Parker Wind

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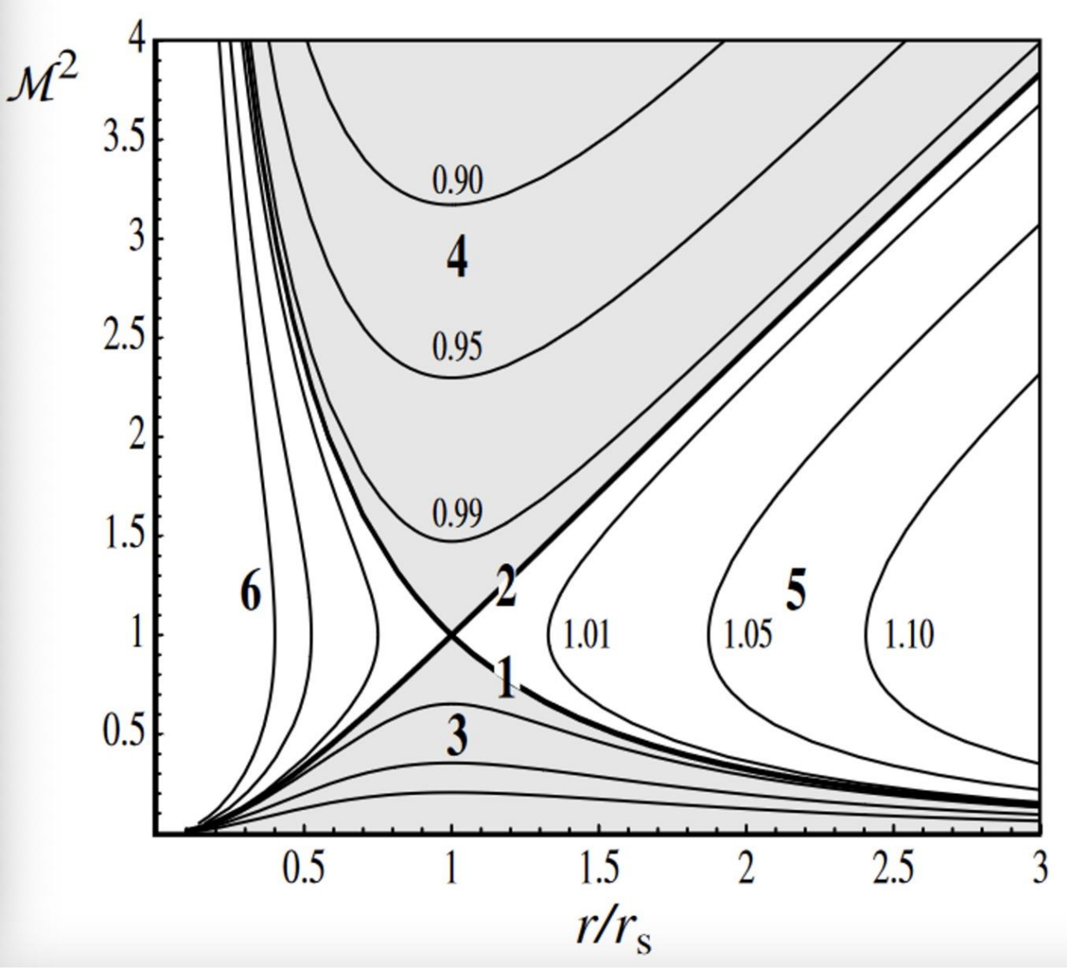
‘*Dynamics of the Interplanetary Gas and Magnetic Fields*’ by E.N.Parker

Observations give:

Gas is often flowing radially outward in all directions from the Sun with velocities ranging from $500 - 1500 km/s$.

Plot the figure $\mathcal{M}^2 = \frac{v^2(r)}{c_s^2(r)}$ - $\frac{r}{r_s}$



Type 1 : *Bondi Accretion*

$$v^2(r_s) = c_s^2(r_s), v \rightarrow 0 \; as \; r \rightarrow \infty$$
$$v^2 < c_s^2, r > r_s$$
$$v^2 > c_s^2, r < r_s$$

Type 2 : *Parker Wind*

$$v^2(r_s) = c_s^2(r_s), v \rightarrow 0 \; as \; r \rightarrow 0$$
$$v^2 < c_s^2, r < r_s$$
$$v^2 > c_s^2, r > r_s$$

