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SSCE 1793 DIFFERENTIAL EQUATIONS **TUTORIAL 3**

1. Use the definition of Laplace transform to determine $F(s)$ for the following functions.

a. $f(t) = 5e^{5t}$. **b.** $f(t) = 3e^{-4t}$.

c. $f(t) = \sinh 4t$. **d.** $f(t) = \cos kt$.

e. $f(t) = \begin{cases} t, & 0 < t < 4 \\ 5, & t > 4. \end{cases}$ **f.** $f(t) = \begin{cases} -1, & 0 < t < 4 \\ 1, & t > 4. \end{cases}$

g. $f(t) = \begin{cases} \sin 2t, & 0 < t < \pi \\ 0, & t > \pi. \end{cases}$ **h.** $f(t) = \begin{cases} e^t, & 0 < t < 2 \\ 0, & 2 < t < 4 \\ 5, & t > 4. \end{cases}$

2. Use the Laplace transform table to find $F(s)$ for the given function.

a. $f(t) = 2\sin t + 3\cos 2t$. **b.** $f(t) = e^{2t}\sinh^2 t$.

c. $f(t) = 2t^2 - 3t + 4$. **d.** $f(t) = (\sin t + \cos t)^2$.

e. $f(t) = e^{-2t}\sin 5t$. **f.** $f(t) = \sin t\, \cos t$.

g. $f(t) = 5e^{2t} + 7e^{-t}$. **h.** $f(t) = (t-1)^2 + t\sinh 2t$.

i. $f(t) = t^2 - t\sinh t - 2e^{-t}\sin 3t$. **j.** $f(t) = t^3 e^{-4t} + t\sin t$.

k. $f(t) = te^{-t}\cos 2t$. **l.** $f(t) = 2t^2 e^{-t}\cosh t$.

3. Sketch the graph of the given function for $t \geq 0$, and find its Laplace transform.

a. $f(t) = (t-4)H(t-4)$. **b.** $f(t) = H(t-2) - H(t-3)$.

c. $f(t) = (t-3)H(t-1)$. **d.** $f(t) = \cos(t-\pi)H(t-\pi)$.

e. $f(t) = e^{-2t}H(t-4)$.

4. Express the given function in terms of unit step functions, and find its Laplace transform.

a. $f(t) = \begin{cases} e^t, & 0 < t < 2\pi \\ \cos t, & t > 2\pi. \end{cases}$ **b.** $f(t) = \begin{cases} 0, & 0 < t < 2 \\ t, & 2 < t < 5 \\ e^{2t}, & t > 5. \end{cases}$

c. $f(t) = \begin{cases} 0, & 0 < t < 1 \\ \sin(t-1), & t > 1. \end{cases}$ **d.** $f(t) = \begin{cases} 0, & 0 < t < 2 \\ t^3 + 1, & t > 2. \end{cases}$

e. $f(t) = \begin{cases} \sin t, & 0 < t < \pi \\ \sin 2t, & \pi < t < 2\pi \\ \sin 3t, & t > 2\pi. \end{cases}$ **f.** $f(t) = \begin{cases} e^{-t}, & 0 < t < 2 \\ 2t - 1, & t > 2. \end{cases}$

5. Determine the following transforms.

a. $\mathcal{L}\left\{e^t\delta(t-2)\right\}$. **b.** $\mathcal{L}\left\{\cos t\,\delta(t-3\pi)\right\}$.

c. $\mathcal{L}\left\{t^3 e^{-3t}\delta(t-1)\right\}$. **d.** $\mathcal{L}\left\{t\,\delta(t-1)\right\}$.

6. Find $\mathcal{L}\{f(t)\}$ for the following periodic functions.

a. $f(t) = \begin{cases} 1, & 0 < t < 1 \\ -1, & 1 < t < 2 \end{cases}$
$f(t) = f(t+2)$.

b. $f(t) = \begin{cases} t, & 0 < t < \pi \\ 2\pi - t, & \pi < t < 2\pi \end{cases}$
$f(t) = f(t+2\pi)$.

c. $f(t) = \begin{cases} t, & 0 < t < 1 \\ 1, & 1 < t < 2 \end{cases}$
$f(t) = f(t+2)$.

d. $f(t) = 1 - t, \quad 0 < t < 2$
$f(t) = f(t+2)$.

