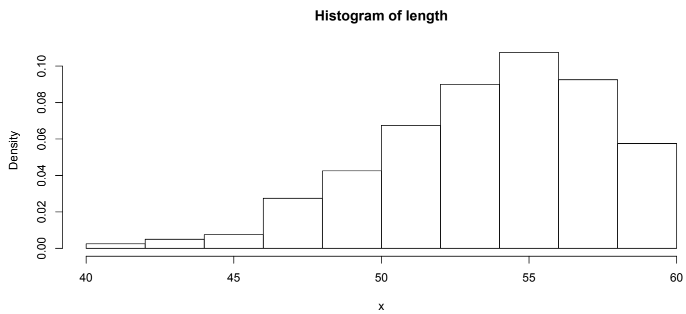
1. The length of items being manufactured in a factory are currently being modeled using a normal distribution with mean μ and standard deviation σ^2. A histogram of the data recording the length of 200 recently manufactured items is given below:



After discussing with an engineer in the factory, it has been discovered that it is impossible for the items to have length < 30 or > 60 due to manufacturing constraints. Thus, it would be more appropriate to model the data using a truncated normal distribution. The truncated normal distribution has pdf given as

$$f(x|\mu, \sigma, a, b) = \frac{\phi\left(\frac{x-\mu}{\sigma}\right)}{\Phi\left(\frac{b-\mu}{\sigma}\right) - \Phi\left(\frac{a-\mu}{\sigma}\right)}$$

where $\phi(\cdot)$ is the pdf of a $N(0,1)$ and $\Phi(\cdot)$ is the cdf of a $N(0,1)$ and $a = 30$ is the lower limit and $b = 60$ is the upper limit.

(a) Concisely outline how the maximum likelihood estimates are found. Clearly define any quantities that you refer to in your description.

(b) The truncated normal model was fitted to the data and the estimated parameters are:

$$\hat{\mu} = 54.73 \text{ and } \hat{\sigma} = 4.50.$$

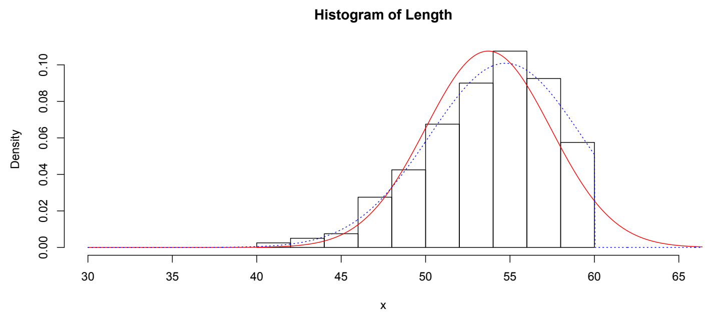
The inverse of the observed information matrix is

$$\hat{I}^{-1} = \begin{pmatrix} 0.25 & 0.12 \\ 0.12 & 0.13 \end{pmatrix}.$$

Further, the maximized likelihood is -532.0.

Find approximate 95% confidence intervals for the parameters.

(c) The model fit ignoring the truncation (ie. fitting a $N(\mu, \sigma^2)$ distribution) gives parameters $(\mu, \sigma) = (53.70, 3.74)$. The maximized likelihood is -546.0. A plot of the fitted models is given as follows:



Comment on why the estimates differ from the estimates found for the normal distribution for the truncated normal.

(d) Which model is preferred? Discuss.