Static Equilibrium

High temperature of solar corona

$\downarrow$

fully ionized gas

$\downarrow$

total gas pressure:

$$p(r) = 2NkT$$

For static equilibrium, the barometric relation:

$$\frac{d}{dr}(2NkT) + \frac{GM_{\odot}MN}{r^2} = 0$$

$$\text{thermal conductivity } \kappa(T) \begin{cases} \approx 5 \times 10^{-7} T^n \ ergs/cm^2 \sec^o K, \ n = \frac{5}{2}, for\ ionized\ hyrdrogen \\ \approx 2.5 \times 10^3 T^n \ ergs/cm^2 \sec^o K, \ n = \frac{1}{2}, for\ neutral\ hydrogen \end{cases}$$

Sufficiently far from the Sun, we have the steady state heat flow:

$$\nabla \cdot [\kappa(T) \nabla T] = 0$$

requires that $T(r)$ fall off with r as $\mathrm{r}^{-\frac{1}{n+1}}$.
Suppose there are no sources of coronal heat beyond the outer corona boundary $r = a$:

$$T(r) = T_0 \left(\frac{a}{r}\right)^{1/(n+1)}$$

Integrate the barometric relation:

$$\frac{d}{dr}(2NkT) + \frac{GM_{\odot}MN}{r^2} = 0$$

$$N(r) = N_0\left(\frac{r}{a}\right)^{1/(n+1)} exp\left\{\left[\frac{\lambda(n+1)}{n}\right]\left[\left(\frac{a}{r^2}\right)^{n/(n+1)} - 1\right]\right\}$$

$$\lambda = \frac{GMM_{\odot}}{2kT_0 a}$$

$$N(r) \sim N_0\left(\frac{r}{a}\right)^{1/(n+1)} \exp[-\frac{\lambda(n+1)}{n}]$$

For $n > 0$:

$N(r)$ becomes infinite as r increases.

For $n < 0$:

$N(r)$ vanishes at infinity.

For $n = 0$:

$$N(r) = N_0\left(\frac{a}{r}\right)^{\gamma-1}$$

When $n \neq 0$:

$$p(\mathrm{r}) = 2NkT$$

$$p(r) = p_0 \exp\{[\frac{\lambda(n+1)}{n}][\left(\frac{a}{r}\right)^{\frac{n}{n+1}} - 1]\}$$

When $n = 0$:

$$p(r) = p_0\left(\frac{a}{r}\right)^{\lambda}$$

Now we've got $T(r), N(r), p(r)$.

With the temperature varying as in $T(r)$ and with n at least as large as $\frac{1}{2}$ for neutral hydrogen, we have the non-vanishing pressure at infinity:

$$p(\infty) = p_0 \exp[\frac{-\lambda(n+1)}{n}]$$

No general pressure at infinity could balance $p(\infty)$ (interstellar gas pressure), we conclude that probably it is not possible for the solar corona, or, indeed, perhaps the atmosphere of any star, to be in complete hydrostatic equilibrium out to large distances.

Hence there is no hydrostatic equilibrium solution with vanishing pressure at infinity.

Stationary Expansion

The equation of motion:

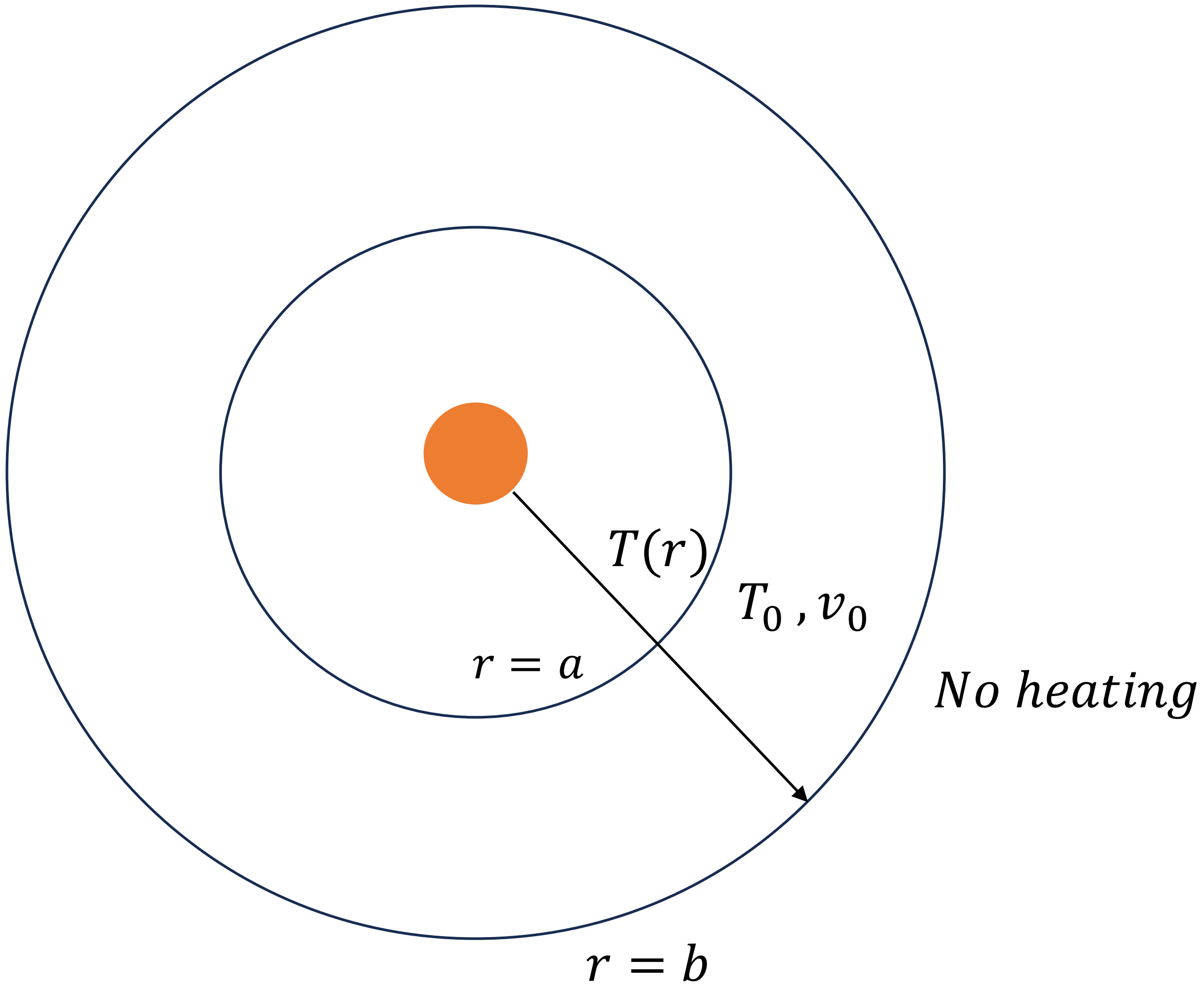
$$NMv\frac{dv}{dr} = -\frac{d}{dr}(2NkT) - GNMM_{\odot}\frac{1}{r^2}$$

The equation of continuity:

$$\frac{d}{dr}(r^2Nv) = 0$$

suppose that the corona is spherically symmetric:

$$N(r)v(r) = N_0v_0\left(\frac{a}{r}\right)^2$$



$T(r)$
T_0 , v_0
$r = a$
No heating
$r = b$

Introduce dimensionless variables:

$$\begin{cases} \xi = \dfrac{r}{a} \\ \tau = \dfrac{T(r)}{T_0} \\ \lambda = \dfrac{GMM_\odot}{2akT_0} \\ \psi = \dfrac{\frac{1}{2}Mv^2}{kT_0} \end{cases}$$

The equation of motion becomes:

$$\frac{d\psi}{d\xi}\left(1-\frac{\tau}{\psi}\right)=-2\xi^2\frac{d}{d\xi}\left(\frac{\tau}{\xi^2}\right)-\frac{2\lambda}{\xi^2}$$

At $a < r < b, \tau = 1,$ the equation yields:

$$\psi - ln\psi = \psi_0 - ln\psi_0 + 4ln\xi - 2\lambda(1-\frac{1}{\xi})$$

At $r = a, \xi = 1 \rightarrow \psi = \psi_0$.

At $r > b, \tau \approx 0$:

$$\psi(\xi) = \psi\left(\frac{b}{a}\right) - \lambda(\frac{a}{b}-\frac{1}{\xi})$$

For steady outward flow v_0, T_0, we let:

$$Y = 4ln\xi - 2\lambda(1 - \frac{1}{\xi})$$

$$Z = \psi - ln\psi$$

At $a < r < b, \tau = 1$:

$$Z = \psi_0 - ln\psi_0 + Y$$

Since the gas start at $r = a$ has already supersonic velocity, we consider the value of ψ at $\xi = 1$ is rather less than unity.