7. Determine the inverse Laplace transform of the following functions.

a. $\dfrac{1}{s+2}$.

b. $\dfrac{1}{s^2+4}$.

c. $\dfrac{1}{2s^2+1}$.

d. $\dfrac{1}{s^2-1}$.

e. $\dfrac{2}{(s-2)^2+9}$.

f. $\dfrac{s}{(s+1)^2+5}$.

g. $\dfrac{2s+1}{(s-1)^2+7}$.

h. $\dfrac{1}{s^2-2s+2}$.

i. $\dfrac{s+3}{s^2+2s+5}$.

j. $\dfrac{s}{s^2-2s+(17/4)}$.

k. $\dfrac{2s+3}{(s+4)^3}$.

l. $\dfrac{1}{(s-3)^2}$.

m. $\dfrac{2s-4}{(s-1)^2}$.

n. $\dfrac{4}{s^2+9}-\dfrac{1}{(s-6)^2}$.

o. $\dfrac{e^{-4s}}{s+2}$.

p. $\dfrac{3se^{-2s}}{s^2+14}$.

q. $\dfrac{se^{-3s}}{s^2+4}$.

r. $\dfrac{e^{-4s}}{s-3}$.

s. $\dfrac{1-e^{-2\pi s}}{s^2+1}$.

t. $e^{-2s}\left(\dfrac{s+2}{s^2-4s+8}\right)$.

u. $e^{-3s}\left(\dfrac{s-5}{s^2+4s+5}\right)$.

8. Determine the inverse Laplace transforms of the following functions.

a. $\dfrac{e^{-s}}{s}-\dfrac{5e^{-3s}}{s}$.

b. $\dfrac{2}{s}-\dfrac{4e^{-2s}}{s^2}+\dfrac{4e^{-4s}}{s^2}$.

c. $\dfrac{se^{-\pi s}}{s^2+4}$.

d. $\dfrac{e^{-s}}{s^3}$.

9. Use the method of partial fractions to find the inverse Laplace transforms for the following functions.

a. $\dfrac{1}{s\,(s^2+4)}$.

b. $\dfrac{s+3}{(s-2)(s+1)}$.

c. $\dfrac{8}{s^3\,(s^2-s-2)}$.

d. $\dfrac{2s-13}{s\,(s^2-4s+13)}$.

e. $\dfrac{2s-3}{s^2+s-2}$.

f. $\dfrac{2s^2-s}{(s^2+4)^2}$.

g. $\dfrac{s^2+2s-4}{s^3-5s^2+2s+8}$.

h. $\dfrac{s^2+1}{(s-1)\,(s^2+2)}$.

i. $\dfrac{s^2+4s+5}{(s+1)\,(s+2)^2}$.

j. $\dfrac{s^2+s}{(s^2-4s+8)\,(s-2)}$.

10. Determine the inverse Laplace transforms of the following functions.

a. $\dfrac{e^{-5s}}{s\,(s^2+9)}$.

b. $\dfrac{-2e^{-2s}(s-4)}{(s-5)^3}$.

c. $\dfrac{(5s-3)e^{-s}}{s^2-7s+10}$.

d. $\dfrac{2e^{-4s}}{s\,(s^3-8)}$.

