

CS330 - Computer Organization and Assembly Language Programming

Lecture 13

-Review-

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Agenda

- **Review**

You are responsible for all the topics

This is a rough review

- **Midterm Exam**

February 24th, 2022 Thursday

Lecture Time

Exam 1

- 16 multiple choice questions
 - ~4 calculations/conversion type
 - ~3 open ended questions
-
- 50+2 points
 - 75 minutes
 - Closed books/notes/etc

Review

Hello World !

A typical *C program* basically consists of the following parts;

- Preprocessor Commands
- Functions
- Variables
- Statements & Expressions
- Comment

```
#include <stdio.h>
int main()
{
    /* the first program in CS330 */
    printf("hello, world\n");
    return 0;
}
```

Information is Bits + Context

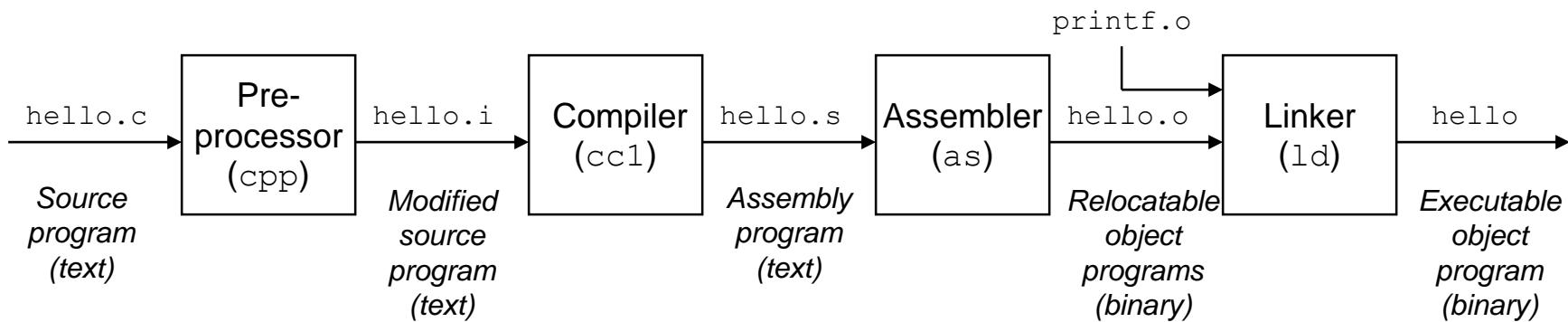
- “hello.c” is a source code
 - Sequence of bits (0 or 1)
 - 8-bit data chunks are called bytes
 - Each byte represents some text character in the program

#	i	n	c	l	u	d	e	<sp>	<	s	t	d	i	o	.
35	105	110	99	108	117	100	101	32	60	115	116	100	105	111	46
h	>	\n	\n	i	n	t	<sp>	m	a	i	n	()	\n	{
104	62	10	10	105	110	116	32	109	97	105	110	40	41	10	123
\n	<sp>	<sp>	<sp>	<sp>	p	r	i	n	t	f	("	h	e	1
10	32	32	32	32	112	114	105	110	116	102	40	34	104	101	108
l	o	,	<sp>	w	o	r	l	d	\	n	")	;	\n	}
108	111	44	32	119	111	114	108	100	92	110	34	41	59	10	125

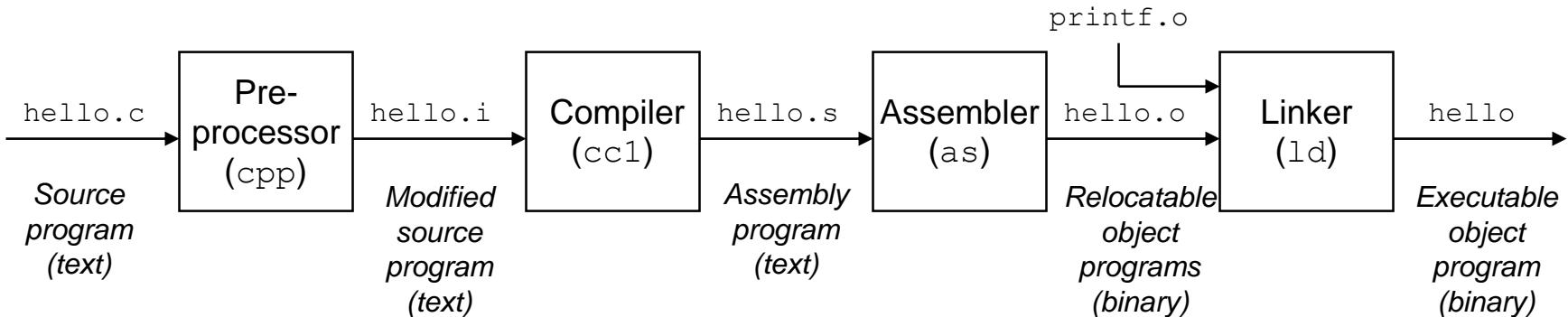
Figure 1.2 The ASCII text representation of hello.c.

Programs translated by other programs

- unix> gcc -o hello hello.c



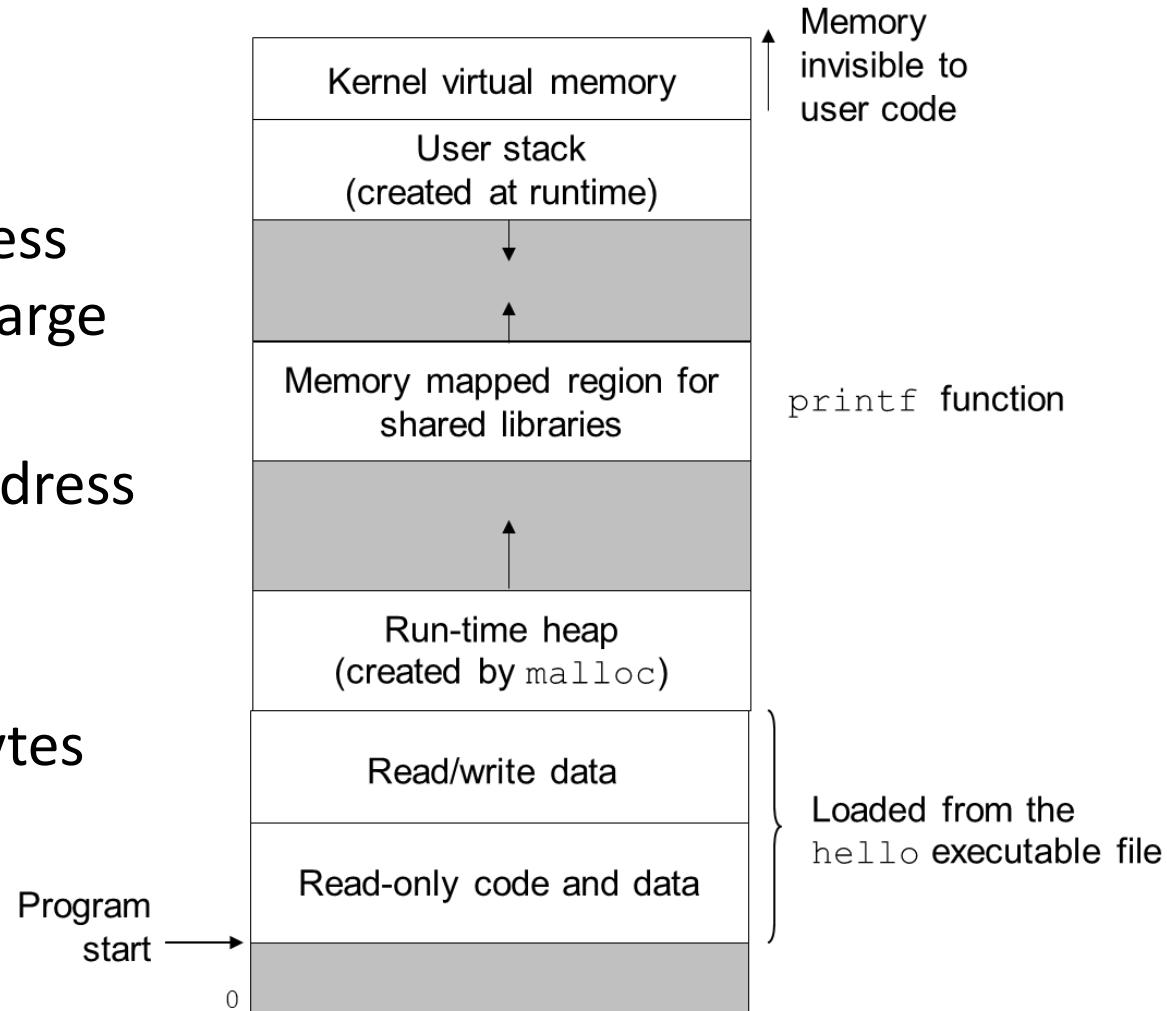
Compilation System



- Pre-processing
 - E.g., `#include<stdio.h>` is inserted into `hello.i`
- Compilation (.s)
 - Each statement is an assembly language program
- Assembly (.o)
 - A binary file whose bytes encode mach. language instructions
- Linking
 - Get `printf()` which resides in a separate precompiled object file

Virtual memory

- Illusion that each process has exclusive use of a large main memory
 - Example: Virtual address space for Linux
- **Files:** A sequence of bytes



Amdahl's Law

- Effectiveness of improving the performance of one part of system
- Speed up one part → Effect on the overall system performance?
- $T_{new} = (1 - \alpha)T_{old} + \frac{\alpha T_{old}}{k}$
- $= T_{old}[(1 - \alpha) + \frac{\alpha}{k}]$
- $S = \frac{T_{old}}{T_{new}}$
- $S = \frac{1}{((1-\alpha)+(\alpha/k))}$

Amdahl's Law / Example

- Consider a system;
 - A part of the system initially consumed 60% of the time ($\alpha = 0.6$)
 - It is sped up by a factor of 3 (k=3)
- Overall improvement ?

Amdahl's Law / Example

- Consider a system;
 - A part of the system initially consumed 60% of the time ($\alpha = 0.6$)
 - It is sped up by a factor of 3 ($k=3$)
- Overall improvement ?
- $S = \frac{1}{((1-\alpha)+(\alpha/k))}$
- $= 1 / [0.4 + 0.6/3] = \mathbf{1.67 \text{ times}}$

Amdahl's Law / Example 2

- Calculate the following improvements on a current system and decide which one is better
- **1)** if we make 90% of a program run 10 times faster.
- **2)** if we make 80% of a program run 20% faster

Amdahl's Law / Example 2

- 1) if we make 90% of a program run 10 times faster.

$$S = \frac{1}{((1-\alpha)+(\alpha/k))} = \frac{1}{((1-0.9)+(0.9/10))} = 5.26$$

- 2) if we make 80% of a program run 20% faster

$$S = \frac{1}{((1-\alpha)+(\alpha/k))} = \frac{1}{((1-0.8)+(0.8/1.2))} = 1.153$$

Data Measurement Chart

Data Measurement Size

Bit	Single Binary Digit (1 or 0)
Byte	8 bits
Kilobyte (KB)	1,024 Bytes
Megabyte (MB)	1,024 Kilobytes
Gigabyte (GB)	1,024 Megabytes
Terabyte (TB)	1,024 Gigabytes
Petabyte (PB)	1,024 Terabytes
Exabyte (EB)	1,024 Petabytes

The Decimal System

- System based on decimal digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9) to represent numbers
- For example the number 83 means eight tens plus three:

$$83 = (8 * 10) + 3$$

- The number 4728 means four thousands, seven hundreds, two tens, plus eight:

$$4728 = (4 * 1000) + (7 * 100) + (2 * 10) + 8$$

- The decimal system is said to have a **base**, or **radix**, of 10. This means that each digit in the number is multiplied by 10 raised to a power corresponding to that digit's position:

$$83 = (8 * 10^1) + (3 * 10^0)$$

$$4728 = (4 * 10^3) + (7 * 10^2) + (2 * 10^1) + (8 * 10^0)$$

The Binary System

- Only two digits, 1 and 0
- Represented to the base 2
- The digits 1 and 0 in binary notation have the same meaning as in decimal notation:

$$0_2 = 0_{10}$$

$$1_2 = 1_{10}$$

- To represent larger numbers each digit in a binary number has a value depending on its position:

$$10_2 = (1 * 2^1) + (0 * 2^0) = 2_{10}$$

$$11_2 = (1 * 2^1) + (1 * 2^0) = 3_{10}$$

$$100_2 = (1 * 2^2) + (0 * 2^1) + (0 * 2^0) = 4_{10}$$

For representing numbers in base 2, there are two possible digits (0, 1) in which column values are a power of two:

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
128	64	32	16	8	4	2	1

$$\begin{array}{cccccccccc} 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 + 64 + 32 + 0 + 0 + 0 + 0 + 2 + 1 = 99 \end{array}$$

Although values represented in base 2 are significantly longer than those in base 10, **binary representation is used in digital computing because of the resulting simplicity of hardware design**

Encoding Byte Values

- Byte = 8 bits
 - Binary 00000000_2 to 11111111_2
 - Decimal: 0_{10} to 255_{10}
 - Hexadecimal 00_{16} to FF_{16}
 - Base 16 number representation
 - Use characters ‘0’ to ‘9’ and ‘A’ to ‘F’
 - Write $FA1D37B_{16}$ in C as
 - $0xFA1D37B$
 - $0xfa1d37b$

Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

$1100\ 1001\ 0111\ 1011 \rightarrow 0xC97B$

Machine Words

- Machine has “word size”
 - Nominal size of integer-valued data
 - More importantly – a virtual address is encoded by such a word
 - Hence, it determines max size of virtual address space
 - Most current machines are 32 bits (4 bytes)
 - Limits addresses to 4GB
 - Becoming too small for memory-intensive applications
 - Newer systems are 64 bits (8 bytes)
 - Potentially address $\approx 1.84 \times 10^{19}$ bytes
 - Machines support multiple data formats
 - Fractions or multiples of word size
 - Always integral number of bytes

Data Sizes

- Each computer has a word size
 - For a machine with w -bit word size
 - The virtual address can range from 0 to $2^w - 1$
 - The program access to at most 2^w bytes
- 32 bit vs 64 bit

Addressing and Byte Ordering

- For objects that span multiple bytes (e.g. integers), we need to agree on two things
 - what would be the address of the object?
 - how would we order the bytes in memory?