Thus Y, Z both decrease from their initial values at $\xi = 1$.

$$Y = 4ln\xi - 2\lambda(1 - \frac{1}{\xi})$$

$$Z = \psi - ln\psi$$

Y reaches a minimum at $\xi = \frac{\lambda}{2}$

Z reaches a minimum at $\psi = 1$

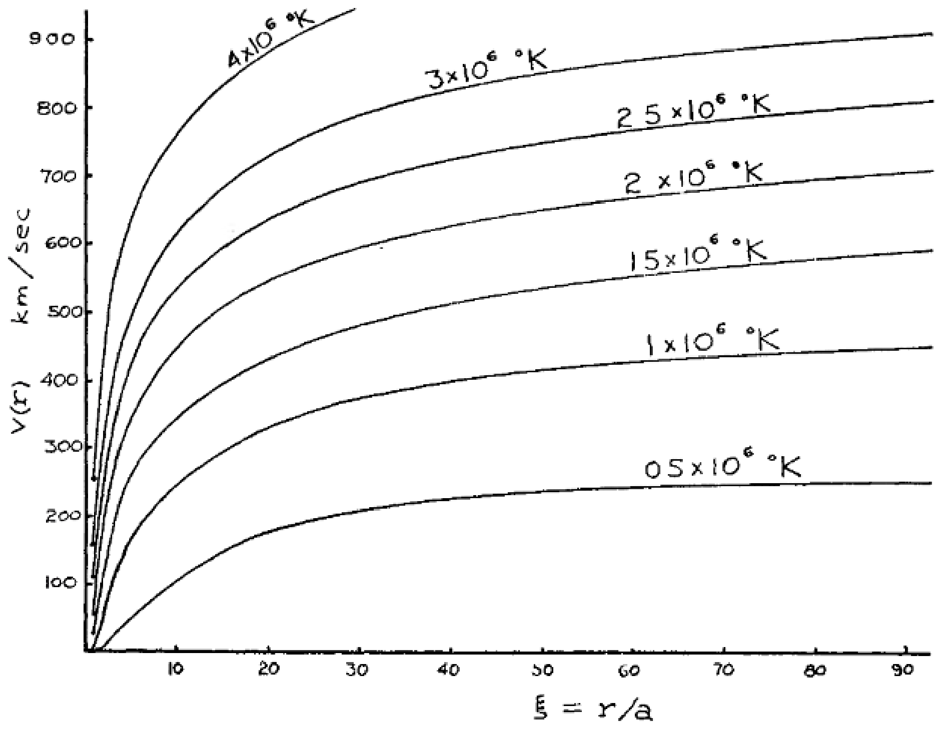
To get real and positive ψ, we must have $\psi = 1$ at $\xi = \frac{\lambda}{2}$:

$$\psi_0 - ln\psi_0 = 2\lambda - 3 - 4\ln\frac{\lambda}{2}$$

It yields a constant values of ψ_0 (or v_0) such that the outward expansion of gas is steady.

Thus at $a < r < b, \tau = 1,$ the equation yields:

$$\psi - ln\psi = -3 - 4\ln\frac{\lambda}{2} + 4ln\xi + \frac{2\lambda}{\xi}$$



$v(\xi)\ for\ various\ T_0$

This effect is result of spherical expansion instead of one –dimensional.

Consider the general equation of continuity in $n -$ dimensions :

$$N(r)v(r) = N_0 v_0 \left(\frac{a}{r}\right)^2$$

New equation of motion yields:

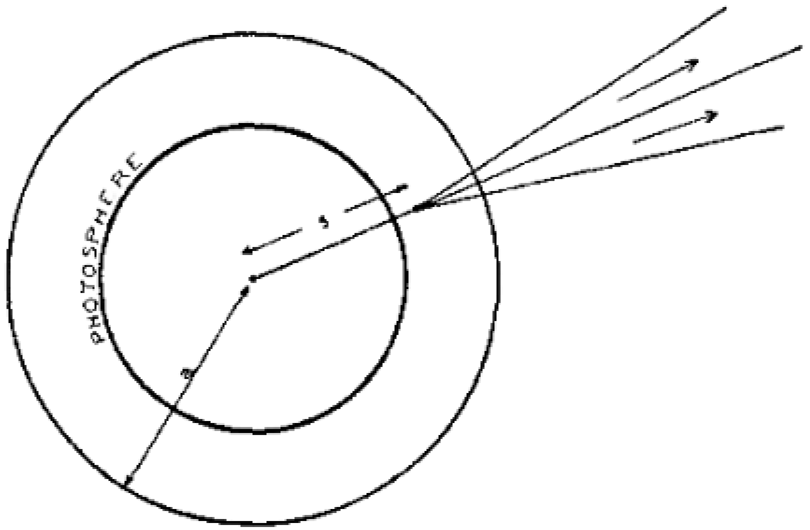
$$\psi - ln\psi = \psi_0 - ln\psi_0 + 2(n-1)ln\xi - 2\lambda(1 - \frac{1}{\xi})$$

