Modelling the Lyapunov exponent of a tracer gradient using a stochastic differential equation.

Ian Towey

Msc Data and Computational Science
School of Mathematics and Statistics, University College Dublin

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Context of work

Modeling chaotic mixing of tracer:

- Numerical Simulation of two-dimensional turbulence.
- Model 2D vorticity using a system of stochastic differential equations.
- Use the equilibrium solution of the SDEs corresponding Fokker-Planck PDE to find the PDF of the orientation angles and growth rate (Lyapunov exponent).
- Compare the PDF of the orientation angles from the vorticity simulation and the SDE model

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$$\boxed{\frac{\partial \omega}{\partial t} + \mathbf{u} \cdot \nabla \omega = 0 + (-1)^p \nu_p \nabla^{2p} \omega + \nabla \times \mathbf{F} - \nabla \times \mathbf{D}}$$

Numerical simulation of 2D Vorticity/advection-diffusion

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- **Convective cooling**, which exploits the adverse temperature gradient created by the pipe and its surroundings, in the presence of gravity, further enhanced by suspending **phase-changing bubbles** in the working fluid.

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Two proposed solutions, both involving passive cooling methods:

- **Convective cooling**, which exploits the adverse temperature gradient created by the pipe and its surroundings, in the presence of gravity, further enhanced by suspending **phase-changing bubbles** in the working fluid.
- **A sieve cooler**

Both approaches can be placed in the same generic mathematical framework.

Modelling Approach

- Region 1 – closed to the pipe surface. Contains the TEG.
- Region 2 – contains the cooling device.
- Region 3 – zone where cooler interacts with ambient air.

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For convective cooling, the adverse temperature gradient is maintained by gravity, meaing that the cooler is mounted on top of the pipe – **Rayleigh–Bénard convection**

Region 1

- Region closed to pipe surface, in contact with pipe through patch of area A.
- Pipe maintained at constant temperature $T_0 = 60°\mathrm{C}$
- Otherwise, pipe is assumed to be **fully insulated**
- TEG located in region 1

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Region 1 is modelled as a thermally conducting region:

- TEG can be assigned an 'effective' thermal conductivity k_1 taking account of thermal conduction and heat generation/consumption due to thermoelectric effects, with $k_1 \approx 1\,\mathrm{W}/(\mathrm{m\,K})$
- Size of region is d_1, with $d_1 \geq d_{TEG}$. Values with $d_1 > d_{TEG}$ could be achieved by embedding the TEG in a medium that is conductivity-matched to the TEG.

Region 2

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Using the theory of Rayleigh–Bénard convection, we shall compute an effective conductivity for region 2, $k_{2,\text{eff}} = k_2 \, \mathrm{Nu}$, where

- k_2 is the ordinary thermal conductivity,
- Nu is the **Nusselt number**, defined in what follows.

Region 3 and matching

- Region 3 is just the surrounding air with thermal conductivity k_3.
- In each region, the heat equation

$$k_{i\,\mathrm{eff}}\,(\mathrm{d}T/\mathrm{d}x) = J$$

is solved for the temperature T ($x = 0$ is the pipe surface). Here, $k_{i\,\mathrm{eff}}$ is the thermal conductivity (effective or otherwise) appropriate for each layer and J_E is the heat flux.

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- **Matching conditions** at the region boundaries close the resulting system of equations.

Rayleigh–Bénard convection

- Refers to the problem of ‘a fluid heated from below’
- Adverse temperature gradient formed due to gravity – hot fluid ends up sitting on the bottom
- Convection pattern sets in to redistribute the hot fluid
- Leads to enhanced heat transfer

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Convection sets in above a critical value of the **Rayleigh number**

$$\mathrm{Ra} = \frac{g\alpha(T_{\mathrm{hot}} - T_{\mathrm{cold}})\ell^3}{\nu\kappa},$$

- g is the acceleration due to gravity
- α is the thermal expansion coefficient in K^{-1}
- $T_{\mathrm{hot}} - T_{\mathrm{cold}}$ is the temperature difference across region 2
- $\ell = d_2 - d_1$ is the vertical extent of region 2
- κ is the thermal diffusivity in m^2/s

Typically, $\mathrm{Ra} \gtrsim 10^3 - 10^4$ for the onset of convection.