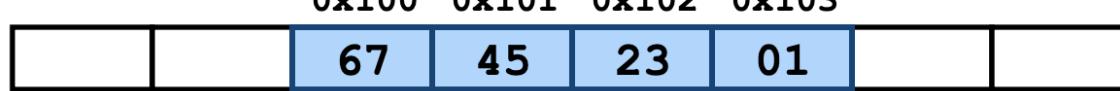
Byte Ordering

- How to order bytes within multi-byte word in memory
- Conventions
 - (most) Sun's, IBMs are “Big Endian” machines
 - Least significant byte has highest address (comes last)
 - (most) Intel's are “Little Endian” machines
 - Least significant byte has lowest address (comes first)
- Example
 - Variable x has 4-byte representation 0x01234567
 - Address given by &x is 0x100 0x100 0x101

BigEndian



LittleEndian



Boolean Variables and Operations

- Developed by George Boole in 19th Century
 - Algebraic representation of logic
 - Encode “True” as 1 and “False” as 0
 - $\langle \{0,1\}, |, \&, \sim, 0, 1 \rangle$
 - | is “sum” operation, & is “product” operation
 - \sim is “complement” operation (not additive inverse)
 - 0 is identity for sum, 1 is identity for product
- Makes use of variables and operations
 - Are logical
 - A variable may take on the value 1 (TRUE) or 0 (FALSE)
 - Basic logical operations are AND, OR, XOR and NOT

Boolean Variables and Operations / 2

- AND
 - Yields true (binary value 1) if and only if both of its operands are true
 - In the absence of parentheses the AND operation takes precedence over the OR operation
 - When no ambiguity will occur the AND operation is represented by simple concatenation instead of the dot operator
- OR
 - Yields true if either or both of its operands are true
- NOT
 - Inverts the value of its operand

Table: Boolean Operators

(a) Boolean Operators of Two Input Variables

P	Q	NOT P (\bar{P})	P AND Q ($P \bullet Q$)	P OR Q ($P + Q$)	P NAND Q ($\overline{P \bullet Q}$)	P NOR Q ($\overline{P + Q}$)	P XOR Q ($P \oplus Q$)
0	0	1	0	0	1	1	0
0	1	1	0	1	1	0	1
1	0	0	0	1	1	0	1
1	1	0	1	1	0	0	0

(b) Boolean Operators Extended to More than Two Inputs (A, B, ...)

Operation	Expression	Output = 1 if
AND	$A \bullet B \bullet \dots$	All of the set {A, B, ...} are 1.
OR	$A + B + \dots$	Any of the set {A, B, ...} are 1.
NAND	$\overline{A \bullet B \bullet \dots}$	Any of the set {A, B, ...} are 0.
NOR	$\overline{A + B + \dots}$	All of the set {A, B, ...} are 0.
XOR	$A \oplus B \oplus \dots$	The set {A, B, ...} contains an odd number of ones.

Table: Basic Identities of Boolean Algebra

Basic Postulates		
$A \cdot B = B \cdot A$	$A + B = B + A$	Commutative Laws
$A \cdot (B + C) = (A \cdot B) + (A \cdot C)$	$A + (B \cdot C) = (A + B) \cdot (A + C)$	Distributive Laws
$1 \cdot A = A$	$0 + A = A$	Identity Elements
$A \cdot \overline{A} = 0$	$A + \overline{A} = 1$	Inverse Elements
Other Identities		
$0 \cdot A = 0$	$1 + A = 1$	
$A \cdot A = A$	$A + A = A$	
$A \cdot (B \cdot C) = (A \cdot B) \cdot C$	$A + (B + C) = (A + B) + C$	Associative Laws
$\overline{A \cdot B} = \overline{A} + \overline{B}$	$\overline{A + B} = \overline{A} \cdot \overline{B}$	DeMorgan's Theorem

Exercise 1

Evaluate the following expression when A =0, B =1, and C= 1

$$F = B + \bar{C}A + B\bar{A} + A\bar{B}$$

Simplify the following functions;

$$F = AB + BC + \bar{B}C$$

$$F = A + \bar{A}B$$

Name	Graphical Symbol	Algebraic Function	Truth Table															
AND		$F = A \cdot B$ or $F = AB$	<table border="1"> <tr> <td>A</td><td>B</td><td>F</td></tr> <tr> <td>0</td><td>0</td><td>0</td></tr> <tr> <td>0</td><td>1</td><td>0</td></tr> <tr> <td>1</td><td>0</td><td>0</td></tr> <tr> <td>1</td><td>1</td><td>1</td></tr> </table>	A	B	F	0	0	0	0	1	0	1	0	0	1	1	1
A	B	F																
0	0	0																
0	1	0																
1	0	0																
1	1	1																
OR		$F = A + B$	<table border="1"> <tr> <td>A</td><td>B</td><td>F</td></tr> <tr> <td>0</td><td>0</td><td>0</td></tr> <tr> <td>0</td><td>1</td><td>1</td></tr> <tr> <td>1</td><td>0</td><td>1</td></tr> <tr> <td>1</td><td>1</td><td>1</td></tr> </table>	A	B	F	0	0	0	0	1	1	1	0	1	1	1	1
A	B	F																
0	0	0																
0	1	1																
1	0	1																
1	1	1																
NOT		$F = \bar{A}$ or $F = A'$	<table border="1"> <tr> <td>A</td><td>F</td></tr> <tr> <td>0</td><td>1</td></tr> <tr> <td>1</td><td>0</td></tr> </table>	A	F	0	1	1	0									
A	F																	
0	1																	
1	0																	
NAND		$F = \overline{AB}$	<table border="1"> <tr> <td>A</td><td>B</td><td>F</td></tr> <tr> <td>0</td><td>0</td><td>1</td></tr> <tr> <td>0</td><td>1</td><td>1</td></tr> <tr> <td>1</td><td>0</td><td>1</td></tr> <tr> <td>1</td><td>1</td><td>0</td></tr> </table>	A	B	F	0	0	1	0	1	1	1	0	1	1	1	0
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XOR		$F = A \oplus B$	<table border="1"> <tr> <td>A</td><td>B</td><td>F</td></tr> <tr> <td>0</td><td>0</td><td>0</td></tr> <tr> <td>0</td><td>1</td><td>1</td></tr> <tr> <td>1</td><td>0</td><td>1</td></tr> <tr> <td>1</td><td>1</td><td>0</td></tr> </table>	A	B	F	0	0	0	0	1	1	1	0	1	1	1	0
A	B	F																
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General Boolean Algebras

- Boolean operations can be extended to work on bit vectors
 - Operations applied bitwise

$$\begin{array}{rcl} \begin{array}{c} 01101001 \\ \& \underline{01010101} \end{array} & \begin{array}{c} 01101001 \\ \mid \underline{01010101} \end{array} & \begin{array}{c} 01101001 \\ \wedge \underline{01010101} \end{array} \\ \begin{array}{c} 01000001 \\ \textcolor{red}{01111101} \end{array} & \begin{array}{c} 01111101 \\ \textcolor{red}{00111100} \end{array} & \begin{array}{c} 00111100 \\ \textcolor{red}{10101010} \end{array} \\ \hline & & \end{array}$$

- All of the properties of Boolean algebra apply
- Now, Boolean \mid , $\&$ and \sim correspond to set union, intersection and complement

Bit-Level Operations in C

- Operations **&**, **|**, **~**, **^** Available in C
 - Apply to any “integral” data type
 - long, int, short, char, unsigned
 - View arguments as bit vectors
 - Arguments applied bit-wise
- Examples (Char data type)
 - $\sim 0x41 \rightarrow 0xBE$
 - $\sim 01000001_2 \rightarrow 10111110_2$
 - $\sim 0x00 \rightarrow 0xFF$
 - $\sim 00000000_2 \rightarrow 11111111_2$
 - $0x69 \& 0x55 \rightarrow 0x41$
 - $01101001_2 \& 01010101_2 \rightarrow 01000001_2$
 - $0x69 | 0x55 \rightarrow 0x7D$
 - $01101001_2 | 01010101_2 \rightarrow 01111101_2$

Contrast: Logic Operations in C

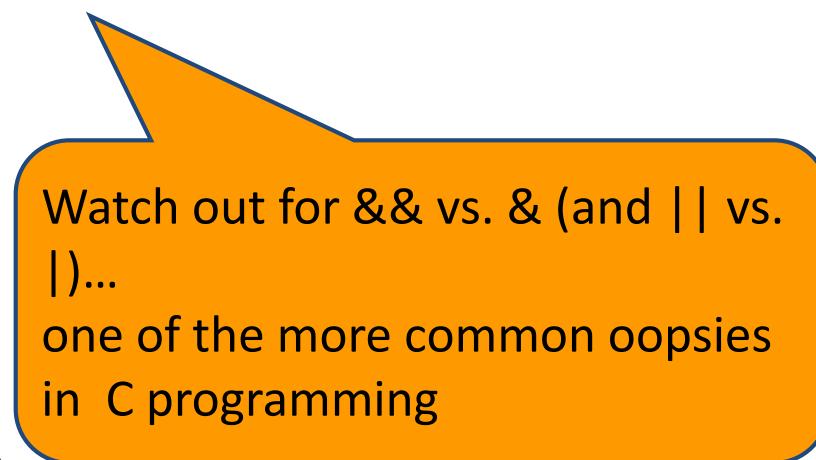
- Contrast to Logical Operators

- **&&, ||, !**

- View 0 as “False”
 - Anything nonzero as “True”
 - Always return 0 or 1
 - Early termination

- Examples (char data type)

- $\text{!}0x41 \rightarrow 0x00$
 - $\text{!}0x00 \rightarrow 0x01$
 - $\text{!!}0x41 \rightarrow 0x01$
 - $0x69 \&\& 0x55 \rightarrow 0x01$
 - $0x69 \mid\mid 0x55 \rightarrow 0x01$
 - $p \&\& *p$ (avoids null pointer access)



Watch out for `&&` vs. `&` (and `||` vs. `|`)...
one of the more common oopsies
in C programming

Shift Operations

- Left Shift: $x \ll y$
 - Shift bit-vector **x** left **y** positions
 - Throw away extra bits on left
 - Fill with 0's on right
- Right Shift: $x \gg y$
 - Shift bit-vector **x** right **y** positions
 - Throw away extra bits on right
 - Logical shift
 - Fill with 0's on left
 - Arithmetic shift
 - Replicate most significant bit on left
 - Useful in two's compliment
- Undefined Behavior
 - Shift amount < 0 or \geq word size

Argument x	01100010
<< 3	00010000
Log. >> 2	00011000
Arith. >> 2	00011000

Argument x	10100010
<< 3	00010000
Log. >> 2	00101000
Arith. >> 2	11101000

Boolean Algebra \approx Integer Ring

Commutative	$A \mid B = B \mid A$ $A \& B = B \& A$	$A + B = B + A$ $A * B = B * A$
Associativity	$(A \mid B) \mid C = A \mid (B \mid C)$ $(A \& B) \& C = A \& (B \& C)$	$(A + B) + C = A + (B + C)$ $(A * B) * C = A * (B * C)$
Product distributes over sum	$A \& (B \mid C) = (A \& B) \mid (A \& C)$	$A * (B + C) = A * B + B * C$
Sum and product identities	$A \mid 0 = A$ $A \& 1 = A$	$A + 0 = A$ $A * 1 = A$
Zero is product annihilator	$A \& 0 = 0$	$A * 0 = 0$
Cancellation of negation	$\sim(\sim A) = A$	$-(-A) = A$

Boolean Algebra \neq Integer Ring

Boolean: Sum distributes over product	$A (B \& C) = (A B) \& (A C)$	$A + (B * C) \neq (A + B) * (B + C)$
Boolean: Idempotency	$A A = A$ $A \& A = A$	$A + A \neq A$ $A * A \neq A$
Boolean: Absorption	$A (A \& B) = A$ $A \& (A B) = A$	$A + (A * B) \neq A$ $A * (A + B) \neq A$
Boolean: Laws of Complements	$A \sim A = 1$	$A + \sim A \neq 1$
Ring: Every element has additive inverse	$A \sim A \neq 0$	$A + \sim A = 0$