11. Use the convolution theorem to find the inverse Laplace transform of the given function.

a. $\dfrac{1}{s(s+2)}$.

b. $\dfrac{9}{s^3(s-2)}$.

c. $\dfrac{4}{s^2(s+1)}$.

d. $\dfrac{s}{(s+4)^2}$.

e. $\dfrac{1}{(s+1)(s-2)}$.

f. $\dfrac{1}{s(s^2+4)}$.

g. $\dfrac{1}{(s-1)^2}$.

h. $\dfrac{1}{(s^2+1)^2}$.

i. $\dfrac{2}{(s-1)(s^2+4)}$.

j. $\dfrac{s}{(s^2+1)^2}$.

k. $\dfrac{s}{(s-3)(s^2+1)}$.

l. $\dfrac{s^2}{(s^2+4)^2}$.

m. $\dfrac{4}{(s^2+4)^2}$.

n. $\dfrac{e^{-3s}}{s(s^2+9)}$.

12. Use the method of Laplace transforms to solve the following problems.

a. $\dfrac{dy}{dt} - 5y = 0;\ y(0) = 2.$

b. $\dfrac{dy}{dt} - 5y = e^{5t};\ y(0) = 0.$

c. $\dfrac{dy}{dt} + y = \sin t;\ y(0) = 1.$

d. $\dfrac{dy}{dt} - 5y = 0;\ y(\pi) = 2.$

e. $\dfrac{dy}{dt} + 2y = e^t;\ y(0) = 1.$

f. $\dfrac{dy}{dt} + 2y = 0;\ y(1) = 1.$

g. $\dfrac{d^2y}{dt^2} - 3\dfrac{dy}{dt} + 4y = 0;$ $\quad y(0) = 1,\ y'(0) = 5.$

h. $\dfrac{d^2y}{dt^2} + \dfrac{dy}{dt} + y = 0;$ $\quad y(0) = 4,\ y'(0) = -3.$

i. $\dfrac{d^2y}{dt^2} - \dfrac{dy}{dt} - 2y = 4t^2;$ $\quad y(0) = 1,\ y'(0) = 4.$

j. $\dfrac{d^2y}{dt^2} + 2\dfrac{dy}{dt} - 3y = \sin 2t;$ $\quad y(0) = 1,\ y'(0) = 0.$

k. $\dfrac{d^2y}{dt^2} - 3\dfrac{dy}{dt} + 2y = e^{-t}$ $\quad y(1) = 1,\ y'(1) = 0.$

l. $\dfrac{d^2y}{dt^2} + y = 0;$ $\quad y(\pi) = 0,\ y'(\pi) = -1.$

m. $\dfrac{d^2y}{dt^2} + 4\dfrac{dy}{dt} + 4y = 6 + e^{-2t};\ \ y(0) = 0,\ y(1) = 0.$

n. $\dfrac{d^2y}{dt^2} - 2\dfrac{dy}{dt} + 2y = e^t \sin t;\ \ y(0) = 0,\ y(\pi/2) = 0.$

o. $\dfrac{d^2y}{dt^2} + y = \delta(t-\pi);$ $\quad y(0) = 0,\ y(3\pi/2) = 1.$

13. Solve the following initial value problems.

a. $\dfrac{d^2y}{dt^2} + 3\dfrac{dy}{dt} + 2y = \begin{cases} t, & 0 < t < 2 \\ 4-t, & t > 2 \end{cases}$

$y(0) = y'(0) = 0.$

b. $\dfrac{d^2y}{dt^2} + \dfrac{dy}{dt} - 12y = \begin{cases} 4, & 0 < t < 2 \\ 0, & 2 < t < 4 \\ -4, & t > 4 \end{cases}$

$y(0) = y'(0) = 0.$

c. $\dfrac{d^2y}{dt^2} + 5\,\dfrac{dy}{dt} + 6y = \begin{cases} -2, & 0 < t < 3 \\ 0, & t > 3 \end{cases}$
$y(0) = y'(0) = 0.$

d. $\dfrac{d^2y}{dt^2} - 4\,\dfrac{dy}{dt} + 4y = \begin{cases} t, & 0 < t < 3 \\ t+2, & t > 3 \end{cases}$
$y(0) = -2,\ y'(0) = 1.$