For 3-dimensions: $\psi - ln\psi = \psi_0 - ln\psi_0 + 4ln\xi - 2\lambda(1 - \frac{1}{\xi})$

For 1-dimension: $\psi - ln\psi = \psi_0 - ln\psi_0 - 2\lambda(1 - \frac{1}{\xi})$

since the R.H.S decreases with increasing ξ, then the L.H.S. must decrease with increasing ψ. Thus, the maximum values of ψ must be smaller than unity at $\xi = \infty$. So v is limited to be less than thermal velocity.

Consider a more local spherical expansion centered around an active region on the surface of the sun:



Continuity condition:

$$N(r)v(r) = N_0 v_0 \left(\frac{a-s}{r-s}\right)^2$$

the equation of motion yields:

$$\psi - ln\psi = \psi_0 - ln\psi_0 + 4\ln\left(\frac{\xi - \frac{s}{a}}{1 - \frac{s}{a}}\right) - 2\lambda(1 - \frac{1}{\xi})$$

And similarly, $\psi = 1$ at $\xi = \frac{\lambda}{4}\left[1 + \left(1 - \frac{8s}{\lambda a}\right)\right]^{\frac{1}{2}}$ where the both sides have the minimum values.

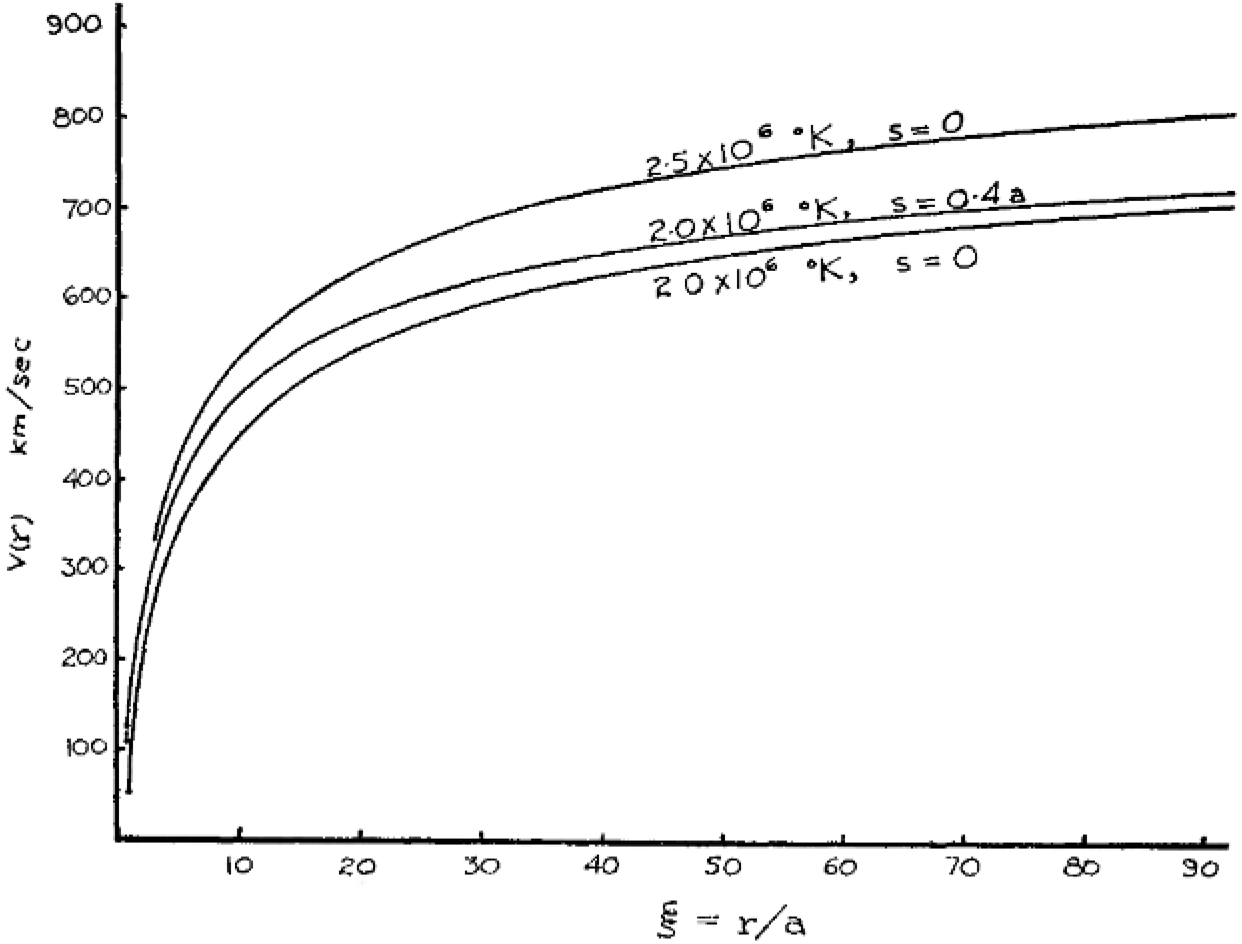
We must have

$$\left(1 - \frac{8s}{\lambda a}\right) > 0$$

$$\frac{s}{a} < \frac{\lambda}{8}$$

to make sure there is solution for the equation of motion
starting with $\psi < 1\ at\ \xi = 1$ and going to $\psi > 1\ at\ \xi = \infty$ because there is no minimum of the R.H.S. of the equation to match the L.H.S. at $\psi = 1$.

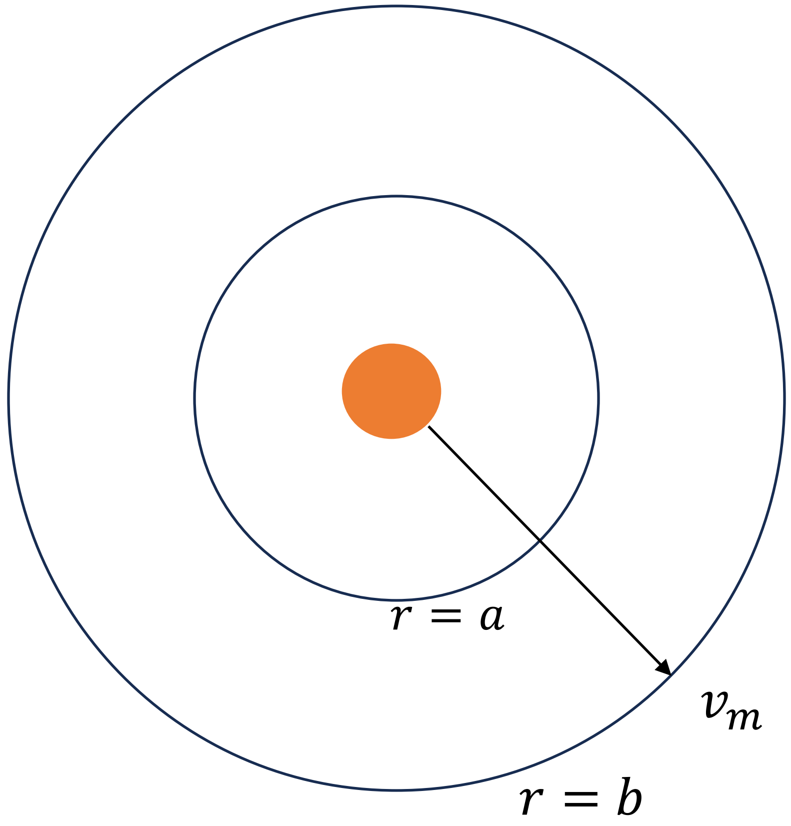
In 1-dimension: $\psi \leq 1$ if $\frac{s}{a} \geq \frac{\lambda}{8}$.