Properties of & and ^

- Boolean ring
 - $\langle \{0,1\}, \wedge, \&, I, 0, 1 \rangle$
 - Identical to integers mod 2
 - I is identity operation: $I(A) = A$
 - $A \wedge A = 0$
- Property: Boolean ring
 - Commutative sum $A \wedge B = B \wedge A$
 - Commutative product $A \& B = B \& A$
 - Associative sum $(A \wedge B) \wedge C = A \wedge (B \wedge C)$
 - Associative product $(A \& B) \& C = A \& (B \& C)$
 - Prod. over sum $A \& (B \wedge C) = (A \wedge B) \wedge (B \& C)$
 - 0 is sum identity $A \wedge 0 = A$
 - 1 is prod. identity $A \& 1 = A$
 - 0 is product annihilator $A \& 0 = 0$
 - Additive inverse $A \wedge A = 0$

Unsigned Encodings

$$B2U(X) = \sum_{i=0}^{w-1} x_i \cdot 2^i$$

(Binary To Unsigned)

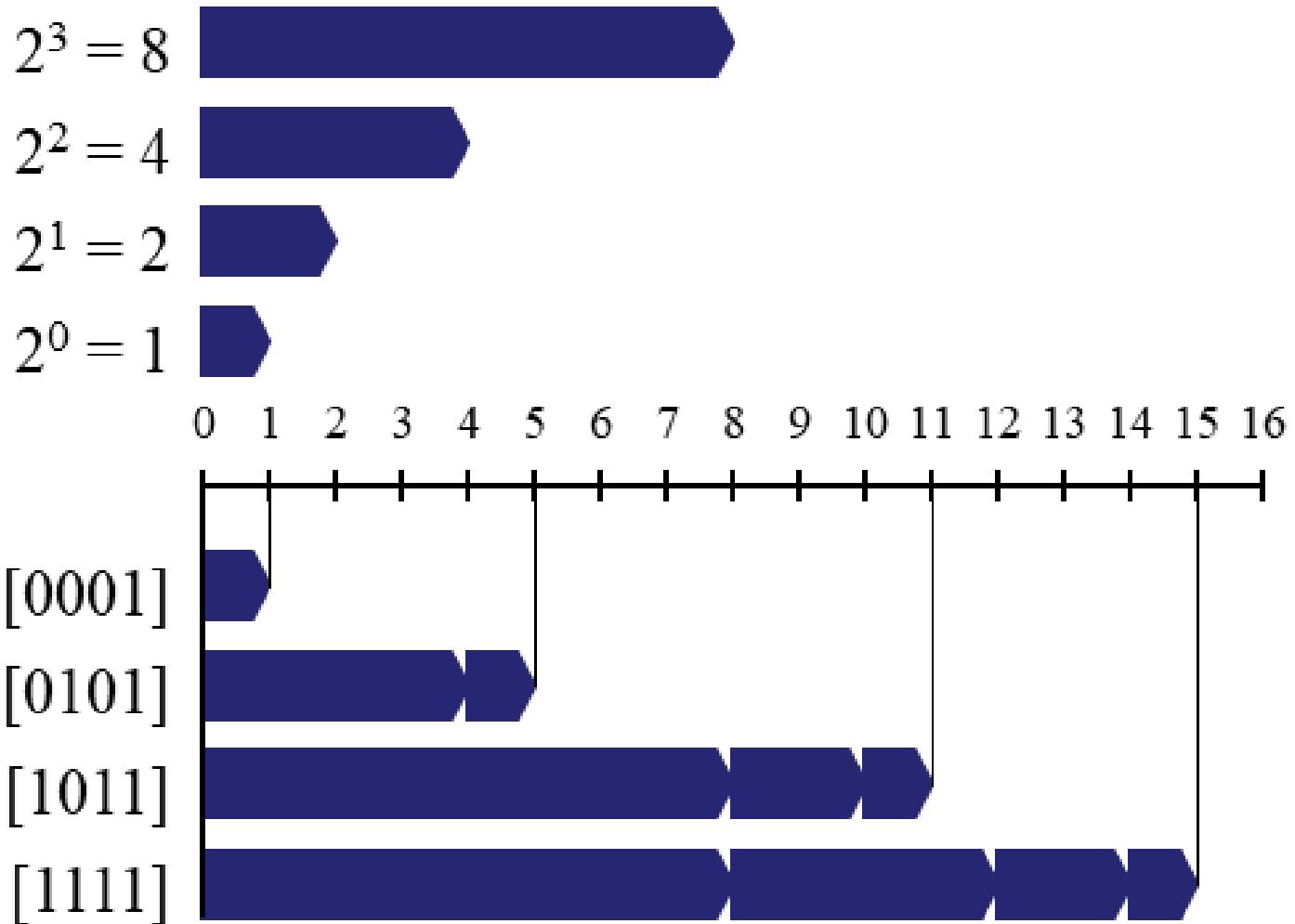
$$\text{e.g. } B2U([1011]) = 1 * 2^3 + 0 * 2^2 + 1 * 2^1 + 1 * 2^0 = 11$$

– C short 2 bytes long

```
short int x = 15213;
```

	Decimal	Hex	Binary
x	15213	3B 6D	00111011 01101101

Examples



Two's Complement

$$B2T(X) = -x_{w-1} \cdot 2^{w-1} + \sum_{i=0}^{w-2} x_i \cdot 2^i$$


Sign Bit

- e.g. $B2T([1011]) = -1 * 2^3 + 0 * 2^2 + 1 * 2^1 + 1 * 2^0 = -5$
- C short 2 bytes long

```
short int y = -15213;
```

	Decimal	Hex	Binary
x	15213	3B 6D	00111011 01101101
y	-15213	C4 93	11000100 10010011

- **Sign bit**

- For 2's complement, most significant bit indicates sign
 - 0 for nonnegative; 1 for negative

Two's Complement

- Invert and add **one**

Suppose we're working with 8 bit quantities and suppose we want to find how **-28** would be expressed in two's complement notation.

- First we write out 28 in **binary form**.

00011100

- Then we **invert the digits**. 0 becomes 1, 1 becomes 0.

11100011

- Then we **add 1**.

11100100

That is how one would write -28 in 8 bit binary.

Numeric Ranges

■ Unsigned Values

- $UMin = 0$
000...0
- $UMax = 2^w - 1$
111...1

■ Two's Complement Values

- $TMin = -2^{w-1}$
100...0
- $TMax = 2^{w-1} - 1$
011...1

■ Other Values

- Minus 1
111...1

Values for $W = 16$

	Decimal	Hex	Binary
UMax	65535	FF FF	11111111 11111111
TMax	32767	7F FF	01111111 11111111
TMin	-32768	80 00	10000000 00000000
-1	-1	FF FF	11111111 11111111
0	0	00 00	00000000 00000000

Signed vs. Unsigned in C

- Constants
 - By default are considered to be signed integers
 - Unsigned if have “U” as suffix
0U, 4294967259U
- Casting
 - Explicit casting between signed & unsigned same as U2T and T2U

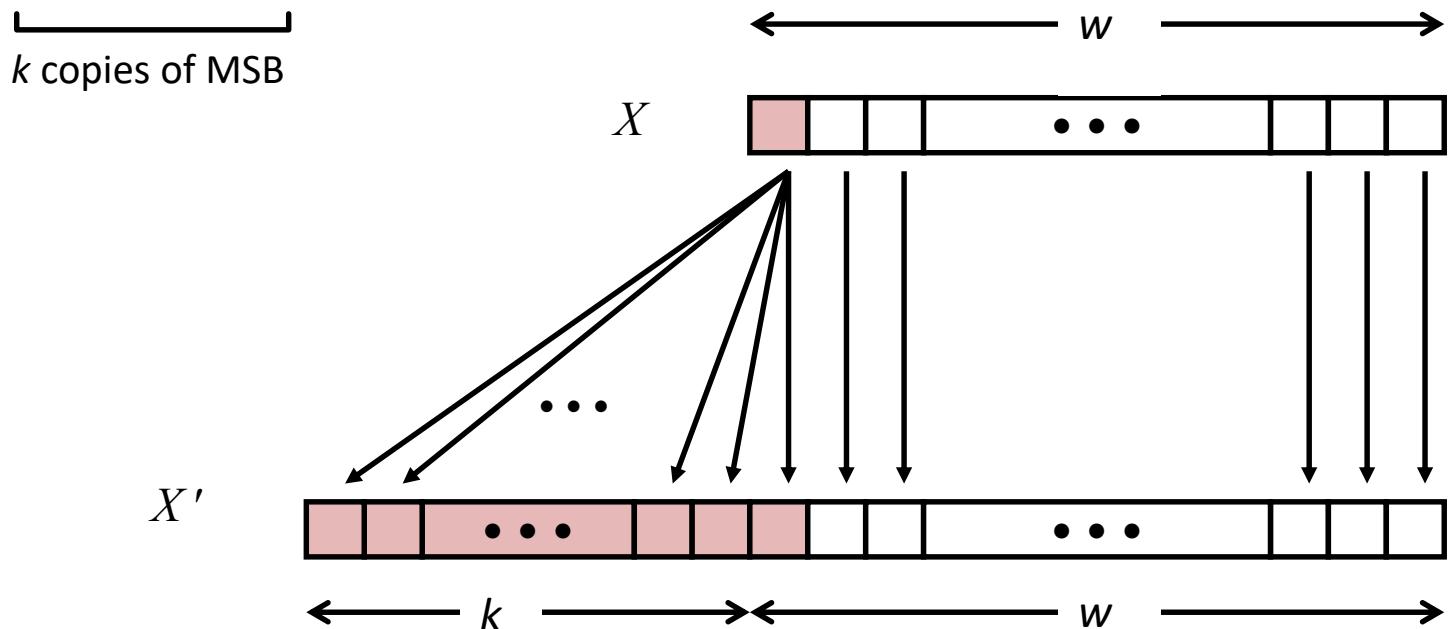
```
int tx, ty;  
unsigned ux, uy;  
tx = (int) ux;  
uy = (unsigned) ty;
```

- Implicit casting also occurs via assignments and procedure calls

```
tx = ux;  
uy = ty;
```

Sign Extension

- **Task:**
 - Given w -bit signed integer x
 - Convert it to $w+k$ -bit integer with same value
- **Rule:**
 - Make k copies of sign bit:
 - $X' = x_{w-1}, \dots, x_{w-1}, x_{w-1}, x_{w-2}, \dots, x_0$



Truncating Numbers

- Reduce the number of bits representing the number
- Truncating w -bit number to a k bit number, we drop the high order $w-k$ bits
 - Can alter its value
 - A form of overflow

Summary: Expanding, Truncating: Basic Rules

- **Expanding (e.g., short int to int)**
 - Unsigned: zeros added
 - Signed: sign extension
 - Both yield expected result
- **Truncating (e.g., unsigned to unsigned short)**
 - Unsigned/signed: bits are truncated
 - Result reinterpreted
 - Unsigned: mod operation
 - Signed: similar to mod
 - For small numbers yields expected behavior

OVERFLOW RULE:

- If two numbers are added, and they are both positive or both negative, then overflow occurs if and only if the result has the opposite sign.

Multiplication

- Goal: Computing Product of w -bit numbers x, y
 - Either signed or unsigned
- But, exact results can be bigger than w bits
 - Unsigned: up to $2w$ bits
 - Result range: $0 \leq x * y \leq (2^w - 1)^2 = 2^{2w} - 2^{w+1} + 1$
 - Two's complement min (negative): Up to $2w-1$ bits
 - Result range: $x * y \geq (-2^{w-1}) * (2^{w-1}-1) = -2^{2w-2} + 2^{w-1}$
 - Two's complement max (positive): Up to $2w$ bits, but only for $(TMin_w)^2$
 - Result range: $x * y \leq (-2^{w-1})^2 = 2^{2w-2}$
- So, maintaining exact results...
 - would need to keep expanding word size with each product computed
 - is done in software, if needed
 - e.g., by “arbitrary precision” arithmetic packages

Unsigned Binary Multiplication

$\begin{array}{r} 1011 \\ \times 1101 \\ \hline 1011 \\ 0000 \\ 1011 \\ \hline 1011 \\ \hline 10001111 \end{array}$	<p>Multiplicand (11) Multiplier (13)</p> <p>Partial products</p> <p>Product (143)</p>
---	---