e. $\dfrac{d^2y}{dt^2} + y = \delta(t - 2\pi);$ $\qquad y(0) = 1,\ y'(0) = 0.$

f. $\dfrac{d^2y}{dt^2} + 4y = 8\delta(t - 2\pi);$ $\qquad y(0) = 3,\ y'(0) = 0.$

g. $\dfrac{d^2y}{dt^2} + 4\,\dfrac{dy}{dt} + 4y = 1 + \delta(t - 2);$ $\qquad y(0) = 0,\ y'(0) = 0.$

h. $\dfrac{d^2y}{dt^2} + 9y = \delta(t - 3\pi) + \cos 3t;$ $\qquad y(0) = 0,\ y'(0) = 0.$

14. **a.** Given $F(s) = \dfrac{1}{s^2\,(s^2+9)} = \dfrac{A}{s} + \dfrac{B}{s^2} + \dfrac{Cs+D}{s^2+9}$. Determine A, B, C and D. Hence, find the inverse Laplace transform of $F(s)$.

b. Solve $\dfrac{d^2y}{dt^2} + 9y = g(t);\ y(0) = 1,\ y'(0) = 0$
where $g(t) = \begin{cases} 4t, & 0 < t < 1 \\ 4, & t > 1. \end{cases}$

15. **a.** Find the inverse Laplace transform of

$$F(s) = \frac{1}{s(s+6)(s-5)}.$$

b. Then, solve the following IVP

$$\frac{d^2y}{dt^2} + \frac{dy}{dt} - 30y = g(t);\ y(0) = 0,\ y'(0) = 0$$

where $g(t) = \begin{cases} 2, & 0 < t < 4 \\ 0, & 4 < t < 8 \\ -2, & t > 8. \end{cases}$

16. **a.** Use the convolution theorem to find the inverse Laplace transform of

$$F(s) = \frac{2}{s^2(s-1)}.$$

b. Use the method of Laplace transforms to solve the IVP

$$\frac{d^2y}{dt^2} + 4y = \delta(t-2);\ y(0) = 2,\ y'(0) = -2.$$

SOLUTIONS TO TUTORIAL 3

1. **a.** $\dfrac{5}{s-5}$. **b.** $\dfrac{3}{s+4}$.

c. $\dfrac{4}{s^2-16}$. **d.** $\dfrac{s}{s^2+k^2}$.

e. $\dfrac{1}{s^2}+\dfrac{e^{-4s}}{s}-\dfrac{e^{-4s}}{s^2}$. **f.** $\dfrac{2e^{-4s}}{s}-\dfrac{1}{s}$.

g. $\dfrac{2(1-e^{-\pi s})}{s^2+4}$. **h.** $\dfrac{1}{s-1}-\dfrac{e^{-2(s-1)}}{s-1}+\dfrac{5e^{-4s}}{s}$.

2. **a.** $\dfrac{2}{s^2+1}+\dfrac{3s}{s^2+4}$. **b.** $\dfrac{1}{4}\left[\dfrac{1}{s-4}-\dfrac{2}{s-2}+\dfrac{1}{s}\right]$.

c. $\dfrac{4}{s^3}-\dfrac{3}{s^2}+\dfrac{4}{s}$. **d.** $\dfrac{1}{s}+\dfrac{2}{s^2+4}$.

e. $\dfrac{5}{(s+2)^2+25}$. **f.** $\dfrac{1}{s^2+4}$.

g. $\dfrac{5}{s-2}+\dfrac{7}{s+1}$. **h.** $\dfrac{2}{s^3}-\dfrac{2}{s^2}+\dfrac{1}{s}+\dfrac{4s}{(s^2-4)^2}$.

i. $\dfrac{2}{s^3}+\dfrac{2s}{(s^2-1)^2}-\dfrac{6}{(s+1)^2+9}$.

j. $\dfrac{6}{(s+4)^4}+\dfrac{2s}{(s^2+1)^2}$. **k.** $\dfrac{(s+1)^2-4}{[(s+1)^2+4]^2}$.

l. $\dfrac{2}{s^3}+\dfrac{2}{(s+2)^3}$.

