(a) Approximate 95% confidence intervals are of the form

$$\hat{\alpha} \pm 1.96\sqrt{I_{11}^{-1}} = (11.66 \pm 6.43)$$
$$= (5.22,\ 18.10)$$

$$\hat{\beta} \pm 1.96\sqrt{I_{22}^{-1}} = (-0.216 \pm 0.107)$$
$$= (-0.323,\ -0.109)$$

The CI for β does not contain zero, so we have evidence that $\beta \neq 0$. If $\beta = 0$ then the number of failures does not depend on temperature. This would correspond to the number of failures being binomial$(6, e^{\alpha})$.

(b) When $t_i = 31$ we get

$$\hat{\alpha} + \hat{\beta} t_i = 4.964$$

$$p_i = 0.993$$

The probability of O-ring failure is 0.993

(c) $AIC_1 = 2l_1 - 2p_1 = -33.674$

$AIC_2 = 2l_2 - 2p_2 = -53.66$

So, the regression model is preferred.

This tallys with the $\beta = 0$ vs $\beta \neq 0$ test from (a)

~~BIC gives same conclusion~~

(d) The intercept is much smaller which indicates a lower failure probability.

Furthermore, the slope is much smaller which indicates a weaker dependence on temperature.

At 31°F we get

$$pi = \frac{\exp(2.161)}{1+\exp(2.161)} = 0.897$$

which is a lower probability of failure.