Figure 10.7 Multiplication of Unsigned Binary Integers

Power-of-2 Multiply with Shift

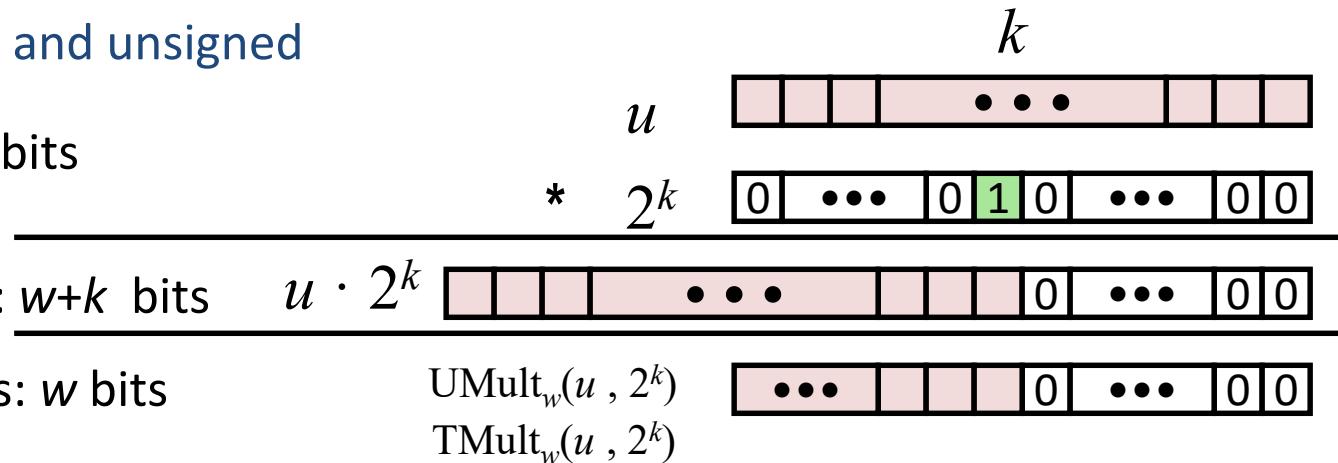
■ Operation

- $u \ll k$ gives $u * 2^k$
- Both signed and unsigned

Operands: w bits

True Product: $w+k$ bits

Discard k bits: w bits



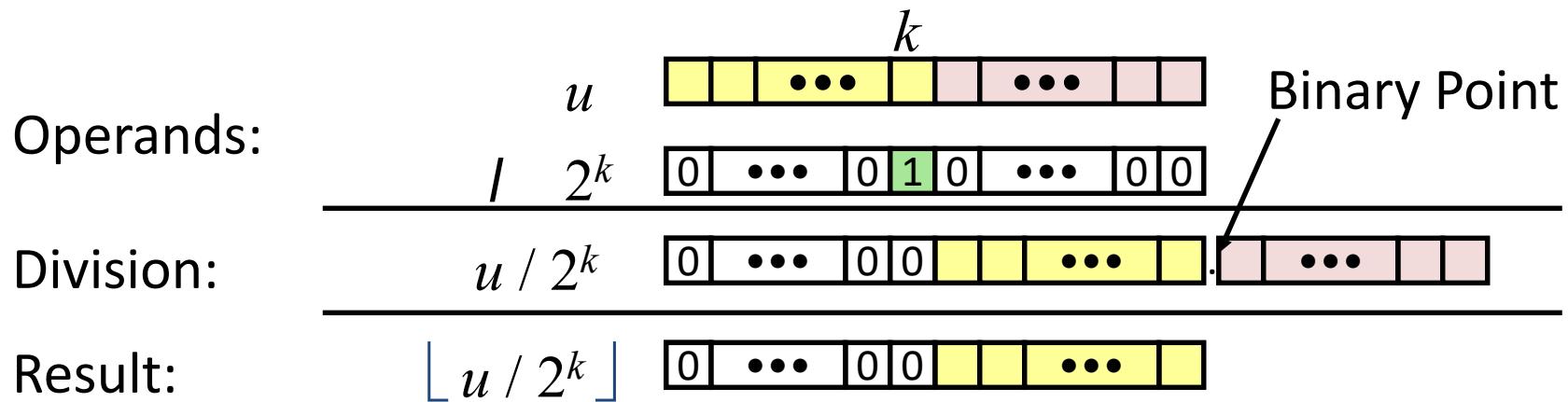
■ Examples

- $u \ll 3 == u * 8$
- $(u \ll 5) - (u \ll 3) == u * 24$
- Most machines shift and add faster than multiply
 - Compiler generates this code automatically

Unsigned Power-of-2 Divide with Shift

■ Quotient of Unsigned by Power of 2

- $u \gg k$ gives $\lfloor u / 2^k \rfloor$
- Uses logical shift



	Division	Computed	Hex	Binary
x	15213	15213	3B 6D	00111011 01101101
x >> 1	7606.5	7606	1D B6	00011101 10110110
x >> 4	950.8125	950	03 B6	00000011 10110110
x >> 8	59.4257813	59	00 3B	00000000 00111011

Arithmetic: Basic Rules

■ Addition:

- Unsigned/signed: Normal addition followed by truncate, same operation on bit level
- Unsigned: addition mod 2^w
 - Mathematical addition + possible subtraction of 2^w
- Signed: modified addition mod 2^w (result in proper range)
 - Mathematical addition + possible addition or subtraction of 2^w

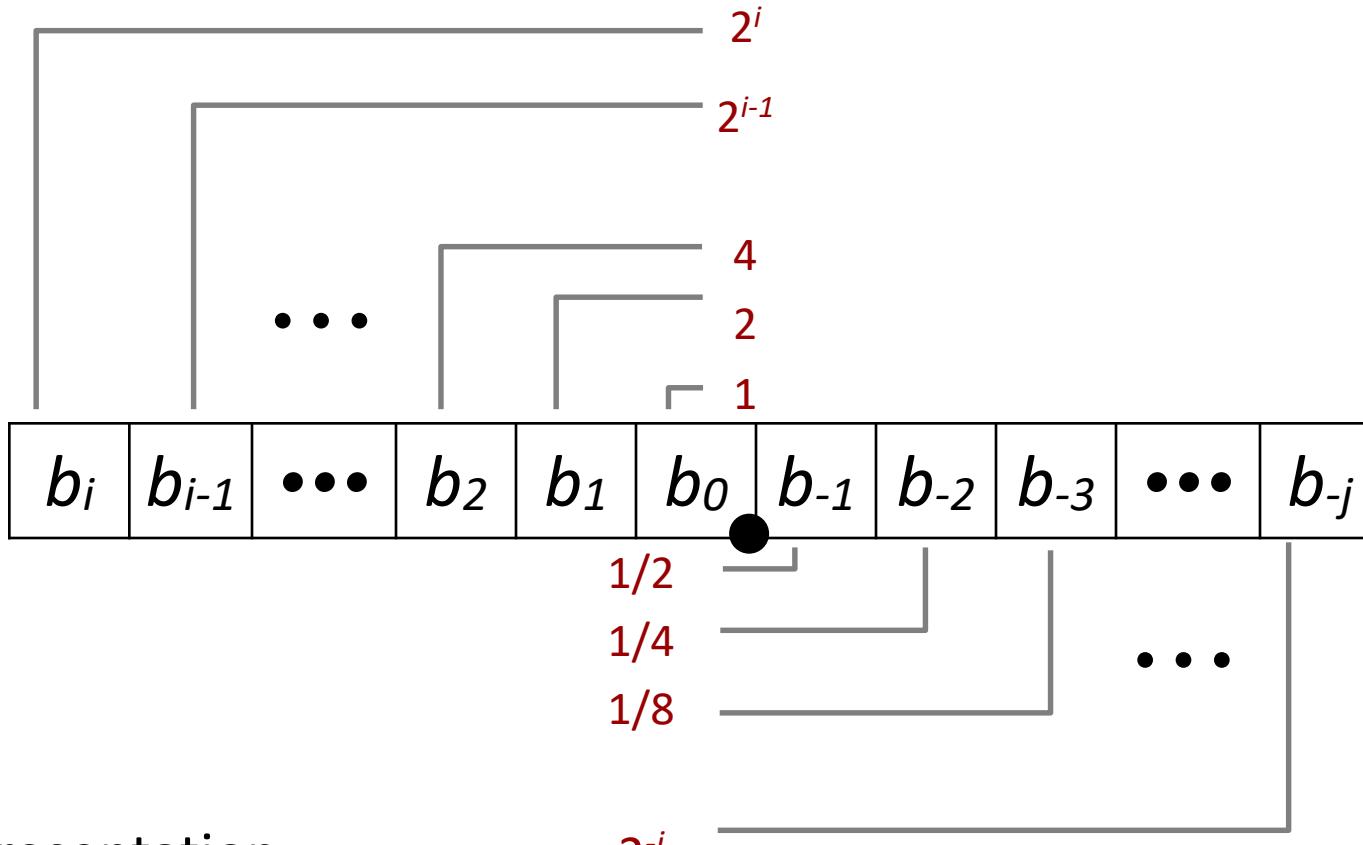
■ Multiplication:

- Unsigned/signed: Normal multiplication followed by truncate, same operation on bit level
- Unsigned: multiplication mod 2^w
- Signed: modified multiplication mod 2^w (result in proper range)

Fractional binary numbers

- What is 1011.101_2 ?

Fractional Binary Numbers



- Representation
 - Bits to right of “binary point” represent fractional powers of 2
 - Represents rational number:

$$\sum_{k=-j}^i b_k \times 2^k$$

Fractional Binary Numbers: Examples

■ Value	Representation
5 3/4	101.11 ₂
2 7/8	10.111 ₂
1 7/16	1.0111 ₂

■ Observations

- Divide by 2 by shifting right (unsigned)
- Multiply by 2 by shifting left
- Numbers of form 0.111111...₂ are just below 1.0
 - $1/2 + 1/4 + 1/8 + \dots + 1/2^i + \dots \rightarrow 1.0$
 - Use notation $1.0 - \epsilon$

Representable Numbers

- **Limitation #1**
 - Can only exactly represent numbers of the form $x/2^k$
 - Other rational numbers have repeating bit representations
 - **Value Representation**
 - $1/3$ $0.0101010101[01]..._2$
 - $1/5$ $0.001100110011[0011]..._2$
 - $1/10$ $0.0001100110011[0011]..._2$
- **Limitation #2**
 - Just one setting of binary point within the w bits
 - Limited range of numbers (very small values? very large?)

IEEE Floating Point

- **IEEE Standard 754**
 - Established in 1985 as uniform standard for floating point arithmetic
 - Before that, many idiosyncratic formats
 - Supported by all major CPUs
- **Driven by numerical concerns**
 - Nice standards for rounding, overflow, underflow
 - Hard to make fast in hardware
 - Numerical analysts predominated over hardware designers in defining standard

Floating Point Representation

- Numerical Form:

$$(-1)^s M \cdot 2^E$$

- Sign bit **s** determines whether number is negative or positive
- Significand **M** normally a fractional value in range [1.0,2.0).
- Exponent **E** weights value by power of two

- Encoding

- MSB **s** is sign bit **s**
- exp field encodes **E** (but is not equal to E)
- frac field encodes **M** (but is not equal to M)

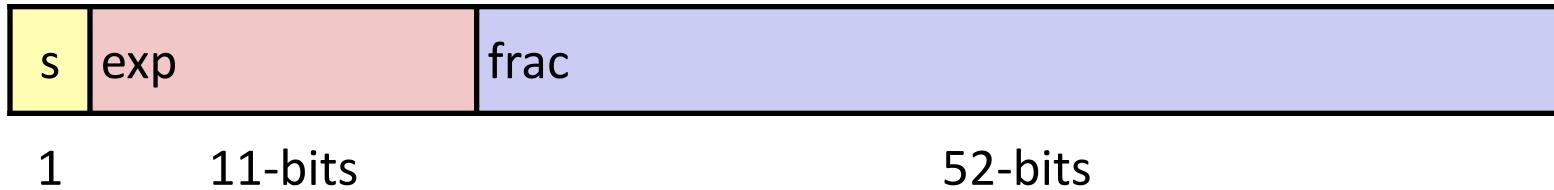


Precision options

- Single precision: 32 bits



- Double precision: 64 bits



- The value encoded by a given bit representation can be divided into three different cases, depending on the value of **exp.**
- Case 1: Normalized Values
- Case 2: Denormalized Values
- Case 3: Special Values

Case 1: “Normalized” Values

$$v = (-1)^s M 2^E$$

- When: $\text{exp} \neq 000\dots 0$ and $\text{exp} \neq 111\dots 1$
- Exponent coded as a *biased* value: $E = \text{Exp} - \text{Bias}$
 - Exp : unsigned value of exp field
 - $\text{Bias} = 2^{k-1} - 1$, where k is number of exponent bits
 - Single precision: 127 (Exp: 1...254, E: -126...127)
 - Double precision: 1023 (Exp: 1...2046, E: -1022...1023)
- Significand coded with implied leading 1: $M = 1.\text{xxx}\dots\text{x}_2$
 - $\text{xxx}\dots\text{x}$: bits of frac field
 - Minimum when frac=000...0 ($M = 1.0$)
 - Maximum when frac=111...1 ($M = 2.0 - \epsilon$)
 - Get extra leading bit for “free”

Case 2: Denormalized Values

$$v = (-1)^s M 2^E$$
$$E = 1 - \text{Bias}$$

- Condition: $\text{exp} = 000\dots0$
- Exponent value: $E = 1 - \text{Bias}$ (instead of $E = 0 - \text{Bias}$)
- Significand coded with implied leading 0: $M = 0.\text{xxx}\dots\text{x}_2$
 - $\text{xxx}\dots\text{x}$: bits of **frac**
- Cases
 - $\text{exp} = 000\dots0, \text{frac} = 000\dots0$
 - Represents zero value
 - Note distinct values: +0 and -0 (why?)
 - $\text{exp} = 000\dots0, \text{frac} \neq 000\dots0$
 - Numbers closest to 0.0
 - Equispaced

Case 3: Special Values

- Condition: **exp = 111...1**
- Case: **exp = 111...1, frac = 000...0**
 - Represents value ∞ (infinity)
 - Operation that overflows
 - Both positive and negative
 - E.g., $1.0/0.0 = -1.0/-0.0 = +\infty$, $1.0/-0.0 = -\infty$
- Case: **exp = 111...1, frac \neq 000...0**
 - Not-a-Number (NaN)
 - Represents case when no numeric value can be determined
 - E.g., $\sqrt{-1}$, $\infty - \infty$, $\infty \times 0$