3. **a.** $\dfrac{e^{-4s}}{s^2}$. **b.** $\dfrac{e^{-2s}}{s}-\dfrac{e^{-3s}}{s}$.

c. $e^{-s}\left(\dfrac{1}{s^2}-\dfrac{2}{s}\right)$. **d.** $\dfrac{e^{-\pi s}s}{s^2+1}$.

e. $\dfrac{e^{-8}e^{-4s}}{s+2}$.

4. **a.** $\dfrac{1}{s-1}\left[1+e^{2\pi(1-s)}\right]+\dfrac{se^{-2\pi s}}{(s^2+1)}$.

b. $e^{-2s}\left(\dfrac{1}{s^2}+\dfrac{2}{s}\right)+e^{-5s}\left(\dfrac{e^{10}}{s-2}-\dfrac{1}{s^2}-\dfrac{5}{s}\right)$.

c. $\dfrac{e^{-s}}{s^2+1}$. **d.** $e^{-2s}\left(\dfrac{6}{s^4}+\dfrac{12}{s^3}+\dfrac{12}{s^2}+\dfrac{9}{s}\right)$.

e. $\dfrac{1}{s^2+1}+e^{-\pi s}\left(\dfrac{2}{s^2+4}+\dfrac{1}{s^2+1}\right)+e^{-2\pi s}\left(\dfrac{3}{s^2+9}-\dfrac{2}{s^2+4}\right)$.

f. $\dfrac{1-e^{-2(1+s)}}{s+1}+e^{-2s}\left(\dfrac{2}{s^2}+\dfrac{3}{s}\right)$.

5. **a.** $e^{2(1-s)}$. **b.** $-e^{-3\pi s}$.

c. $e^{-(3+s)}$. **d.** e^{-s}.

6. **a.** $\dfrac{(e^{-s}-1)^2}{s\,(1-e^{-2s})}$. **b.** $\dfrac{(e^{-\pi s}-1)^2}{s^2\,(1-e^{-2\pi s})}$.

c. $\dfrac{1-e^{-s}-se^{-2s}}{s^2\,(1-e^{-2s})}$. **d.** $\dfrac{(s+1)e^{-2s}+s-1}{s^2\,(1-e^{-2s})}$.

7. **a.** e^{-2t}. **b.** $\frac{1}{2}\sin 2t$.

c. $\frac{1}{\sqrt{2}}\sin\frac{1}{\sqrt{2}}t$. **d.** $\sinh t$.

e. $\frac{2}{3}e^{2t}\sin 3t$. **f.** $e^{-t}\cos\sqrt{5}t - \frac{1}{\sqrt{5}}\,e^{-t}\sin\sqrt{5}t$.

g. $2e^t\cos\sqrt{7}t + \frac{3}{\sqrt{7}}e^t\sin\sqrt{7}t$. **h.** $e^t\sin t$.

i. $e^{-t}(\cos 2t + \sin 2t)$. **j.** $e^t\cos\frac{\sqrt{13}}{2}t + \frac{2e^t}{\sqrt{13}}\sin\frac{\sqrt{13}}{2}t$.

k. $e^{-4t}\left(2t - \frac{5}{2}t^2\right)$. **l.** te^{3t}.

m. $2e^t - 2te^t$. **n.** $\frac{4}{3}\sin 3t - te^{6t}$.

o. $e^{-2(t-4)}H(t-4)$. **p.** $3\cos\left[\sqrt{14}(t-2)\right]H(t-2)$.

q. $\cos\left[2(t-3)\right]H(t-3)$. **r.** $e^{3(t-4)}H(t-4)$.

s. $[1 - H(t-2\pi)]\sin t$.

t. $e^{2(t-2)}\left[\cos\{2(t-2)\} + 2\sin\{2(t-2)\}\right]H(t-2)$.

u. $e^{-2(t-3)}\left[\cos(t-3) - 7\sin(t-3)\right]H(t-3)$.