Rayleigh–Bénard convection – enhanced heat transfer

Fluid motion enhances heat transfer. This is quantified by the **Nusselt number**

$$\mathrm{Nu} = \frac{\langle \int_0^1 \left(wT - \kappa_2 \frac{\partial T}{\partial x} \right) \mathrm{d}x \rangle}{\kappa_2 \left(T_{\mathrm{hot}} - T_{\mathrm{cold}} \right)}.$$

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Not known *a priori* but useful correlations exist from experiments / DNS:

$$\mathrm{Nu} = \begin{cases} 0.54\,\mathrm{Ra}^{1/4}, & 10^3 \leq \mathrm{Ra} \leq 10^7, \qquad \mathrm{Pr} \geq 0.7, \\ 0.15\,\mathrm{Ra}^{1/3}, & 10^7 \leq \mathrm{Ra} \leq 10^{11}, \qquad \text{all}\mathrm{Pr}, \end{cases}$$

where $\mathrm{Pr} = \nu/\kappa$ is the **Prandtl number**.

Rayleigh–Bénard convection – added phase change

To further enhance the heat transfer over and above what is achievable by ordinary RBC, it is proposed to use a working fluid such that

- The fluid is a liquid-phase suspension mixed in with gas-phase boiling bubbles of the same substance.
- The fluid is maintained close to the boiling temperature.
- The bubbles condense on the cold plate, are transported to the hot plate, where they (partially) evaporate, releasing latent heat.
- This further enhances the heat transfer.

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Enhanced heat transfer is quantified by a modified Nusselt-number correlation obtainable from DNS studies in the literature:

$$\mathrm{Nu} = A(\xi)\mathrm{Ra}^{\gamma(\xi)}, \qquad \xi = \frac{T_{\mathrm{hot}} - T_{\mathrm{sat}}}{T_{\mathrm{hot}} - T_{\mathrm{cold}}}$$

where $A(\xi) \approx 1 + 66.31\xi$ and $\gamma(\xi) = (1/3) - 0.26\xi$, and $0 \leq \xi \leq 0.5$.

Convection in the plume

Recall, region 3 consists of the ambient air in contact with the surface of the convective cooler. This region can therefore be modelled as a **convective plume**.

Convection in the plume – model

We have taken the plume model of Morton, Taylor, and Turner and added a thermal boundary layer (to allow for a non-pointlike heat source). Result is an expression for the heat flux into region 3:

$$J_E = \mu_0 \frac{\theta_0 k_3}{R_0} \left(\mathrm{Ra}_3 \mathrm{Pr}_3\right)^{1/7}, \qquad \mathrm{Ra}_3 = \frac{g\alpha\theta_0 R_0^3}{\kappa_3 \nu_3}, \qquad \mathrm{Pr}_3 = \frac{\nu_3}{\kappa_3}$$

where $\theta_0 = T_{\mathrm{cold}} - T_{\mathrm{amb}}$,

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where $\theta_0 = T_{\mathrm{cold}} - T_{\mathrm{amb}}$, and

- $\nu_3 = 1.568 \times 10^{-5}\,\mathrm{m}^2/\mathrm{s}$
- $\kappa_3 = 1.9 \times 10^{-5}\,\mathrm{m}^2/\mathrm{s}$, hence $\mathrm{Pr} = 0.8253$
- $\alpha_3 = 3.43 \times 10^{-3}\,\mathrm{K}^{-1}$,
- $R_0 = 10^{-2}\,\mathrm{m} = 1\,\mathrm{cm}$ – patch radius
- $T_{\mathrm{amb}} = 20^\circ\,\mathrm{C}$, $T_{\mathrm{hot}} = 60^\circ\,\mathrm{C}$
- $\mu = 0.14$

Upper bound

In practice, θ_0 has to be found by matching across the regions. But this is not necessary if we want to find an **upper bound** on the temperature drop achievable across region 1, since $\theta_0 \leq T_{\mathrm{amb}} - T_0$, hence

$$\frac{(\Delta T)_{\mathsf{Reg\,1}}}{d_1} \leq \max\left[\frac{(\Delta T)_{\mathsf{Reg\,1}}}{d_1}\right] := \mu_0 \frac{k_3}{k_1} \frac{(T_0 - T_{\mathrm{amb}})}{R_0} \left(\mathrm{Ra}_{T,3}\, \mathrm{Pr}_3\right)^{1/7}.$$

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But $k_3/k_1 = k_{\mathrm{air}}/k_{TEG,\mathrm{eff}} = O(10^{-2})$, meaning that

$$\frac{(\Delta T)_{\mathsf{Reg\,1}}}{d_1} \lesssim 10\,\mathrm{K/m}.$$

This is off target – heat transfer into the plume gives a very low **limiting value** on the region-1 temperature gradient achievable.