Special Properties of the IEEE Encoding

- **FP Zero Same as Integer Zero**
 - All bits = 0
- **Can (Almost) Use Unsigned Integer Comparison**
 - Must first compare sign bits
 - Must consider $-0 = 0$
 - NaNs problematic
 - Will be greater than any other values
 - What should comparison yield?
 - Otherwise OK
 - Denorm vs. normalized
 - Normalized vs. infinity

Floating Point Operations: Basic Idea

- $x +_f y = \text{Round}(x + y)$
- $x \times_f y = \text{Round}(x \times y)$
- **Basic idea**
 - First **compute exact result**
 - Make it fit into desired precision
 - Possibly overflow if exponent too large
 - Possibly **round to fit into `frac`**

Rounding

- Rounding Modes (illustrate with \$ rounding)

	\$1.40	\$1.60	\$1.50	\$2.50	-\$1.50
– Towards zero	\$1	\$1	\$1	\$2	-\$1
– Round down ($-\infty$)	\$1	\$1	\$1	\$2	-\$2
– Round up ($+\infty$)	\$2	\$2	\$2	\$3	-\$1
– Nearest Even (default)	\$1	\$2	\$2	\$2	-\$2

x86-64 Integer Registers

%rax	%eax
------	------

%r8	%r8d
-----	------

%rbx	%ebx
------	------

%r9	%r9d
-----	------

%rcx	%ecx
------	------

%r10	%r10d
------	-------

%rdx	%edx
------	------

%r11	%r11d
------	-------

%rsi	%esi
------	------

%r12	%r12d
------	-------

%rdi	%edi
------	------

%r13	%r13d
------	-------

%rsp	%esp
------	------

%r14	%r14d
------	-------

%rbp	%ebp
------	------

%r15	%r15d
------	-------

- Can reference low-order 4 bytes (also low-order 1 & 2 bytes)

Some History: IA32 Registers



Moving Data

- Moving Data

movq Source, Dest:

- Operand Types

– ***Immediate:*** Constant integer data

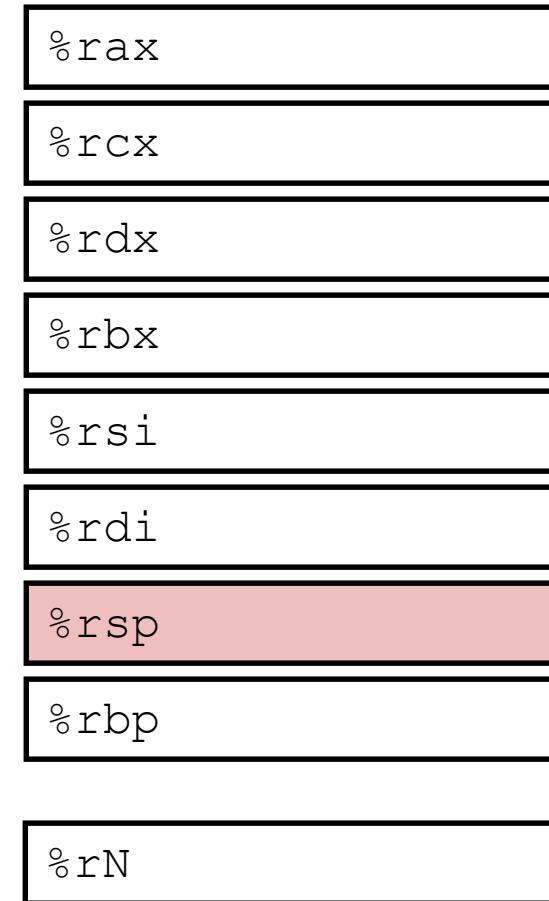
- Example: `$0x400`, `$-533`
- Like C constant, but prefixed with '\$'
- Encoded with 1, 2, or 4 bytes

– ***Register:*** One of 16 integer registers

- Example: `%rax`, `%r13`
- But `%rsp` reserved for special use
- Others have special uses for particular instructions

– ***Memory:*** 8 consecutive bytes of memory at address given by register

- Simplest example: (`%rax`)
- Various other “address modes”



movq Operand Combinations

	Source	Dest	Src,Dest	C Analog
movq	Imm	Reg	movq \$0x4,%rax	temp = 0x4;
		Mem	movq \$-147,(%rax)	*p = -147;
	Reg	Reg	movq %rax,%rdx	temp2 = temp1;
	Reg	Mem	movq %rax,(%rdx)	*p = temp;
	Mem	Reg	movq (%rax),%rdx	temp = *p;

Cannot do memory-memory transfer with a single instruction

Simple Memory Addressing Modes

- **Normal (R) Mem[Reg[R]]**

- Register R specifies memory address
 - Aha! Pointer dereferencing in C

```
movq (%rcx), %rax
```

- **Displacement D(R) Mem[Reg[R]+D]**

- Register R specifies start of memory region
 - Constant displacement D specifies offset

```
movq 8(%rbp), %rdx
```

Example of Simple Addressing Modes

```
void swap
    (long *xp, long *yp)
{
    long t0 = *xp;
    long t1 = *yp;
    *xp = t1;
    *yp = t0;
}
```

```
swap:
    movq    (%rdi), %rax
    movq    (%rsi), %rdx
    movq    %rdx, (%rdi)
    movq    %rax, (%rsi)
    ret
```

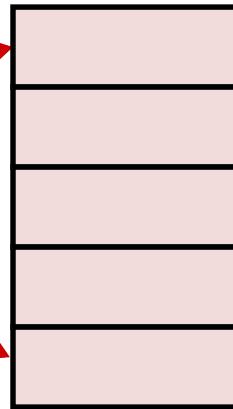
Understanding Swap()

```
void swap
    (long *xp, long *yp)
{
    long t0 = *xp;
    long t1 = *yp;
    *xp = t1;
    *yp = t0;
}
```

Registers

%rdi	
%rsi	
%rax	
%rdx	

Memory



Register	Value
%rdi	xp
%rsi	yp
%rax	t0
%rdx	t1

swap:

```
    movq    (%rdi), %rax    # t0 = *xp
    movq    (%rsi), %rdx    # t1 = *yp
    movq    %rdx, (%rdi)    # *xp = t1
    movq    %rax, (%rsi)    # *yp = t0
    ret
```

Understanding Swap()

Registers

%rdi	0x120
%rsi	0x100
%rax	
%rdx	

Memory

Address
0x120
0x118
0x110
0x108
0x100

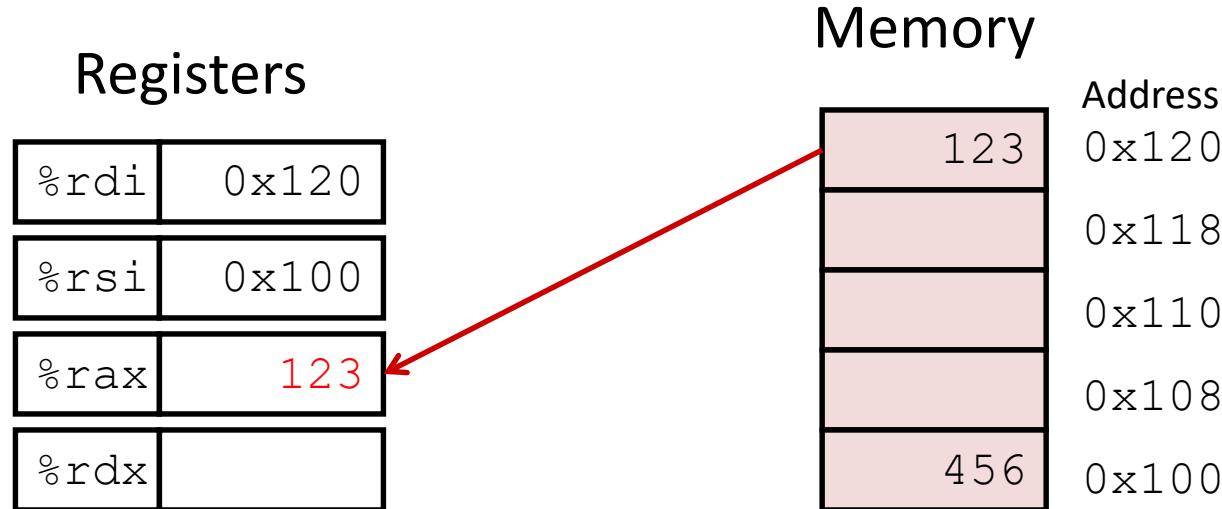
123

456

swap:

```
    movq    (%rdi), %rax    # t0 = *xp
    movq    (%rsi), %rdx    # t1 = *yp
    movq    %rdx, (%rdi)    # *xp = t1
    movq    %rax, (%rsi)    # *yp = t0
    ret
```

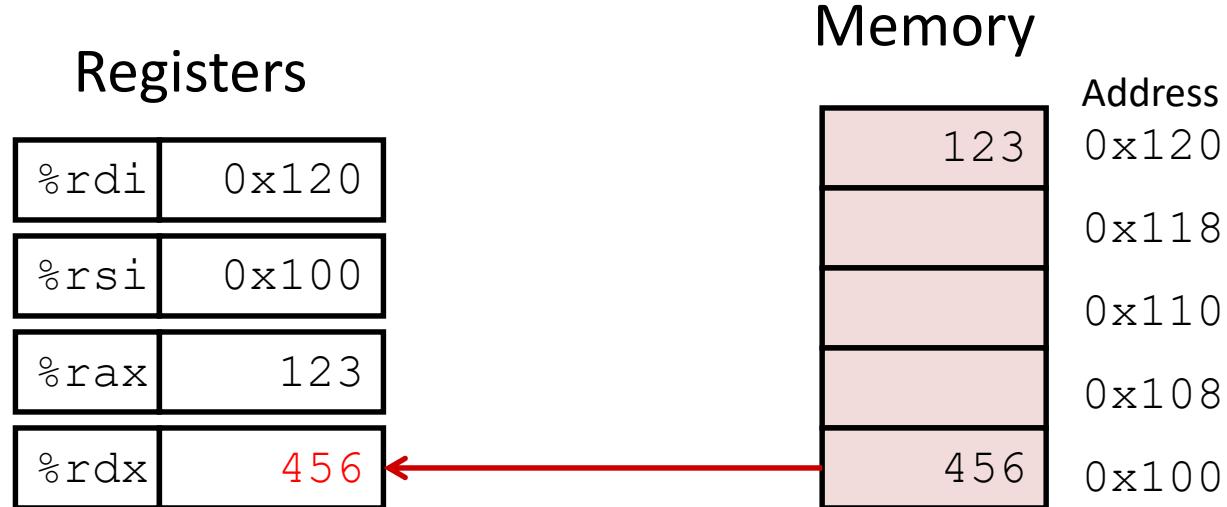
Understanding Swap()



swap:

```
    movq    (%rdi), %rax    # t0 = *xp
    movq    (%rsi), %rdx    # t1 = *yp
    movq    %rdx, (%rdi)    # *xp = t1
    movq    %rax, (%rsi)    # *yp = t0
    ret
```

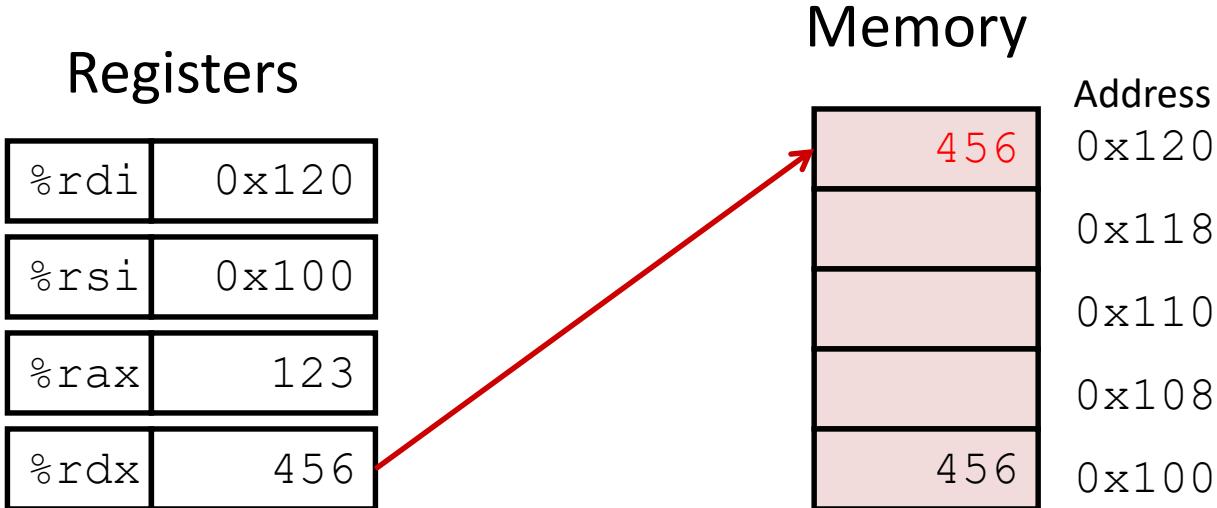
Understanding Swap()



swap:

```
    movq    (%rdi), %rax    # t0 = *xp
    movq    (%rsi), %rdx    # t1 = *yp
    movq    %rdx, (%rdi)    # *xp = t1
    movq    %rax, (%rsi)    # *yp = t0
    ret
```

