b^3 + ... - ab^{2n-1} + b^{2n}\right)$ |

## 3.3 Euclidische Deling

We gaan de derdegraadsveelterm $2x^3+3x^2-4x+5$ delen door de eerstegraadsveelterm $x+2$ met behulp van de praktische werkwijze van lange deling.

| | |
|---:|:---|
| $2x^3+3x^2-4x+5$ | $x+2$ |
| $-2x^3-4x^2+0x+0$ | $2x^2$ |
| $-1x^2-4x+5$ | |
| $+1x^2+2x+0$ | $-x$ |
| $-2x+5$ | |
| $2x+4$ | $-2$ |
| $9$ | |

We kunnen de deling als volgt uitdrukken:

$$2x^3+3x^2-4x+5=(x+2)(2x^2-x-2)+9$$

De rest is 25, wat een graad heeft die kleiner is dan de graad van de deler $x+2$.

## 3.4 Schema van Horner

$$\frac{(3x^3-5x^2+10x-52)}{(x-2)}$$

| | | | | |
|---|---|---|---|---|
| | 3 | -5 | +10 | -5 |
| 2 | | +6 | +2 | +24 |
| | 3 | +1 | +12 | +19 |

# 4 Complexe getallen

## 4.1 Rechthoekige coordinaten

| **Bewerking** | **Formule** |
|---|---|
| $Optelling/Aftrekking$ | $(a+j.b) \pm (c+j.d) = (a+c) \pm j(b+d)$ |
| $Vermenigvuldiging$ | $(a+j.b) \cdot (c+j.d) = (ac-bd) + j(ad+bc)$ |
| $Deling$ | $\frac{(a+j.b)}{(c+j.d)} = \frac{(a+j.b)\cdot(c-j.d)}{(c+j.d)\cdot(c-j.d)} = \left(\frac{ac+bd}{c^2+d^2}\right) + j\left(\frac{bc-ad}{c^2+d^2}\right)$ |
| $Toegevoegde\ van$ | $\overline{(a+j.b)} = (a-j.b)$<br>$\overline{Z_1+Z_2} = \overline{Z_1} + \overline{Z_2}, \quad \overline{Z_1 \cdot Z_2} = \overline{Z_1} \cdot \overline{Z_2}$ |
| $Inverse$ | $z = a+bi \implies z^{-1} = \frac{a-bi}{a^2+b^2}$ |
| $Wortel$ | $\sqrt{a} \wedge a<0 \implies \sqrt{a} = \pm i\sqrt{-a}$<br>$\sqrt{a+bi} = x+yi \iff (x+yi)^2 = a+bi$ |
| $Macht$ | $(a+bi)^0 = 1 \quad \forall n \in \mathbb{N}_0 :$<br>$(a+bi)^n = (a+bi)\cdot(a+bi)\cdots(a+bi)$ |
| $Machten\ of\ i$ | $i^1 = i, \quad i^2 = -1, \quad i^3 = -i, \quad i^4 = 1$ |

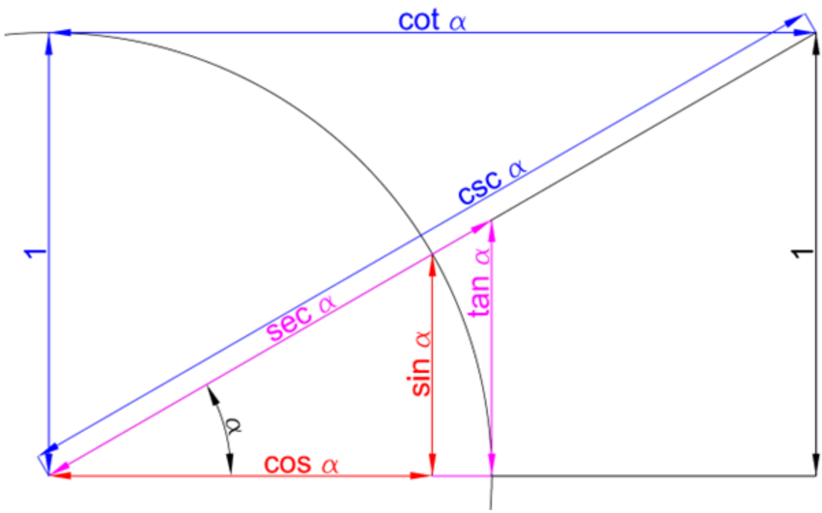
## 4.2 Poolcoördinaten

$$z = a + i.b = r\left(\cos(\varphi) + i.\sin(\varphi)\right) = r\angle\varphi, \quad \tan(\varphi) = \frac{b}{a}, \quad r = \sqrt{a^2+b^2}$$