Reduction to two-layer model

- We can sidestep the limiting value by imposing that the cold plate be temperature-matched to the ambient environment – **two-layer model**.
- Solution temperature profiles read

$$
\begin{aligned}
T_1 &= T_0 - \frac{J_E x}{k_1}, \\
T_2 &= T_0 - \frac{J_E d_1}{k_1} - \frac{J_E}{k_2 \mathrm{Nu}}(x - d_1).
\end{aligned}
$$

  and we require $T_2(d_2) = T_{\mathrm{amb}}$.
- The matching procedure is **nonlinear**, as Nu depends on $T_2(d_2) - T_2(d_1)$.

Parameter study – question to be asked

- We fix

$$(\Delta T)_{\textsf{Reg 1}} = d_1 \left(\mathrm{d}T/\mathrm{d}x\right)_*$$

and ask the question, for what values of ℓ can $T_2(d_2) = T_{\mathrm{amb}}$ be achieved?

- In practice, we want to know the value of $(\Delta T)_{\mathrm{tot}} = T_{\mathrm{amb}} - T_0$ for a given value of ℓ – **can $(\Delta T)_{\mathrm{tot}} = 40\,\mathrm{K}$ be achieved for a sensible value of ℓ?**
- We have other constraints on region 1 – temperature drop in region 1 can be at most the total temperature drop:

$$d_1 \leq \frac{(\Delta T)_{\mathrm{tot}}}{\left(\mathrm{d}T/\mathrm{d}x\right)_*},$$

hence

$$d_{TEG} \leq d_1 \leq \frac{(\Delta T)_{\mathrm{tot}}}{\left(\mathrm{d}T/\mathrm{d}x\right)_*},$$

Parameter study – no phase change

We fix

- $d_1 = 2 \times 10^{-5}\,\mathrm{m}$–$\left[40\,\mathrm{K}/(2.5 \times 10^5\,\mathrm{K/m})\right] = 1.6 \times 10^{-4}\,\mathrm{m}$
- $k_1 = 1\,\mathrm{W/(m\,K)}$ – an estimated 'effective' value for the TEG.
- $k_2 = 0.6\,\mathrm{W/(m\,K)}$ – this is the value for water but should be broadly similar for many working fluids.

Parameter study – with phase change

Electrical circuit analogy

Re-write $J_E \equiv I$, $R_1 = d_1/k_1$, and the region-1 temperature drop as ΔV_1. Then,

$$\Delta V_1 = IR_1 \qquad \text{(Ohm's Law)}.$$

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Region 2 is a nonlinear resistor, with

$$\Delta V_2 = IR_2, \qquad R_2 = \frac{\ell}{k_2}\frac{1}{\mathrm{Nu}}.$$

Nu (hence R_2) depends on I:

$$R_2 = R_{20}(I/I_0)^{-c_2/(1+c_2)}, \qquad R_{20},\, I_0 = \text{Constants}$$

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Total temperature drop is

$$\Delta V = \Delta V_1 + \Delta V + 2 = I\left[R_1 + R_{20}\left(I_0/I\right)^{c_2/(1+c_2)}\right]$$

And we want to know for a given I, **what values of the resistances give us the required total temperature drop ΔV?**

The sieve cooler

- A rectangular metal of similar cross section as the TEG, but with thickness $d_2 = 1\,\mathrm{mm}$.
- A large number of tiny holes are punctured in the cooler converting it into a sieve. In a preferred embodiment the holes are filled with a material of extremely high conductivity.
- We have shown that such a composite can cool the TEG to the desired temperature with reasonable physical parameters.

