

Uncertainty Quantification (ACM41000)

Exercises – Set 2

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1. Let $P(t)$ be the average concentration of a pollutant in a particular domain at time t . The pollutant naturally degrades over time at a rate k but the domain is subject to a pollution source, so that pollutant enters the domain at a constant (positive) rate s . These effects are summarised in the following ordinary differential equation (ODE):

$$\frac{dP}{dt} = -k(P - P_0) + s, \quad k, s \in \mathbb{R}^+, \quad (1)$$

where P_0 is the background level of pollution. Let $P(0) = P_0$ be the initial pollution level. Using the integrating-factor technique, show that

$$P(t) = \frac{s}{k} (1 - e^{-kt}) + P_0. \quad (2)$$

2. Show that

$$\lim_{t \rightarrow \infty} P(t) = P_0 + \frac{s}{k},$$

independent of the initial level of pollution. Hence, make a rough sketch of the solution $P(t)$ coming from Part (a).

3. A powerplant emits nitrous oxides (NO_x) at a rate s according to Equation (1). The factory is fined by the Environmental Protection Agency if the pollution level (even momentarily) exceeds twice the background level, i.e. a fine is imposed if $P(t) > 2P_0$. Show that the factory should emit at a rate

$$s < P_0 k$$

to avoid the fine.