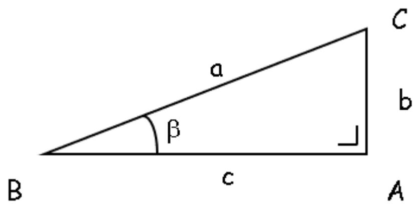
| **Bewerking** | **Formule** |
|---|---|
| $Vermenigvuldiging$ | $z_1 \cdot z_2 = r_1 \cdot r_2 \angle \varphi_1 + \varphi_2$ |
| $Deling$ | $\frac{z_1}{z_2} = \frac{r_1\angle\varphi_1}{r_2\angle\varphi_2} = \frac{r_1}{r_2}\angle\varphi_1 - \varphi_2$ |
| $Inverse$ | $z^{-1} = \frac{1}{r}\angle - \varphi$ |
| $Macht$ | $z^n = r^n\left[\cos(n\cdot\varphi) + i\sin(n\cdot\varphi)\right] \quad n \in \mathbb{N}$ |
| $Wortel$ | $\sqrt{r(\cos\varphi + i\sin\varphi)} = \pm\sqrt{r}\left(\cos\frac{\varphi}{2} + i\sin\frac{\varphi}{2}\right)$ |
| $\sqrt[n]{r\left(\cos\varphi + i\sin\varphi\right)} = \sqrt[n]{r}\left(\cos\frac{\varphi+k\cdot2\pi}{n} + i\sin\frac{\varphi+k\cdot2\pi}{n}\right) \quad \wedge \quad k = 0,1,\cdots,n-1$ | |

# 5 Goniometrie

## 5.1 De Goniometrische Cirkel



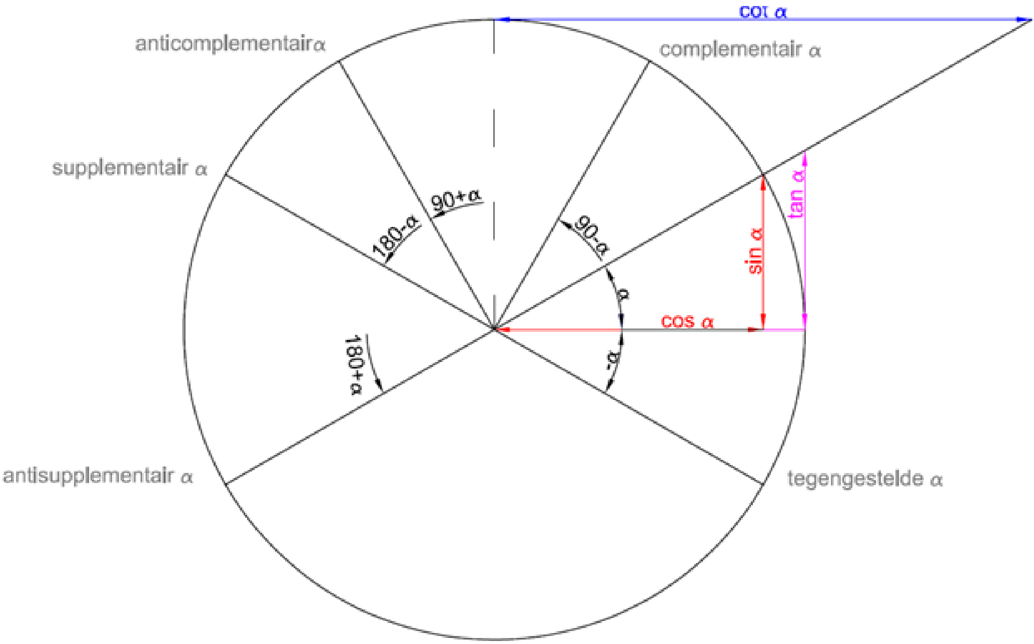

## 5.2 formules uit de goniometrie




| | |
|---|---|
| $\begin{array}{ccc} \csc\beta & sec\beta & \cot\beta \\ \leftarrow & \leftarrow & \leftarrow \\ os & as & oa \\ \rightarrow & \rightarrow & \rightarrow \\ \sin\beta & \cos\beta & \tan\beta \end{array}$ | waarin: $\begin{cases} o: & \text{overstaande rechthoekszijde} \\ s: & \text{schuine zijde (hypotenusa)} \\ a: & \text{aanliggende rechthoekszijde} \end{cases}$ |