8. **a.** $f(t) = \begin{cases} 0, & 0 < t < 1 \\ 1, & 1 < t < 3 \\ -4, & t > 3. \end{cases}$ **b.** $f(t) = \begin{cases} 2, & 0 < t < 2 \\ 10 - 4t, & 2 < t < 4 \\ -6, & t > 4. \end{cases}$

c. $f(t) = \begin{cases} 0, & 0 < t < \pi \\ \cos 2t, & t > \pi. \end{cases}$ **d.** $f(t) = \begin{cases} 0, & 0 < t < 1 \\ \frac{1}{2}(t-2)^2, & t > 1. \end{cases}$

9. **a.** $\frac{1}{4} - \frac{1}{4}\cos 2t$. **b.** $\frac{5}{3}e^{2t} - \frac{2}{3}e^{-t}$.

c. $\frac{8}{3}e^{-t} + \frac{1}{3}e^{2t} - 3 + 2t - 2t^2$. **d.** $e^{2t}\cos 3t - 1$.

e. $\frac{7}{3}e^{-2t} - \frac{1}{3}e^t$. **f.** $\frac{1}{2}\sin 2t - \frac{1}{4}t\sin 2t + t\cos 2t$.

g. $2e^{4t} - \frac{2}{3}e^{2t} - \frac{1}{3}e^{-t}$. **h.** $\frac{2}{3}e^t + \frac{1}{3}\cos\sqrt{2}t + \frac{1}{3\sqrt{2}}\sin\sqrt{2}t$.

i. $2e^{-t} - e^{-2t} - te^{-2t}$. **j.** $\frac{1}{2}e^{2t}(3 - \cos 2t + 5\sin 2t)$.

10. **a.** $\frac{1}{9}\left[1 - \cos\{3(t-5)\}\right]H(t-5)$.

b. $2e^{5(t-2)}\left[(2-t) + \frac{1}{2}(t-2)^2\right]H(t-2)$.

c. $\left[\frac{22}{3}e^{5(t-1)} - \frac{7}{3}e^{2(t-1)}\right]H(t-1)$.

d. $\left[\frac{1}{12}e^{2(t-4)} - \frac{1}{4} + \frac{1}{6}e^{-(t-4)}\cos\sqrt{3}(t-4)\right]H(t-4)$.

11. **a.** $\frac{1}{2}\left(1-e^{-2t}\right)$. **b.** $\frac{9}{4}\left(\frac{1}{2}e^{2t}-t^2-t-\frac{1}{2}\right)$.

c. $4\left(t-1+e^{-t}\right)$. **d.** $\frac{1}{4}t\sin 2t$.

e. $\frac{1}{3}\left(e^{2t}-e^{-t}\right)$. **f.** $\frac{1}{4}(1-\cos 2t)$.

g. te^t. **h.** $\frac{1}{2}(\sin t-t\cos t)$.

i. $\frac{1}{5}\left(2e^t-\sin 2t-2\cos 2t\right)$. **j.** $\frac{1}{2}t\sin t$.

k. $\frac{1}{10}\left(3e^{3t}-3\cos t+\sin t\right)$. **l.** $\frac{1}{4}(\sin 2t+2t\cos 2t)$.

m. $\frac{1}{4}\left(\sin 2t-2t\cos 2t\right)$. **n.** $\frac{1}{9}(1-\cos 3(t-3))H(t-3)$.

12. **a.** $2e^{5t}$. **b.** te^{5t}.

c. $\frac{1}{2}\left(3e^{-t}-\cos t+\sin t\right)$. **d.** $2e^{5(t-\pi)}$.

e. $\frac{1}{3}\left(2e^{-2t}+e^t\right)$. **f.** $e^{-2(t-1)}$.

g. $e^{3t/2}\left(\cos\frac{\sqrt{7}}{2}t+\sqrt{7}\sin\frac{\sqrt{7}}{2}t\right)$.

