1. The 1986 crash of the space shuttle Challenger was linked to failure of O-ring seals in the rocket engines. Data was collected on the 23 previous shuttle missions. The shuttle had six O-ring seals and the number that failed on each launch was recorded. Furthermore, the temperature at each launch was also recorded (in degrees F).

| | temp ($t_i$) | damage ($x_i$) |
|---|---|---|
| 1 | 53.00 | 5.00 |
| 2 | 57.00 | 1.00 |
| 3 | 58.00 | 1.00 |
| 4 | 63.00 | 1.00 |
| 5 | 66.00 | 0.00 |
| 6 | 67.00 | 0.00 |
| 7 | 67.00 | 0.00 |
| 8 | 67.00 | 0.00 |
| 9 | 68.00 | 0.00 |
| 10 | 69.00 | 0.00 |
| 11 | 70.00 | 1.00 |
| 12 | 70.00 | 0.00 |
| 13 | 70.00 | 1.00 |
| 14 | 70.00 | 0.00 |
| 15 | 72.00 | 0.00 |
| 16 | 73.00 | 0.00 |
| 17 | 75.00 | 0.00 |
| 18 | 75.00 | 1.00 |
| 19 | 76.00 | 0.00 |
| 20 | 76.00 | 0.00 |
| 21 | 78.00 | 0.00 |
| 22 | 79.00 | 0.00 |
| 23 | 81.00 | 0.00 |

A binomial model was fitted to the data where $X_i \sim \text{binomial}(6, p_i)$ and where

$$p_i = \frac{\exp(\alpha + \beta t_i)}{1 + \exp(\alpha + \beta t_i)}.$$

The fitted parameter values and inverse information matrix are

$$(\hat{\alpha}, \hat{\beta}) = (11.660, -0.216) \text{ and } I^{-1}(\hat{\alpha}, \hat{\beta}) = \begin{pmatrix} 10.786 & -0.173 \\ -0.173 & 0.003 \end{pmatrix}.$$

(a) Provide approximate 95% confidence intervals for $\alpha$ and $\beta$ and comment on whether there is evidence that $\beta = 0$ or $\beta \neq 0$. What is the relevance of testing $\beta = 0$?

(b) The temperature on the launch day was 31 degrees F, estimate the probability of O-ring failure at this temperature.

(c) The maximized likelihood has the value $\hat{\ell} = -14.837$ and the maximized likelihood for a binomial model where $X_i \sim \text{binomial}(6, p)$ is $\hat{\ell}_0 = -25.830$. Comment on which model is more appropriate.

(d) The engineers at the launch of the shuttle ignored any data where there were zero failures and only analyzed data where $x_i = 1, 2, \ldots, 6$. The same model fitted to this reduced dataset gives

$$(\hat{\alpha}, \hat{\beta}) = (5.323, -0.102) \text{ and } I^{-1}(\hat{\alpha}, \hat{\beta}) = \begin{pmatrix} 10.814 & -0.175 \\ -0.175 & 0.003 \end{pmatrix}.$$

Comment on the different conclusions that the engineers would draw from this analysis.