Uncertainty Quantification (ACM41000)
Exercises – Set 3

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1. Solve the diffusion equation

$$u_t = Du_{xx}, \qquad 0 < x < L, \qquad t > 0,$$

with homogeneous Neumann boundary conditions

$$u_x(x = 0, t > 0) = u_x(x = L, t > 0) = 0,$$

subject to the initial condition

$$u(x, t = 0) = f(x).$$

2. Prove that the solution to the diffusion equation with Neumann boundary conditions and arbitrary initial conditions conserves the quantity

$$\int_0^L u(x, t)\mathrm{d}x.$$

To what kind of system might these boundary conditions correspond?

3. Find the Fourier series of

$$F(x) = x^4,$$

on the interval $[-\pi, \pi]$. By setting $x = \pi$, evaluate

$$\sum_{n=1}^{\infty} \frac{1}{n^4}.$$

Note the hints at the end of the exam.

Some useful formulas

- Integration:

$$\int x^4 \cos(nx)\mathrm{d}x = \frac{4x(n^2x^2 - 6)\cos(nx)}{n^4} + \frac{(n^4x^4 - 12n^2x^2 + 24)\sin(nx)}{n^5}$$

- Euler's summation formula;

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}.$$