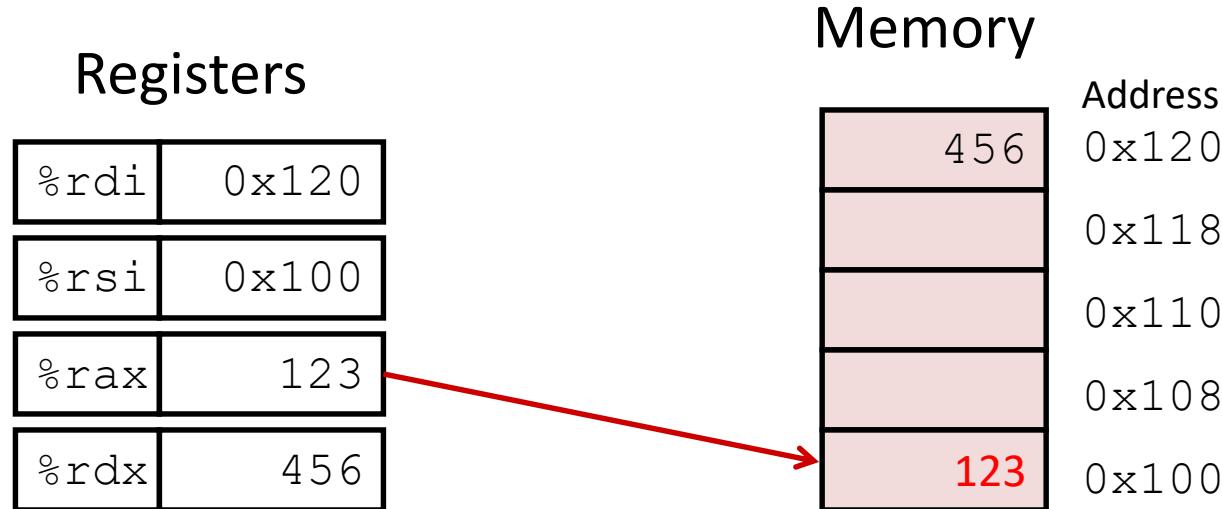
Understanding Swap()



swap:

```
    movq    (%rdi), %rax    # t0 = *xp
    movq    (%rsi), %rdx    # t1 = *yp
    movq    %rdx, (%rdi)    # *xp = t1
    movq    %rax, (%rsi)    # *yp = t0
    ret
```

Understanding Swap()



swap:

```
    movq    (%rdi), %rax    # t0 = *xp
    movq    (%rsi), %rdx    # t1 = *yp
    movq    %rdx, (%rdi)    # *xp = t1
    movq    %rax, (%rsi)    # *yp = t0
    ret
```

Simple Memory Addressing Modes

- Normal (R) $\text{Mem}[\text{Reg}[R]]$

- Register R specifies memory address
 - Aha! Pointer dereferencing in C

```
movq (%rcx), %rax
```

- Displacement D(R) $\text{Mem}[\text{Reg}[R]+D]$

- Register R specifies start of memory region
 - Constant displacement D specifies offset

```
movq 8(%rbp), %rdx
```

Complete Memory Addressing Modes

- Most General Form

D(Rb,Ri,S)

Mem[Reg[Rb]+S*Reg[Ri]+ D]

- D: Constant “displacement” 1, 2, or 4 bytes
- Rb: Base register: Any of 16 integer registers
- Ri: Index register: Any, except for %rsp
- S: Scale: 1, 2, 4, or 8

- Special Cases

(Rb,Ri)

Mem[Reg[Rb]+Reg[Ri]]

D(Rb,Ri)

Mem[Reg[Rb]+Reg[Ri]+D]

(Rb,Ri,S)

Mem[Reg[Rb]+S*Reg[Ri]]

AT&T Syntax vs Intel Syntax

- AT&T Syntax

movl %esp, %ebp

Instruction source destination

- Intel Syntax

movq ebp, esp

Instruction destination source

No percent signs

Pushing and Popping Stack Data

- Stack plays a vital role in the handling of procedure calls.
- Stack = a data structure where values can be added or deleted → according to last in first out discipline
- Add data to stack → push
- Remove data from stack → pop

Push	Source	Push source onto stack
Pop	Destination	Pop top of stack into destination

EXAMPLES:

pushq %rbp

Popq M[R[%rsp]]

Initially

%rax	0x123
%rdx	0
%rsp	0x108

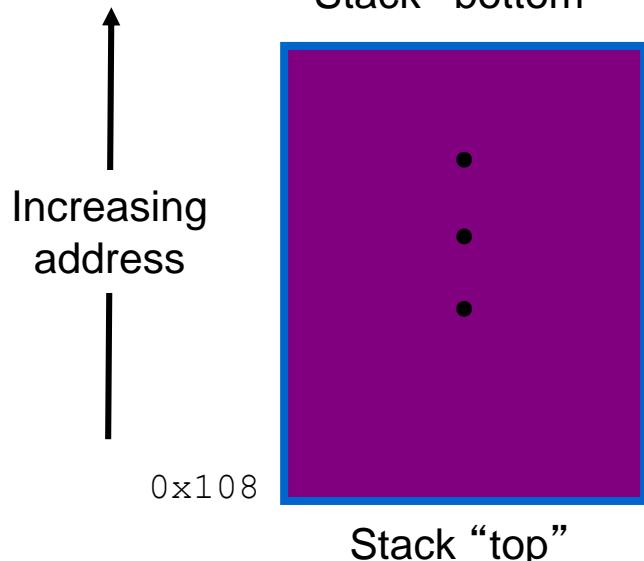
pushq %rax

%rax	0x123
%rdx	0
%rsp	0x100

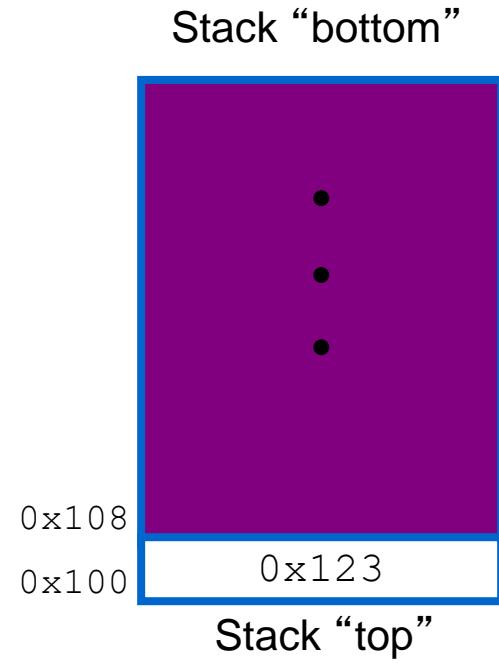
popq %rdx

%rax	0x123
%rdx	0x123
%rsp	0x108

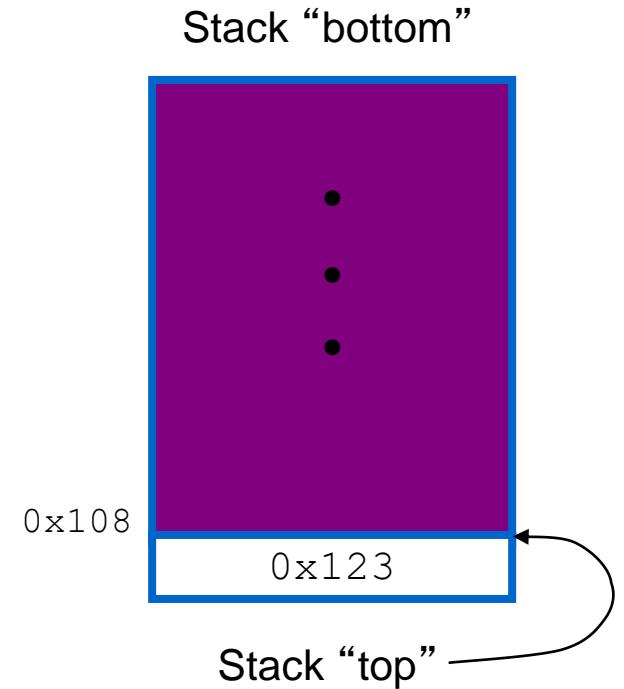
Stack “bottom”



Stack “bottom”



Stack “bottom”



- The stack pointer %rsp holds the address of the top stack element.
- Pushing a quad word value onto the stack involves first decrementing the stack pointer by 8 and then writing the value at the new top of stack address.
- `pushq %rbp` is equal to
`subq &8, %rsp`
`movq %rbp, (%rsp)`

recall -Complete Memory Addressing Modes

- Most General Form

$D(Rb,Ri,S)$

$\text{Mem}[\text{Reg}[Rb]+S*\text{Reg}[Ri]+ D]$

- D: Constant “displacement” 1, 2, or 4 bytes
- Rb: Base register: Any of 16 integer registers
- Ri: Index register: Any, except for %rsp
- S: Scale: 1, 2, 4, or 8

- Special Cases

(Rb,Ri)

$\text{Mem}[\text{Reg}[Rb]+\text{Reg}[Ri]]$

$D(Rb,Ri)$

$\text{Mem}[\text{Reg}[Rb]+\text{Reg}[Ri]+D]$

(Rb,Ri,S)

$\text{Mem}[\text{Reg}[Rb]+S*\text{Reg}[Ri]]$

Address Computation Examples

%rdx	0xf000
%rcx	0x0100

Expression	Address Computation	Address
0x8(%rdx)		
(%rdx,%rcx)		
(%rdx,%rcx,4)		
0x80(,%rdx,2)		

Address Computation Examples

%rdx	0xf000
%rcx	0x0100

Expression	Address Computation	Address
0x8(%rdx)	0xf000 + 0x8	0xf008
(%rdx,%rcx)	0xf000 + 0x100	0xf100
(%rdx,%rcx,4)	0xf000 + 4*0x100	0xf400
0x80(,%rdx,2)	2*0xf000 + 0x80	0x1e080

Arithmetic

&

Logical Operations

Address Computation Instruction

- **leaq *Source, Destination***
 - *Source* is address mode expression
 - Set *Destination* to address denoted by expression
- **leaq S, D D $\leftarrow \&S$ Load effective address**
- Uses
 - Computing addresses without a memory reference
 - E.g., translation of $p = \&x[i];$
 - Computing arithmetic expressions of the form $x + k*y$
 - $k = 1, 2, 4, \text{ or } 8$
- Example

```
long m12(long x)
{
    return x*12;
}
```

Converted to ASM by compiler:

```
leaq (%rdi,%rdi,2), %rax # t <- x+x*2
salq $2, %rax           # return t<<2
```

More on LEAQ

- The destination operand must be a register

```
leaq 7(%rdx, %rdx, 4), %rax
```

→ set register %rax to $5x + 7$

If %rax has the value x, and %rcx holds the value y

```
leaq 6(%rax), %rdx
```

```
leaq (%rax, %rcx), %rdx
```

More on LEAQ

- The destination operand must be a register

```
leaq 7(%rdx, %rdx, 4), %rax
```

→ set register %rax to $5x + 7$

If %rax has the value x, and %rcx holds the value y

```
leaq 6(%rax), %rdx → 6 + x
```

```
leaq (%rax, %rcx), %rdx → x + y
```

Basic Arithmetic Operations

- ADD
- SUB
- IMUL
- IDIV

ADD

ADDQ %rbx, %rax

- adds %rbx to %rax, and overwrites the result in %rax

ADDQ &8, %rsp

adds 8 to the stack pointer %rsp, (incrementing)

- $c = a + b;$

MOVQ a, %rax

MOVQ b, %rbx

ADDQ %rbx, %rax

MOVQ %rax, c

leaq for basic arithmetic operations

```
long scale(long x, long y, long z) {  
    long t= x + 4 * y + 12 * z;  
    return t;  
}
```

scale:

```
leaq (%rdi, %rsi, 4), %rax  
leaq (%rdx, %rdx, 2), %rdx  
leaq (%rax, %rdx, 4), %rax
```

leaq for basic arithmetic operations

```
long scale(long x, long y, long z) {  
    long t= x + 4 * y + 12 * z;  
    return t;  
}
```

x in %rdi, y in %rsi, z in %rdx

scale:

leaq (%rdi, %rsi, 4), %rax	$x+4*y$
leaq (%rdx, %rdx, 2), %rdx	$z+2*z=3*z$
leaq (%rax, %rdx, 4), %rax	$(x+4*y)+4*(3*z)$

Some Arithmetic Operations

- Two Operand Instructions:

Format Computation

addq	<i>Src,Dest</i>	Dest = Dest + Src
subq	<i>Src,Dest</i>	Dest = Dest – Src
imulq	<i>Src,Dest</i>	Dest = Dest * Src
salq	<i>Src,Dest</i>	Dest = Dest << Src <i>Also called shlq</i>
sarq	<i>Src,Dest</i>	Dest = Dest >> Src <i>Arithmetic</i>
shrq	<i>Src,Dest</i>	Dest = Dest >> Src <i>Logical</i>
xorq	<i>Src,Dest</i>	Dest = Dest ^ Src
andq	<i>Src,Dest</i>	Dest = Dest & Src
orq	<i>Src,Dest</i>	Dest = Dest Src

imulq

Imul S, D D \leftarrow D*S

return a*b + c*d;

*Assume a in %edi, b in %sil, c in %rdx, d in %ecx

```
movslq %ecx, %rcx
movsb1 %sil, %esi
imulq %rdx, %rcx
imull %edi, %esi
leal (%rsi,%rcx), %eax
ret
```

salq, shrq, sarg, shlq... .

