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① . $\frac{dz}{dx} = a, \quad \frac{dz}{dy} = b. \quad v = \begin{bmatrix} a \\ b \end{bmatrix}$

② . $v = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_N \end{bmatrix}.$

③ . $f_x(x,y) = 2A(x - x_0)$

$f_y(x,y) = 2B(y - y_0)$

④ . $x^T = [3 \ 1 \ 4].$

$y^T = \begin{bmatrix} 2 \\ 5 \\ 1 \end{bmatrix}.$

$B^T = \begin{bmatrix} 3 & 5 & 1 \\ 5 & 2 & 4 \end{bmatrix}$

$x \cdot x = \begin{bmatrix} 9 \\ 1 \\ 16 \end{bmatrix}.$

$x \cdot y^T = \begin{bmatrix} 6 \\ 5 \\ 4 \end{bmatrix}.$

$x \times y$ not-defined

$y \times x = 2\times3 + 1\times5 + 1\times4 = 15$

$A \times x = \begin{bmatrix} 25 \\ 30 \\ 34 \end{bmatrix}$

$A \times B = \begin{bmatrix} 39 & 38 \\ 19 & 37 \\ 41 & 50 \end{bmatrix}$

B. reshape(1,6) $= [3 \ 5 \ 5 \ 2 \ 1 \ 4].$

Linear least squares: Single-variable

$y = M(x|p) = mx + b.$

objective := $\arg\min \sum_{i=1}^{n} (y_i - mx_i - b)^2 \Rightarrow J(m,b) = \sum_{i=1}^{n} (y_i - mx_i - b)^2$

$\nabla_m J(m,b) = -2\sum_{i=1}^{n} (y_i - mx_i - b)x_i = -2\sum_{i=1}^{n} x_i y_i + 2m\sum_{i=1}^{n} x_i^2 + 2b \cdot \sum_{i=1}^{n} x_i$

$\nabla_b J(m,b) = -2\sum_{i=1}^{n} (y_i - mx_i - b) = -2\sum_{i=1}^{n} y_i + 2m\sum_{i=1}^{n} x_i + 2nb.$

Set $\begin{cases} \nabla_m J(m,b) = 0 & \Rightarrow \sum_{i=1}^{n} y_i = m\sum_{i=1}^{n} x_i + nb. \Rightarrow \underline{\bar{y} = m\bar{x} + b.} \\ \nabla_b J(m,b) = 0. & \Rightarrow \sum_{i=1}^{n} x_i y_i = m\sum_{i=1}^{n} x_i^2 + b\sum_{i=1}^{n} x_i \end{cases}$

$$m = \frac{\sum_{i=1}^{n} (x_i y_i - \bar{y} x_i)}{\sum_{i=1}^{n} (x_i^2 - \bar{x} x_i)} = \frac{\sum_{i=1}^{n} x_i y_i - \frac{1}{n} \cdot (\sum_{i=1}^{n} x_i) \cdot (\sum_{i=1}^{n} y_i)}{\sum_{i=1}^{n} x_i^2 - \frac{1}{n} (\sum_{i=1}^{n} x_i)(\sum_{i=1}^{n} x_i)}$$

$$= \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n} (x_i - \bar{x})^2} = \frac{\mathrm{cov}(x,y)}{\mathrm{Var}(x)}$$

$$b = \bar{y} - \frac{\mathrm{cov}(x,y)}{\mathrm{var}(x)} \cdot \bar{x}$$

Multi-variables.

Define $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_p x_p + \varepsilon.$

$$y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} \quad X = \begin{bmatrix} 1 & x_{11} & \cdots & x_{1p} \\ 1 & x_{21} & & x_{2p} \\ \vdots & \vdots & & \vdots \\ 1 & x_{n1} & & x_{np} \end{bmatrix}$$

$Q(\beta_0, \beta_1, \cdots, \beta_p) = \sum_{i=1}^{n} (y_i - \beta_0 - \beta_1 x_{i1} - \cdots - \beta_p x_{ip})^2$

Derivatives := 0.

$$\begin{cases} \sum (y_i - \beta_0 - \beta_1 x_{i1} - \cdots - \beta_p x_{ip}) = 0. \\ \sum (y_i - \beta_0 - \beta_1 x_{i1} - \cdots - \beta_p x_{ip}) x_{i1} = 0 \\ \cdots \\ \sum (y_i - \beta_0 - \beta_1 x_{i1} - \cdots - \beta_p x_{ip}) x_{ip} = 0. \end{cases} \Rightarrow e^T [1 \ X_1 \ X_2 \cdots X_n] = 0$$

$e^T X = 0.$

$e^T = y - X\hat{\beta}$: $(y - X\hat{\beta})X = 0.$

$y^T X = \hat{\beta}^T X^T X.$

$X^T y = X^T X \hat{\beta} \Rightarrow \hat{\beta} = (X^T X)^{-1} X^T y.$