| | | |
|---|---|---|
| $\sin\beta = \frac{b}{a}$ | $\cos\beta = \frac{c}{a}$ | $\tan\beta = \frac{b}{c}$ |
| $\csc\beta = \frac{a}{b}$ | $\sec\beta = \frac{a}{c}$ | $\cot\beta = \frac{c}{b}$ |
| $\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$ | $\cot\alpha = \frac{\cos\alpha}{\sin\alpha}$ | $\cot\alpha = \frac{1}{\tan\alpha}$ |
| $\sec\alpha = \frac{1}{\cos\alpha}$ | $\csc\alpha = \frac{1}{\sin\alpha}$ | |

$\boxed{\sin^2\alpha + \cos^2\alpha = 1}$ $\boxed{\tan^2\alpha + 1 = \sec^2\alpha}$ $\boxed{1 + \cot^2\alpha = \csc^2\alpha}$




| gelijkehoeken | supplementairehoeken | complementairehoeken |
|---|---|---|
| $\sin(\alpha + k2\pi) = \sin\alpha$ | $\sin(\pi - \alpha) = \sin\alpha$ | $\sin\left(\frac{\pi}{2} - \alpha\right) = \cos\alpha$ |
| $\cos(\alpha + k2\pi) = \cos\alpha$ | $\cos(\pi - \alpha) = -\cos\alpha$ | $\cos\left(\frac{\pi}{2} - \alpha\right) = \sin\alpha$ |
| $\tan(\alpha + k2\pi) = \tan\alpha$ | $\tan(\pi - \alpha) = -\tan\alpha$ | $\tan\left(\frac{\pi}{2} - \alpha\right) = \cot\alpha$ |
| $\cot(\alpha + k2\pi) = \cot\alpha$ | $\cot(\pi - \alpha) = -\cot\alpha$ | $\cot\left(\frac{\pi}{2} - \alpha\right) = \tan\alpha$ |
| $\sec(\alpha + k2\pi) = \sec\alpha$ | $\sec(\pi - \alpha) = -\sec\alpha$ | $\sec\left(\frac{\pi}{2} - \alpha\right) = \csc\alpha$ |
| $\csc(\alpha + k2\pi) = \csc\alpha$ | $\csc(\pi - \alpha) = \csc\alpha$ | $\csc\left(\frac{\pi}{2} - \alpha\right) = \sec\alpha$ |

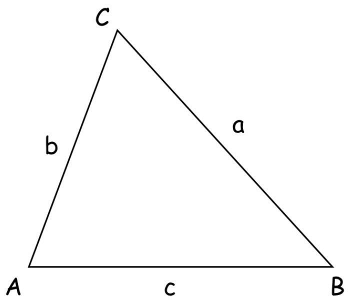
| tegengesteldehoeken | antisupplementairehoeken | anticomplementairehoeken |
|---|---|---|
| $\sin(-\alpha) = -\sin\alpha$ | $\sin(\pi + \alpha) = -\sin\alpha$ | $\sin\left(\frac{\pi}{2} + \alpha\right) = \cos\alpha$ |
| $\cos(-\alpha) = \cos\alpha$ | $\cos(\pi + \alpha) = -\cos\alpha$ | $\cos\left(\frac{\pi}{2} + \alpha\right) = -\sin\alpha$ |
| $\tan(-\alpha) = -\tan\alpha$ | $\tan(\pi + \alpha) = \tan\alpha$ | $\tan\left(\frac{\pi}{2} + \alpha\right) = -\cot\alpha$ |
| $\cot(-\alpha) = -\cot\alpha$ | $\cot(\pi + \alpha) = \cot\alpha$ | $\cot\left(\frac{\pi}{2} + \alpha\right) = -\tan\alpha$ |
| $\sec(-\alpha) = \sec\alpha$ | $\sec(\pi + \alpha) = -\sec\alpha$ | $\sec\left(\frac{\pi}{2} + \alpha\right) = -\csc\alpha$ |
| $\csc(-\alpha) = -\csc\alpha$ | $\csc(\pi + \alpha) = -\csc\alpha$ | $\csc\left(\frac{\pi}{2} + \alpha\right) = \sec\alpha$ |

