Drop size distribution from the breakup rate

Ianto Cannon, December 9, 2025

We follow the model of Garrett, Li, and Farmer [2], in which drops of diameter d break into m equally sized daughter drops. Volume conservation sets the daughter-drop diameter to $d' = d/m^{1/3}$. In steady state, the rate of drop removal equals the rate of creation,

$$N(d')\,\tau(d')\,d' = m\,N(d)\,\tau(d)\,d, \tag{1}$$

where $\tau(d)$ is the drop lifetime before breakup, and $N(d)\Delta d$ is the number of drops with diameters in the range d to $d + \Delta d$.

The classical inertial-range estimate for the breakup lifetime is the eddy turnover time,

$$\tau_d = \frac{d^{2/3}}{\epsilon^{1/3}}, \tag{2}$$

which applies when surface tension is negligible.

The model proposed by Coulaloglou and Tavlarides [1] and measured by Vela-Martín and Avila [3] accounts for the suppression of breakup near the Hinze scale $d_H = 0.725\,\sigma^{3/5}\rho^{-3/5}\epsilon^{-2/5}$. This leads to an exponential breakup lifetime,

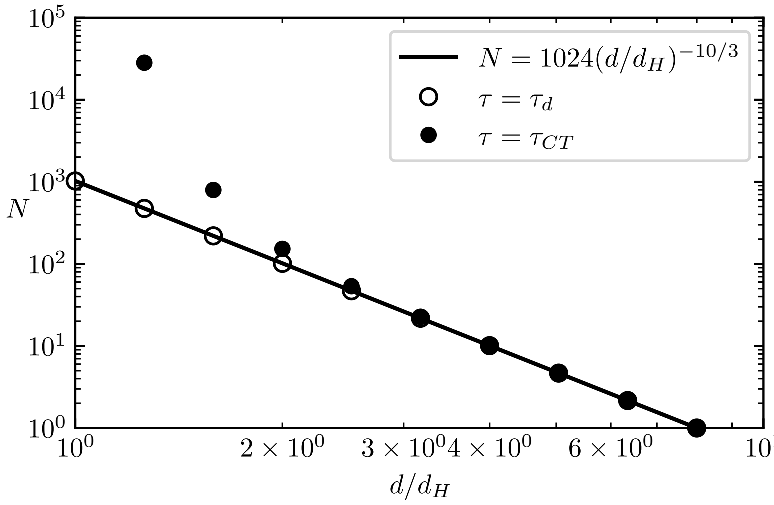
$$\tau_{CT} = \frac{d^{2/3}}{14.8\,\epsilon^{1/3}} \exp\left(\frac{-7.8\,\sigma}{\rho\,\epsilon^{2/3} d^{5/3}}\right). \tag{3}$$

We start with a single drop of size $8d_H$, which lies in the inertial range, so that the subsequent cascade is insensitive to the precise initial size. At each step, equation 1 gives $N(d')$ from the known $N(d)$, allowing us to iteratively solve toward smaller diameters using $\tau = \tau_d$ (empty circles) and $\tau = \tau_{CT}$ (filled circles). This procedure is analogous to injecting drops of size $8d_H$ at a constant rate and allowing them to breakup in a turbulent flow.

Both models converge to the inertial-range prediction

$$N(d) \sim (d/d_H)^{-10/3}, \tag{4}$$

because far above d_H both lifetimes scale as $d^{2/3}$, so the steady state condition necessarily produces the classical $d^{-10/3}$ drop size distribution. Near d_H, however, the Coulaloglou and Tavlarides [1] lifetime predicts thousands of times more drops because surface tension greatly prolongs the breakup time. In this analysis we choose binary breakups ($m = 2$), but the resulting distributions are in fact insensitive to the choice of m and to any overall prefactor in the breakup rate (such as the 14.8 in equation 2). This invariance arises because $N(d')$ depends on the ratio $\tau(d)/\tau(d')$, causing such constants to cancel.



References

[1] C. A. Coulaloglou and L. L. Tavlarides. “Description of Interaction Processes in Agitated Liquid-Liquid Dispersions”. In: *Chemical Engineering Science* 32.11 (1977), pp. 1289–1297. ISSN: 0009-2509. DOI: 10.1016/0009-2509(77)85023-9.

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[3] Alberto Vela-Martín and Marc Avila. “Memoryless Drop Breakup in Turbulence”. In: *Science Advances* 8.50 (Dec. 2022), eabp9561. DOI: 10.1126/sciadv.abp9561.