- Left/right shift operations
- salq \$4, %rax → left shift rax by 4 bits
- For example

```
movw $ff00,%ax      # ax=1111.1111.0000.0000 (0xff00, unsigned 65280, signed -256)
shrw $3,%ax        # ax=0001.1111.1110.0000 (0x1fe0, signed and unsigned 8160)
                  # (logical shifting unsigned numbers right by 3
                  #   is like integer division by 8)
shlw $1,%ax        # ax=0011.1111.1100.0000 (0x3fc0, signed and unsigned 16320)
                  # (logical shifting unsigned numbers left by 1
                  #   is like multiplication by 2)
```

Binary Operations

xorq, andq, orq, notq...

```
movl $0x1, %eax ; eax := 1
```

```
movl $0x0, %ebx ; ebx := 0
```

```
orl %eax, %ebx ; ebx := eax V ebx
```

ebx would be 1 because $1 \vee 0 \Leftrightarrow 1$

```
notl %ebx ; ebx := 0
```

Unary Operations

- Single operand serving as both source and destination
- Operand can be either a register or memory location

incq (%rsp)

similar to ++ or – in C

decq (%rcx)

Other Arithmetic Operations

- One Operand Instructions

incq *Dest* $Dest = Dest + 1$

decq *Dest* $Dest = Dest - 1$

negq *Dest* $Dest = -Dest$

notq *Dest* $Dest = \sim Dest$

- See book for more instructions

Arithmetic Expression Example

```
long arith  
(long x, long y, long z)  
{  
    long t1 = x+y;  
    long t2 = z+t1;  
    long t3 = x+4;  
    long t4 = y * 48;  
    long t5 = t3 + t4;  
    long rval = t2 * t5;  
    return rval;  
}
```

arith:

leaq	(%rdi,%rsi), %rax
addq	%rdx, %rax
leaq	(%rsi,%rsi,2), %rdx
salq	\$4, %rdx
leaq	4(%rdi,%rdx), %rcx
imulq	%rcx, %rax
ret	

Interesting Instructions

- **leaq**: address computation
- **salq**: shift
- **imulq**: multiplication
 - But, only used once

Understanding Arithmetic Expression Example

```
long arith  
(long x, long y, long z)  
{  
    long t1 = x+y;  
    long t2 = z+t1;  
    long t3 = x+4;  
    long t4 = y * 48;  
    long t5 = t3 + t4;  
    long rval = t2 * t5;  
    return rval;  
}
```

arith:

```
leaq    (%rdi,%rsi), %rax    # t1  
addq    %rdx, %rax          # t2  
leaq    (%rsi,%rsi,2), %rdx  
salq    $4, %rdx            # t4  
leaq    4(%rdi,%rdx), %rcx  # t5  
imulq   %rcx, %rax          # rval  
ret
```

Register	Use(s)
%rdi	Argument x
%rsi	Argument y
%rdx	Argument z
%rax	t1, t2, rval
%rdx	t4
%rcx	t5

```
long arith(long x, long y, long z){  
    long t1 = x ^ y;  
    long t2= z*48;  
    long t3= t1 & 0x0FOFOFOF;  
    long t4= t2-t3;  
    return t4;  
} // Assume x in %rdi, y in %rsi , and z in %rdx
```

arith:

```
    xorq %rsi, %rdi      long t1 = x ^ y;  
    leaq   (%rdx, %rdx, 2), %rax      3*z  
    salq   $4, %rax      t2 = 16*(3*z)=48*z  
    andl   $252645135, %edi      t3 = t1 & 0x0FOFOFOF  
    subq   %rdi, %rax      return t2-t3;  
    ret
```

Machine Programming I: Summary

- History of Intel processors and architectures
 - Evolutionary design leads to many quirks and artifacts
- C, assembly, machine code
 - New forms of visible state: program counter, registers,
...
 - Compiler must transform statements, expressions,
procedures into low-level instruction sequences
- Assembly Basics: Registers, operands, move
 - The x86-64 move instructions cover wide range of
data movement forms
- Arithmetic
 - C compiler will figure out different instruction
combinations to carry out computation

Arithmetic Expression Example

```
long arith  
(long x, long y, long z)  
{  
    long t1 = x+y;  
    long t2 = z+t1;  
    long t3 = x+4;  
    long t4 = y * 48;  
    long t5 = t3 + t4;  
    long rval = t2 * t5;  
    return rval;  
}
```

arith:

leaq	(%rdi,%rsi), %rax
addq	%rdx, %rax
leaq	(%rsi,%rsi,2), %rdx
salq	\$4, %rdx
leaq	4(%rdi,%rdx), %rcx
imulq	%rcx, %rax
ret	

Interesting Instructions

- **leaq**: address computation
- **salq**: shift
- **imulq**: multiplication
 - But, only used once

Understanding Arithmetic Expression Example

```
long arith  
(long x, long y, long z)  
{  
    long t1 = x+y;  
    long t2 = z+t1;  
    long t3 = x+4;  
    long t4 = y * 48;  
    long t5 = t3 + t4;  
    long rval = t2 * t5;  
    return rval;  
}
```

arith:

```
leaq    (%rdi,%rsi), %rax    # t1  
addq    %rdx, %rax          # t2  
leaq    (%rsi,%rsi,2), %rdx  
salq    $4, %rdx            # t4  
leaq    4(%rdi,%rdx), %rcx  # t5  
imulq   %rcx, %rax          # rval  
ret
```

Register	Use(s)
%rdi	Argument x
%rsi	Argument y
%rdx	Argument z
%rax	t1, t2, rval
%rdx	t4
%rcx	t5

```
long arith(long x, long y, long z){  
    long t1 = x ^ y;  
    long t2= z*48;  
    long t3= t1 & 0x0FOFOFOF;  
    long t4= t2-t3;  
    return t4;  
} // Assume x in %rdi, y in %rsi , and z in %rdx
```

arith:

```
    xorq %rsi, %rdi      long t1 = x ^ y;  
    leaq   (%rdx, %rdx, 2), %rax      3*z  
    salq   $4, %rax        t2 = 16*(3*z)=48*z  
    andl   $252645135, %edi      t3 = t1 & 0x0FOFOFOF  
    subq   %rdi, %rax        return t2-t3;  
    ret
```

Machine Programming I: Summary

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data movement forms
- Arithmetic
 - C compiler will figure out different instruction
combinations to carry out computation

Processor State (x86-64, Partial)

- Information about currently executing program
 - Temporary data (**%rax**, ...)
 - Location of runtime stack (**%rsp**)
 - Location of current code control point (**%rip**, ...)
 - Status of recent tests (**CF, ZF, SF, OF**)

Current stack top

Registers

%rax	%r8
%rbx	%r9
%rcx	%r10
%rdx	%r11
%rsi	%r12
%rdi	%r13
%rsp	%r14
%rbp	%r15

%rip

Instruction pointer

CF

ZF

SF

OF

Condition codes

Condition Codes (Implicit Setting)

- **Single bit registers**
 - CF** Carry Flag (for unsigned) **SF** Sign Flag (for signed)
 - ZF** Zero Flag **OF** Overflow Flag (for signed)
- **Implicitly set (think of it as side effect) by arithmetic operations**

Example: **addq Src,Dest** $\leftrightarrow t = a+b$

CF set if carry out from most significant bit (unsigned overflow)

ZF set if $t == 0$

SF set if $t < 0$ (as signed)

OF set if two's-complement (signed) overflow
 $(a>0 \ \&\& \ b>0 \ \&\& \ t<0) \ || \ (a<0 \ \&\& \ b<0 \ \&\& \ t>=0)$
- **Not set by leaq instruction**

Condition Codes (Explicit Setting: Compare)

- **Explicit Setting by Compare Instruction**

- cmpq** $Src2, Src1$

- cmpq b, a** like computing $a-b$ without setting destination

- CF set** if carry out from most significant bit (used for unsigned comparisons)

- ZF set** if $a == b$

- SF set** if $(a-b) < 0$ (as signed)

- OF set** if two's-complement (signed) overflow

- $(a>0 \ \&\& \ b<0 \ \&\& \ (a-b)<0) \ \|\ (a<0 \ \&\& \ b>0 \ \&\& \ (a-b)>0)$

Condition Codes (Explicit Setting: Test)

- **Explicit Setting by Test instruction**
 - testq Src2, Src1**
 - **testq b, a** like computing **a&b** without setting destination
 - Sets condition codes based on value of *Src1* & *Src2*
 - Useful to have one of the operands be a mask
- ZF set** when **a&b == 0**
- SF set** when **a&b < 0**

Reading Condition Codes

- **SetX Instructions**
 - Set low-order byte of destination to 0 or 1 based on combinations of condition codes
 - Does not alter remaining 7 bytes

SetX	Condition	Description
sete	ZF	Equal / Zero
setne	$\sim ZF$	Not Equal / Not Zero
sets	SF	Negative
setns	$\sim SF$	Nonnegative
setg	$\sim (SF^OF) \ \& \ \sim ZF$	Greater (Signed)
setge	$\sim (SF^OF)$	Greater or Equal (Signed)
setl	(SF^OF)	Less (Signed)
setle	$(SF^OF) \ \ ZF$	Less or Equal (Signed)
seta	$\sim CF \ \& \ \sim ZF$	Above (unsigned)
setb	CF	Below (unsigned)

x86-64 Integer Registers

%rax	%al	
%rbx	%bl	
%rcx	%cl	
%rdx	%dl	
%rsi	%sil	
%rdi	%dil	
%rsp	%spl	
%rbp	%bp1	
%r8		%r8b
%r9		%r9b
%r10		%r10b
%r11		%r11b
%r12		%r12b
%r13		%r13b
%r14		%r14b
%r15		%r15b

– Can reference low-order byte

Reading Condition Codes (Cont.)

- SetX Instructions:
 - Set single byte based on combination of condition codes
- One of addressable byte registers
 - Does not alter remaining bytes
 - Typically use **movzbl** to finish job
 - 32-bit instructions also set upper 32 bits to 0

```
int gt (long x, long y)
{
    return x > y;
}
```

Register	Use(s)
%rdi	Argument x
%rsi	Argument y
%rax	Return value

```
cmpq    %rsi, %rdi    # Compare x:y
setg    %al             # Set when >
movzbl  %al, %eax     # Zero rest of %rax
ret
```

Jumping

- **jX Instructions**
 - Jump to different part of code depending on condition codes

jX	Condition	Description
jmp	1	Unconditional
je	ZF	Equal / Zero
jne	~ZF	Not Equal / Not Zero
js	SF	Negative
jns	~SF	Nonnegative
jg	~(SF^OF) & ~ZF	Greater (Signed)
jge	~(SF^OF)	Greater or Equal (Signed)
jl	(SF^OF)	Less (Signed)
jle	(SF^OF) ZF	Less or Equal (Signed)
ja	~CF & ~ZF	Above (unsigned)
jb	CF	Below (unsigned)

Conditional Branch Example (Old Style)

- Generation

```
gcc -Og -S -fno-if-conversion control.c
```

```
long absdiff
    (long x, long y)
{
    long result;
    if (x > y)
        result = x-y;
    else
        result = y-x;
    return result;
}
```

absdiff:

```
    cmpq    %rsi, %rdi  # x:y
    jle     .L4
    movq    %rdi, %rax
    subq    %rsi, %rax
    ret
.L4:      # x <= y
    movq    %rsi, %rax
    subq    %rdi, %rax
    ret
```

Register	Use(s)
%rdi	Argument x
%rsi	Argument y
%rax	Return value

Expressing with Goto Code

- C allows `goto` statement
- Jump to position designated by label

```
long absdiff
    (long x, long y)
{
    long result;
    if (x > y)
        result = x-y;
    else
        result = y-x;
    return result;
}
```

```
long absdiff_j
    (long x, long y)
{
    long result;
    int ntest = x <= y;
    if (ntest) goto Else;
    result = x-y;
    goto Done;
Else:
    result = y-x;
Done:
    return result;
}
```

General Conditional Expression Translation (Using Branches)

C Code

```
val = Test ? Then_Expr : Else_Expr;
```

```
val = x>y ? x-y : y-x;
```

Goto Version

```
ntest = !Test;  
if (ntest) goto Else;  
val = Then_Expr;  
goto Done;  
Else:  
    val = Else_Expr;  
Done:  
    . . .
```

- Create separate code regions for then & else expressions
- Execute appropriate one

Using Conditional Moves

- Conditional Move Instructions
 - Instruction supports:
if (Test) Dest \leftarrow Src
 - Supported in post-1995 x86 processors
 - GCC tries to use them
 - But, only when known to be safe
- Why?
 - Branches are very disruptive to instruction flow through pipelines
 - Conditional moves do not require control transfer

C Code

```
val = Test  
    ? Then_Expr  
    : Else_Expr;
```

Goto Version

```
result = Then_Expr;  
eval = Else_Expr;  
nt = !Test;  
if (nt) result = eval;  
return result;
```

Conditional Move Example

```
long absdiff
    (long x, long y)
{
    long result;
    if (x > y)
        result = x-y;
    else
        result = y-x;
    return result;
}
```