h. $e^{-t/2}\left(4\cos\frac{\sqrt{3}}{2}t+\frac{2}{\sqrt{3}}\sin\frac{\sqrt{3}}{2}t\right)$.

i. $2\left(e^{2t}+e^{-2t}-t^2+t\right)-3$.

j. $\frac{1}{10}e^t-\frac{1}{26}e^{-3t}-\frac{4}{65}\cos 2t-\frac{78}{65}\sin 2t$.

k. $\frac{1}{6}e^{-t}+\frac{1}{3}e^{2t-3}-\frac{1}{2}e^{t-2}$. **l.** $\sin t$.

m. $\frac{1}{4}(2t^2-3t+1)e^{-2t}+\frac{1}{4}(t-1)$. **n.** $\frac{1}{2}te^t\cos t$.

o. $\sin t\, H(t-\pi)$.

13. **a.**
$$y(t)=e^{-t}-\frac{1}{4}e^{-2t}-\frac{3}{4}+\frac{1}{2}t+\frac{3}{2}H(t-2)$$
$$-(t-2)H(t-2)-2e^{-(t-2)}H(t-2)$$
$$+\frac{1}{2}e^{-2(t-2)}H(t-2).$$

b.
$$y(t)=4\left[-\frac{1}{12}+\frac{1}{28}e^{-4t}+\frac{1}{21}e^{3t}\right]$$
$$-4\left[-\frac{1}{12}+\frac{1}{28}e^{-4(t-2)}+\frac{1}{21}e^{3(t-2)}\right]H(t-2)$$
$$-4\left[-\frac{1}{2}+\frac{1}{28}e^{-4(t-4)}+\frac{1}{21}e^{3(t-4)}\right]H(t-4).$$

c.
$$y(t)=-\frac{1}{3}-\frac{2}{3}e^{-3t}+e^{-2t}+\left[\frac{1}{3}+\frac{2}{3}e^{-3(t-3)}-e^{-2(t-3)}\right]H(t-3).$$

d. $y(t) = \frac{1}{4}\left(1 + t - 9e^{2t} + 21te^{2t}\right)$
$$+ \left[\frac{1}{2} - \frac{1}{2}e^{2(t-3)} + (t-3)e^{2(t-3)}\right] H(t-3).$$

e. $y(t) = \sin(t - 2\pi)H(t - 2\pi) - \cos t.$

f. $y(t) = 3\cos 2t + 4\sin 2(t - 2\pi)H(t - 2\pi).$

g. $y(t) = \frac{1}{4}\left(1 - e^{-2t}\right) - \frac{1}{2}te^{-2t} + (t-2)e^{-2(t-2)}H(t-2).$

h. $y(t) = -\frac{1}{3}\sin 3t\, H(t - 3\pi) + \frac{1}{3}t\sin t.$

14. a. $A = 0, B = \frac{1}{9}, C = 0, D = -\frac{1}{9}.\quad f(t) = \frac{1}{9}t - \frac{1}{27}\sin 3t.$

b. $y(t) = \frac{4}{9}t + \cos 3t - \frac{4}{27}\sin 3t$
$$- \left[\frac{4}{9}(t-1) - \frac{4}{27}\sin 3(t-1)\right] H(t-1).$$

15. a. $f(t) = \frac{1}{55}e^{5t} + \frac{1}{66}e^{-6t} - \frac{1}{30}.$

b. $y(t) = -\frac{1}{15} + \frac{1}{33}e^{-6t} + \frac{2}{55}e^{5t}$
$$+ \left[\frac{1}{15} - \frac{1}{33}e^{-6(t-4)} - \frac{2}{55}e^{5(t-4)}\right] H(t-4)$$
$$+ \left[\frac{1}{15} - \frac{1}{33}e^{-6(t-8)} - \frac{2}{55}e^{5(t-8)}\right] H(t-8).$$

16. a. $f(t) = 2\left(e^t - t - 1\right).$

b. $y(t) = 2\cos 2t - \sin 2t + \frac{1}{2}\sin 2(t-2)\, H(t-2).$