- **De sinusregel:**

$$\frac{a}{\sin \hat{A}} = \frac{b}{\sin \hat{B}} = \frac{c}{\sin \hat{C}}$$

- **De cosinusregel:**

$$a^2 = b^2 + c^2 - 2bc\cos \hat{A}$$
$$b^2 = c^2 + a^2 - 2ca\cos \hat{B}$$
$$c^2 = a^2 + b^2 - 2ab\cos \hat{C}$$




| | |
|---|---|
| $\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$ | $\sin 2\alpha = 2\sin\alpha\cos\alpha$ |
| $\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$ | $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$ $= 1 - 2\sin^2\alpha \quad (*)$ $= 2\cos^2\alpha - 1 \quad (**)$ |
| $\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$ | $\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha}$ |

| | | |
|---|---|---|
| $\sin^2\alpha = \frac{1-\cos 2\alpha}{2} \quad (*)$ | $\sin 2\alpha = \frac{2\tan\alpha}{1+\tan^2\alpha}$ | $\sin\alpha = \frac{2t}{1+t^2} \quad \wedge \quad \tan\frac{\alpha}{2} = t$ |
| $\cos^2\alpha = \frac{1+\cos 2\alpha}{2} \quad (**)$ | $\cos 2\alpha = \frac{1-\tan^2\alpha}{1+\tan^2\alpha}$ | $\cos\alpha = \frac{1-t^2}{1+t^2}$ |
| $\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}$ | $\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha}$ | $\tan\alpha = \frac{2t}{1-t^2}$ |
| $\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$ | | |

## 5.3 Omgekeerde formules van Simpson

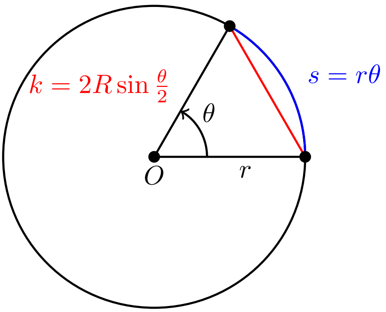
| | |
|---|---|
| $\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$ | $\sin(\alpha+\beta) + \sin(\alpha-\beta) = 2\sin\alpha\cos\beta$ $\sin(\alpha+\beta) - \sin(\alpha-\beta) = 2\cos\alpha\sin\beta$ |
| $\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$ | $\cos(\alpha+\beta) + \cos(\alpha-\beta) = 2\cos\alpha\cos\beta$ $\cos(\alpha+\beta) - \cos(\alpha-\beta) = -2\sin\alpha\sin\beta$ |

## 5.4 Formules van Simpson

| | |
|---|---|
| $\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$ | $\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$ |
| $\sin\alpha - \sin\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$ | $\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$ |

## 5.5 Belangrijke goniometrische waarden

| Angle | 0° | 30° | 45° | 60° | 90° | 180° | 270° | 360° |
|---|---|---|---|---|---|---|---|---|
| $\alpha$ | $0$ | $\frac{\pi}{6}$ | $\frac{\pi}{4}$ | $\frac{\pi}{3}$ | $\frac{\pi}{2}$ | $\pi$ | $\frac{3\pi}{2}$ | $2\pi$ |
| $\sin\alpha$ | $0$ | $\frac{1}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{3}}{2}$ | $1$ | $0$ | $-1$ | $0$ |
| $\cos\alpha$ | $1$ | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{1}{2}$ | $0$ | $-1$ | $0$ | $1$ |
| $\tan\alpha$ | $0$ | $\frac{1}{\sqrt{3}}$ | $1$ | $\sqrt{3}$ | / | $0$ | / | $0$ |

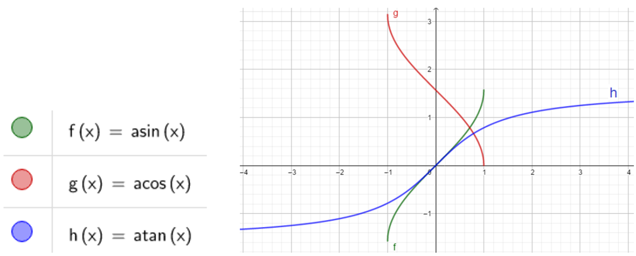



## 5.6 Cyclometrische formules

$$y = \text{Bgsin}x \Leftrightarrow \left(x = \sin y \quad \wedge y \in \left[-\tfrac{\pi}{2}, \tfrac{\pi}{2}\right],\ x \in [-1,1]\right)$$