The sieve cooler – modelling I

- The number N and radius R of the holes are scaled according to $N = |\log \epsilon|$, $R = \alpha\epsilon$. Here ϵ is a small parameter that the engineers will need to select.
- Deep mathematical analysis (Kac, Ozawa, ...) of the small-ϵ limit shows that in the presence of such hole distribution, the spectrum of the Laplacian is shifted by $O(\alpha)$.
- This is equivalent to $O(\alpha)$ heat removal. Notice that in this scaling, although the heat removal is significant, the total volume of the holes is very small.

The sieve cooler – modelling II

- The resulting model equations read

$$\frac{\mathrm{d}^2 T_1}{\mathrm{d}x^2} = 0, \qquad x \in (0, d_1),$$
$$\frac{\mathrm{d}^2 T_2}{\mathrm{d}x^2} - \underbrace{\mu^2 T_2}_{\text{Sieve cooler}} = 0, \qquad x \in (d_1, d_2).$$

- The parameter μ^2 measures the heat removal, and is determined from the number and radius of the holes. It is the same as the spectrum shift of the Laplacian in this perforated domain:

$$\mu^2 := 2\pi\alpha/|\Omega|.$$

- Notice that the equations are an averaged (or homogenized) model for heat transfer in the cooler, that already takes into account the many tiny holes filled with highly conductive material.
- This model can be embedded into our previous 'matching' framework – the results indicate that the desired temperature gradient can be achieved by the sieve cooler also.

Summary / Conclusions I

Using the theory of Rayleigh–Bénard convection with phase change, we can give concrete answers to most of the questions posed at the start of the study group:

- Is such a system [enhanced heat transfer with phase-changing convective cells] thermodynamically feasible?
  Yes – provided the cell is temperature-matched to the surroundings
- What performance/size enhancements might be expected compared to metallic cooling fins?
  We have not yet quantified performance enhancements but can say that the size of the cell can be made smaller than that for fins: $< 1\,\mathrm{cm}$ *compared to* $4\,\mathrm{cm}$ *for fins.*
- What properties should the working fluid possess?
  The fluid should be a suspension of liquid phase surrounding phase-changing bubbles of the same substance. The bubbles should be maintained near the boiling point. The boiling point should therefore be between $T_{\mathrm{amb}} = 20°\mathrm{C}$ *and* $T_0 = 60°\mathrm{C}$. *The Rayleigh number must be sufficiently large so as to ensure self-sustaining convection (ideally* $\mathrm{Ra} \geq O(10^4)$*).*

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Summary / Conclusions II

- What are the practical candidates for such a fluid?
  We don't know yet, but the search can be narrowed by looking at which fluids give the required phase-change, Nusselt number, and Rayleigh number characteristics.
- What pressure should the closed system work at?
  This can be inferred from the phase diagram once the boiling temperature is specified; the latter must be between $T_{\mathrm{amb}} = 20^\circ\mathrm{C}$ *and* $T_0 = 60^\circ\mathrm{C}$
- What size and shape should the vessel be?
  The size and shape should be chosen so as to maximize the Nusselt number for a given Rayleigh number – this can be explored further using the existing literature on convection.
- Can the system self-start and under what conditions?
  Yes, convection self-starts once the critical Rayleigh number is attained. However, it can take minutes for the convection to become fully developed. This is an important consideration for using this kind of device.

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Summary / Conclusions III

- Such a system will need to move the liquid phase back to the evaporation site, how does the energy required to do this effect the efficiency of the system? Will it impact scale?
  Because the device is entirely passive – powered by the adverse temperature gradient – no energy source is needed to move the liquid phase back to the evaporation site. The boiling bubbles are carried around the cell by the convective velocity field. There are also no impacts on the scale of the device.
- Ideally, the hot-water pipe should be fully insulated with the TEG/Cooler puncturing the insulation thereby providing a leakage path. What would be the impact of using uninsulated pipes?
  Interaction with the environment (as in the three-region model) severely hampers the functionality of the cooling device. Therefore, appropriate insulation (and in other parts of the device, temperature-matching) would appear to be essential.
- Is it more appropriate to re-design the TEG to match constraints of a predefined heat-source (water-pipe)and working fluid?
  This would not seem to be necessary. There is already some flexibility concerning the scale of region 1 – this can be increased by embedding the TEG in a medium that is conductivity-matched to the TEG itself.

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