Register	Use(s)
%rdi	Argument x
%rsi	Argument y
%rax	Return value

```
absdiff:
    movq    %rdi, %rax    # x
    subq    %rsi, %rax    # result = x-y
    movq    %rsi, %rdx
    subq    %rdi, %rdx    # eval = y-x
    cmpq    %rsi, %rdi    # x:y
    cmovle %rdx, %rax    # if <=, result = eval
    ret
```

Bad Cases for Conditional Move

Expensive Computations

```
val = Test(x) ? Hard1(x) : Hard2(x);
```

- Both values get computed
- Only makes sense when computations are very simple

Risky Computations

```
val = p ? *p : 0;
```

- Both values get computed
- May have undesirable effects

Computations with side effects

```
val = x > 0 ? x*=7 : x+=3;
```

- Both values get computed
- Must be side-effect free

“Do-While” Loop Example

C Code

```
long pcount_do
(unsigned long x) {
    long result = 0;
    do {
        result += x & 0x1;
        x >>= 1;
    } while (x);
    return result;
}
```

Goto Version

```
long pcount_goto
(unsigned long x) {
    long result = 0;
loop:
    result += x & 0x1;
    x >>= 1;
    if(x) goto loop;
    return result;
}
```

- Count number of 1's in argument x (“popcount”)
- Use conditional branch to either continue looping or to exit loop

“Do-While” Loop Compilation

Goto Version

```
long pcount_goto
(unsigned long x) {
    long result = 0;
loop:
    result += x & 0x1;
    x >>= 1;
    if(x) goto loop;
    return result;
}
```

Register	Use(s)
%rdi	Argument x
%rax	result

```
        movl    $0, %eax      #  result = 0
.L2:                           #  loop:
        movq    %rdi, %rdx
        andl    $1, %edx      #  t = x & 0x1
        addq    %rdx, %rax    #  result += t
        shrq    %rdi          #  x >>= 1
        jne     .L2          #  if (x) goto loop
        rep; ret
```

General “Do-While” Translation

C Code

```
do  
  Body  
  while (Test);
```

Goto Version

```
loop:  
  Body  
  if (Test)  
    goto loop
```

- Body:

```
{  
  Statement1;  
  Statement2;  
  ...  
  Statementn;  
}
```

General “While” Translation #1

- “Jump-to-middle” translation
- Used with `-Og`

While version

```
while (Test)
  Body
```



Goto Version

```
goto test;
loop:
  Body
test:
  if (Test)
    goto loop;
done:
```

While Loop Example #1

C Code

```
long pcount_while
(unsigned long x) {
    long result = 0;
    while (x) {
        result += x & 0x1;
        x >>= 1;
    }
    return result;
}
```

Jump to Middle

```
long pcount_goto_jtm
(unsigned long x) {
    long result = 0;
    goto test;
loop:
    result += x & 0x1;
    x >>= 1;
test:
    if(x) goto loop;
    return result;
}
```

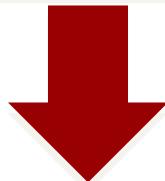
- Compare to do-while version of function
- Initial goto starts loop at test

General “While” Translation #2

While version

```
while (Test)
  Body
```

- “Do-while” conversion
- Used with `-O1`



Do-While Version

```
if (!Test)
  goto done;
do
  Body
  while(Test);
done:
```



Goto Version

```
if (!Test)
  goto done;
loop:
  Body
  if (Test)
    goto loop;
done:
```

While Loop Example #2

C Code

```
long pcount_while
(unsigned long x) {
    long result = 0;
    while (x) {
        result += x & 0x1;
        x >>= 1;
    }
    return result;
}
```

Do-While Version

```
long pcount_goto_dw
(unsigned long x) {
    long result = 0;
    if (!x) goto done;
loop:
    result += x & 0x1;
    x >>= 1;
    if(x) goto loop;
done:
    return result;
}
```

- Compare to do-while version of function
- Initial conditional guards entrance to loop

“For” Loop Form

General Form

```
for (Init; Test; Update)  
  
Body
```

```
#define WSIZE 8*sizeof(int)  
long pcount_for  
(unsigned long x)  
{  
    size_t i;  
    long result = 0;  
    for (i = 0; i < WSIZE; i++)  
    {  
        unsigned bit =  
            (x >> i) & 0x1;  
        result += bit;  
    }  
    return result;  
}
```

Init

```
i = 0
```

Test

```
i < WSIZE
```

Update

```
i++
```

Body

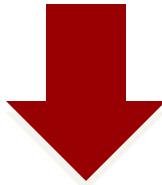
```
{  
    unsigned bit =  
        (x >> i) & 0x1;  
    result += bit;  
}
```

“For” Loop → While Loop

For Version

```
for (Init; Test; Update)
```

Body



While Version

```
Init;
```

```
while (Test) {
```

Body

Update;

```
}
```

For-While Conversion

Init

```
i = 0
```

Test

```
i < WSIZE
```

Update

```
i++
```

Body

```
{  
    unsigned bit =  
        (x >> i) & 0x1;  
    result += bit;  
}
```

```
long pcount_for_while  
(unsigned long x)  
{  
    size_t i;  
    long result = 0;  
    i = 0;  
    while (i < WSIZE)  
    {  
        unsigned bit =  
            (x >> i) & 0x1;  
        result += bit;  
        i++;  
    }  
    return result;  
}
```

“For” Loop Do-While Conversion

C Code

```
long pcount_for
(unsigned long x)
{
    size_t i;
    long result = 0;
    for (i = 0; i < WSIZE; i++)
    {
        unsigned bit =
            (x >> i) & 0x1;
        result += bit;
    }
    return result;
}
```

Goto Version

```
long pcount_for_goto_dw
(unsigned long x) {
    size_t i;
    long result = 0;
    i = 0;
    if (! (i < WSIZE))           Init
        goto done;                 ! Test
loop:
{
    unsigned bit =
        (x >> i) & 0x1;           Body
    result += bit;
}
i++; Update
if (i < WSIZE)           Test
    goto loop;
done:
    return result;
}
```

- Initial test can be optimized away

```
long switch_eg
    (long x, long y, long z)
{
    long w = 1;
    switch(x) {
        case 1:
            w = y*z;
            break;
        case 2:
            w = y/z;
            /* Fall Through */
        case 3:
            w += z;
            break;
        case 5:
        case 6:
            w -= z;
            break;
        default:
            w = 2;
    }
    return w;
}
```

Switch Statement Example

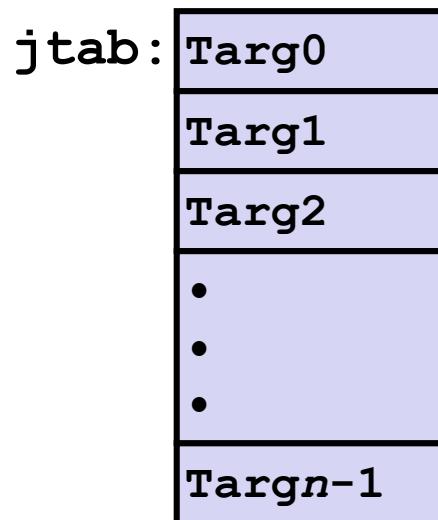
- Multiple case labels
 - Here: 5 & 6
- Fall through cases
 - Here: 2
- Missing cases
 - Here: 4

Jump Table Structure

Switch Form

```
switch(x) {  
    case val_0:  
        Block 0  
    case val_1:  
        Block 1  
        . . .  
    case val_{n-1}:  
        Block n-1  
}
```

Jump Table



Jump Targets

Targ0:

Code Block 0

Targ1:

Code Block 1

Targ2:

Code Block 2

•
•
•

Targ{n-1}:

Code Block n-1

Translation (Extended C)

```
goto *JTab[x];
```

Switch Statement Example

```
long switch_eg(long x, long y, long z)
{
    long w = 1;
    switch(x) {
        . . .
    }
    return w;
}
```

Setup:

`switch_eg:`

```
    movq    %rdx, %rcx
    cmpq    $6, %rdi    # x:6
    ja     .L8
    jmp    * .L4(,%rdi,8)
```

What range of values
takes default?

Register	Use(s)
%rdi	Argument x
%rsi	Argument y
%rdx	Argument z
%rax	Return value

Note that **w** not
initialized here

Switch Statement Example

```
long switch_eg(long x, long y, long z)
{
    long w = 1;
    switch(x) {
        . . .
    }
    return w;
}
```

Setup:

```
switch_eg:
    movq    %rdx, %rcx
    cmpq    $6, %rdi      # x:6
    ja     .L8           # Use default
    jmp    * .L4(,%rdi,8) # goto *JTab[x]
```

Jump table

```
.section  .rodata
.align 8
.L4:
.quad   .L8  # x = 0
.quad   .L3  # x = 1
.quad   .L5  # x = 2
.quad   .L9  # x = 3
.quad   .L8  # x = 4
.quad   .L7  # x = 5
.quad   .L7  # x = 6
```

*Indirect
jump*



Assembly Setup Explanation

- Table Structure
 - Each target requires 8 bytes
 - Base address at `.L4`
- Jumping
 - **Direct:** `jmp .L8`
 - Jump target is denoted by label `.L8`
 - **Indirect:** `jmp * .L4(,%rdi,8)`
 - Start of jump table: `.L4`
 - Must scale by factor of 8 (addresses are 8 bytes)
 - Fetch target from effective Address `.L4 + x*8`
 - Only for $0 \leq x \leq 6$

Jump table

```
.section    .rodata
.align 8
.L4:
.quad     .L8    # x = 0
.quad     .L3    # x = 1
.quad     .L5    # x = 2
.quad     .L9    # x = 3
.quad     .L8    # x = 4
.quad     .L7    # x = 5
.quad     .L7    # x = 6
```

Jump Table

Jump table

```
.section    .rodata
.align 8
.L4:
.quad      .L8  # x = 0
.quad      .L3  # x = 1
.quad      .L5  # x = 2
.quad      .L9  # x = 3
.quad      .L8  # x = 4
.quad      .L7  # x = 5
.quad      .L7  # x = 6
```

```
switch(x) {
    case 1:          // .L3
        w = y*z;
        break;
    case 2:          // .L5
        w = y/z;
        /* Fall Through */
    case 3:          // .L9
        w += z;
        break;
    case 5:
    case 6:          // .L7
        w -= z;
        break;
    default:         // .L8
        w = 2;
}
```

Code Blocks ($x == 1$)

```
switch(x) {  
    case 1: // .L3  
        w = y*z;  
        break;  
    . . .  
}
```

```
.L3:  
    movq    %rsi, %rax # y  
    imulq   %rdx, %rax # y*z  
    ret
```

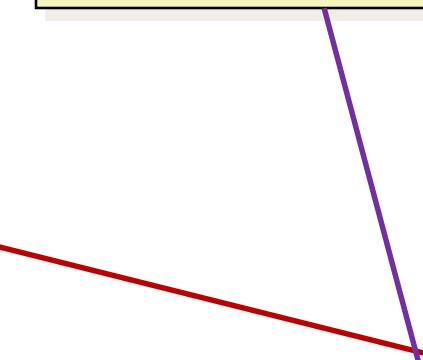
Register	Use(s)
%rdi	Argument x
%rsi	Argument y
%rdx	Argument z
%rax	Return value

Handling Fall-Through

```
long w = 1;  
.  
.  
switch(x) {  
.  
.case 2:  
    w = y/z;  
    /* Fall Through */  
case 3:  
    w += z;  
    break;  
.  
.  
}
```

```
case 2:  
    w = y/z;  
    goto merge;
```

```
case 3:  
    w = 1;  
  
merge:  
    w += z;
```



Code Blocks ($x == 2$, $x == 3$)

```
long w = 1;  
.  
.  
switch(x) {  
.  
. . .  
case 2:  
    w = y/z;  
    /* Fall Through */  
case 3:  
    w += z;  
    break;  
.  
.  
}
```

```
.L5:          # Case 2  
    movq    %rsi, %rax  
    cqto  
    idivq   %rcx      # y/z  
    jmp     .L6        # goto merge  
.L9:          # Case 3  
    movl    $1, %eax    # w = 1  
.L6:          # merge:  
    addq    %rcx, %rax # w += z  
    ret
```

Register	Use(s)
%rdi	Argument x
%rsi	Argument y
%rdx	Argument z
%rax	Return value

Code Blocks ($x == 5$, $x == 6$, default)

```
switch(x) {  
    . . .  
    case 5: // .L7  
    case 6: // .L7  
        w -= z;  
        break;  
    default: // .L8  
        w = 2;  
}
```

```
.L7:                      # Case 5,6  
    movl $1, %eax      # w = 1  
    subq %rdx, %rax   # w -= z  
    ret  
.L8:                      # Default:  
    movl $2, %eax      # 2  
    ret
```

Register	Use(s)
%rdi	Argument x
%rsi	Argument y
%rdx	Argument z
%rax	Return value

Summarizing

- C Control
 - if-then-else
 - do-while
 - while, for
 - switch
- Assembler Control
 - Conditional jump
 - Conditional move
 - Indirect jump (via jump tables)
 - Compiler generates code sequence to implement more complex control
- Standard Techniques
 - Loops converted to do-while or jump-to-middle form
 - Large switch statements use jump tables
 - Sparse switch statements may use decision trees (if-elseif-elseif-else)