$$y = \text{Bgcos}x \Leftrightarrow (x = \cos y \quad \wedge y \in [0,\pi],\ x \in [-1,1])$$

$$y = \text{Bgtan}\ x \Leftrightarrow \left(x = \tan y \quad \wedge y \in \left]-\tfrac{\pi}{2}, \tfrac{\pi}{2}\right[,\ x \in\right)$$




$$\sin(\text{Bgsin}x) = x \forall x \in [-1,1]$$

$$\cos(\text{Bgcos}x) = x \forall x \in [-1,1]$$

$$tg(\text{Bgtan}) = x \forall x \in$$

$$cotg(\text{Bgcot}x) = x \forall x \in$$

$$\cos(\text{Bgsin}x) = \sqrt{1-x^2} \forall x \in [-1,1]$$

$$\sin(\text{Bgcos}x) = \sqrt{1-x^2} \forall x \in [-1,1]$$

$$Bgsin(-x) = -\text{Bgsin}x \forall x \in [-1,1]$$

$$\cos(\text{Bgtan}x) = \frac{1}{\sqrt{1+x^2}} \forall x \in$$

$$\sin(\text{Bgtan}x) = \frac{x}{\sqrt{1+x^2}} \forall x \in$$

$$\text{Bgsin}x + \text{Bgcos}x = \frac{\pi}{2} \forall x \in [-1,1]$$

$$\text{Bgcot}x + \text{Bgtan}x = \frac{\pi}{2} \forall x \in$$

$$\text{Bgtan}(-x) = -\text{Bgtan}x \forall x \in$$

$$\text{Bgcot}(-x) = -\text{Bgcot}x \forall x \in$$

$$\text{Bgcos}(-x) = \pi - \text{Bgcos}x \forall x \in [-1,1]$$

# 6 Diversen

## 6.1 Wiskundige Symbolen (ISO 31/XI)

| | |
|---|---|
| $x \in A$ | is een element van de verzameling |
| $x \notin A$ | is geen element van de verzameling |
| $\{x_1, x_2, \ldots, x_n\}$ | de verzameling door opsomming |
| $\{x \in A \mid p(x)\}$ | de verzameling waar de elementen voldoen aan de eigenschap $p(x)$ |
| $\emptyset$ | de lege verzameling |
| $\mathbb{N}$ | de natuurlijke getallen $(0, 1, 2, \ldots)$ |
| $\mathbb{Z}$ | de gehele getallen $(\ldots, -2, -1, 0, 1, 2, \ldots)$ |
| $\mathbb{Q}$ | de rationale getallen (breuken van $\mathbb{Z}$) |
| $\mathbb{R}$ | de reële getallen |
| $\mathbb{C}$ | de complexe getallen |
| $B \subseteq A$ | $B$ behoort tot $A$ (kan er mee samenvallen) |
| $B \subset A$ | $B$ behoort strikt tot $A$ |
| $A \cup B$ | samenvoeging van $A$ en $B$ (unie) |
| $A \cap B$ | doorsnede van $A$ en $B$ (de gemeenschappelijke elementen) |
| $A \setminus B$ | $A$ verschilt $B$, wat tot $A$ behoort en niet tot $B$ |
| $\mathcal{C}_U A$ | het complement van $A$ in het universum $U$ |
| $(a, b)$ | het geordend paar |
| $(a_1, a_2, \ldots, a_n)$ | een geordend $n$-tal |
| $A \times B$ | de productverzameling van $A$ en $B$ |
| $\#$ | rangnummer of aantal |

## 6.2 Logische symbolen

| | |
|---|---|
| $p \wedge q$ | conjunctie, de beweringen $p$ en $q$ zijn geldig |
| $p \vee q$ | disjunctie, de bewering $p$ of $q$ is geldig |
| $\neg p$ | negatie, de bewering $p$ is niet geldig |
| $p \Rightarrow q$ | implicatie, als $p$ dan $q$ |
| $p \Leftrightarrow q$ | equivalentie, de beweringen $p$ en $q$ zijn gelijkwaardig |
| $\forall x$ | universele kwantor, voor alle elementen geldt |
| $\exists x$ | existentiële kwantor, er zijn elementen die voldoen aan |