Uncertainty Quantification (ACM41000)
Exercises – Set 1

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1. Solve the following first-order **elementary** ODEs and sketch the family of solution curves.
$$\frac{dy}{dx} = \mathrm{e}^x \sin x, \qquad \frac{dy}{dx} = \frac{1}{1+x^2}.$$

2. Let $T(t)$ be the temperature of a hot object at time t. The temperature cools to the background temperature $T_0 < T(t)$ according to Newton's Law of cooling:
$$\frac{dT}{dt} = -k\left(T - T_0\right), \qquad k \in \mathbb{R}^+.$$
Notice that this ODE is separable – hence or otherwise, solve for $T(t)$. Leave your answer in terms of $T(0)$, the initial temperature of the object.

3. Solve the following **separable** initial-value problem:
$$\frac{du}{dt} = u^2 t, \qquad u(0) = 1.$$

4. Sketch the one-dimensional vector field for the following autonomous ODEs:
$$\frac{\mathrm{d}y}{\mathrm{d}t} = y\cos(y), \qquad \frac{\mathrm{d}y}{\mathrm{d}t} = y(y-1)\left(1 - \tfrac{1}{2}y\right).$$

5. Solve the following ODEs using the integrating-factor technique:
$$\frac{dy}{dx} + \frac{y}{x+1} = 1, \qquad x\frac{dy}{dx} + 2y = x\mathrm{e}^x.$$