When $s > 0$, larger outward velocities result from the same temperature

Coronal Heating and Mass Loss

Suppose that the energy is transported from the photosphere out through the corona and is finally absorbed in thermal motions beyond $r = a$ to heat the coronal gas (Hydromagnetic wavs by Fermi acceleration of ions), and the corona is heated only out to $r = b$ where the velocity achieves v_m.



total energy transport at $r = b$:

$$e_b = \left(\frac{1}{2} N_m M v_m^3\right) \times (4\pi b^2)$$

at smaller r:

$$e_r = \left(\frac{1}{2} N M v^3\right) \times (4\pi r^2)$$

The energy flux transported by the coronal heating mechanism:

$$I(r) = e_b - e_r = \frac{1}{2} M \left(N_m v_m^3 \frac{b^2}{r^2} - N v^3 \right) erg/cm^3 \ sec$$

with

$$N(r)v(r) = N_0 v_0 \left(\frac{a}{r}\right)^2$$

we have

$$I(r) = N_0 v_0 \left(\frac{a}{r}\right)^2 kT_0(\psi_m - \psi)$$

If the rate of transport of thermal energy density has a characteristic velocity w, then the total transport is

$$I = 2wNkT + 2vNkT$$

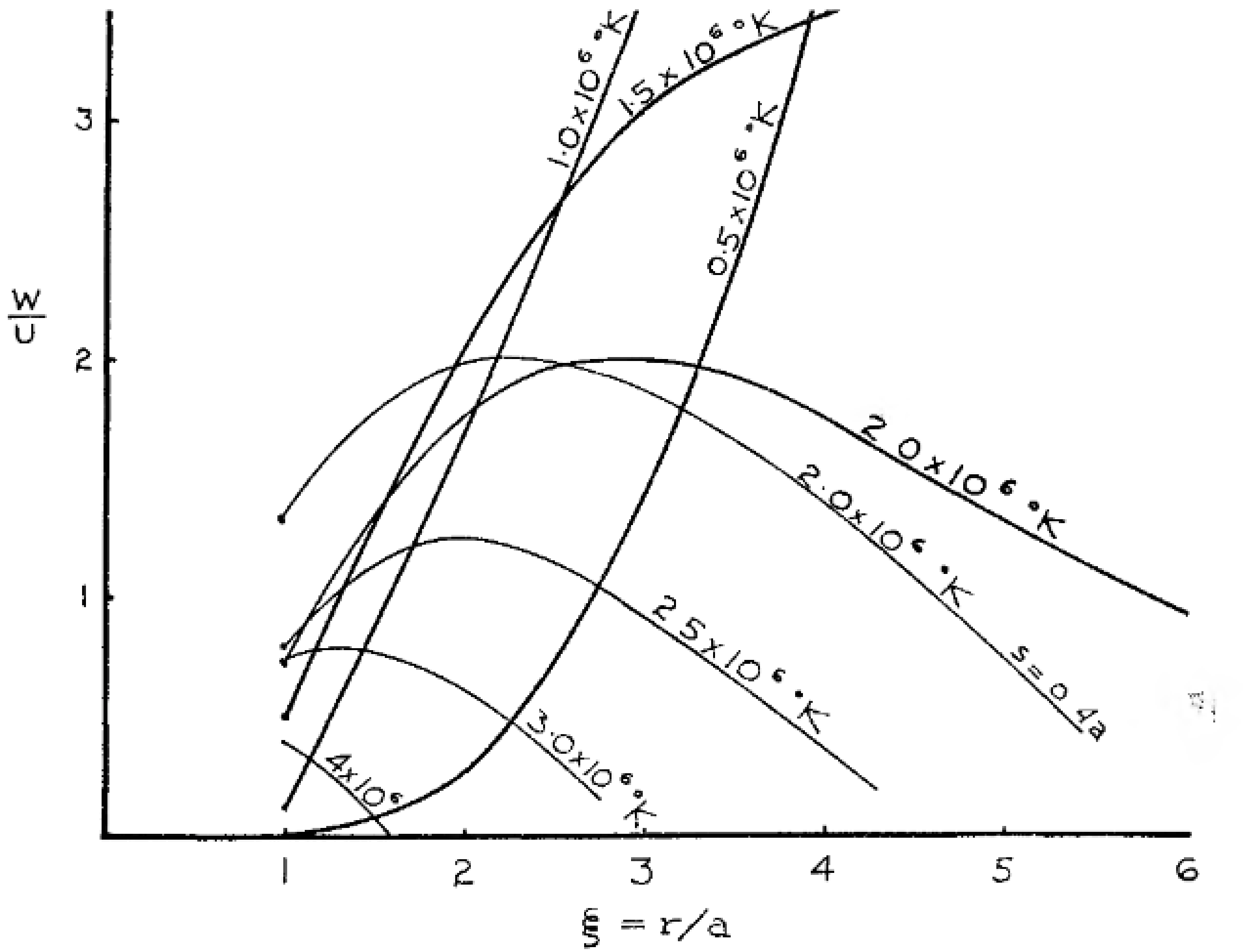
neglecting gravitational potential energy,

$$I = I(r)$$

$$w = v[\frac{1}{2}(\psi_m - \psi) - 1]$$

the ratio of w to the thermal ion velocity $u = \left(\frac{3kT_0}{M}\right)^{1/2}$ is:

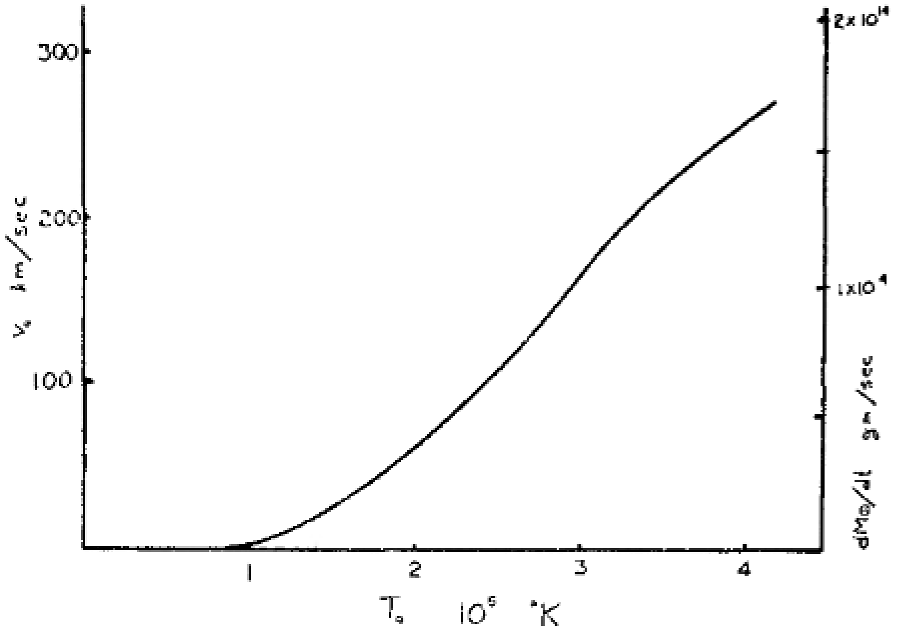
$$\frac{w}{u} = \left(\frac{3}{2}\right)^{\frac{1}{2}} \psi^{\frac{1}{2}}[\frac{1}{2}(\psi_m - \psi) - 1]$$



Ratio of the effective transport velocity to thermal velocity necessary for maintaining the indicated coronal temperatures

Consider the solar mass loss:

$$\frac{dM_{\odot}}{dt} = 4\pi a^2 N_0 M v_0$$



General Solar Magnetic Field

Observations suggests that there are no field-free regions in the sun.

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each cubic meter of gas flowing outward from the sun is threaded by magnetic lines of force from the main bulk of the sun.

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the outward-streaming gas, because it is ionized, will carry the imbedded lines of force with it. The lines, being imbedded in both the sun and the ejected gas, will be stretched out radially as the gas moves away from the sun.

↓

If beyond some distance $r = b$ from the sun the steady efflux of gas has some semblance of spherical symmetry, then after a time the lines of force will be entirely radial.

Consider a steady-state magnetic field resulting from a spherically symmetric outflow of gas from a rotating star. And suppose that beyond $r = b$ both the solar gravitation and outward acceleration by high coronal temperature may be neglected, so that the outward velocity is constant v_m:

$$v_r = v_m$$

$$v_\theta = 0$$

$$v_\phi = \omega(r - b) sin\theta$$

where ω is the angular velocity of the Sun.

The streamline with ϕ_0 at $r = b$ is :

$$\frac{r}{b} - 1 - \ln\left(\frac{r}{b}\right) = \frac{v_m}{b\omega}(\phi - \phi_0)$$

Consider only steady-state conditions and one end of each line of force is fixed in the Sun, then the lines of force of the magnetic field coincide with the streamlines.

$$\nabla \cdot \boldsymbol{B} = 0$$

$$B_r(r, \theta, \phi) = B(\theta, \phi_0)\left(\frac{b}{r}\right)^2$$

$$B_\theta(r, \theta, \phi) = 0$$

$$B_\phi(r, \theta, \phi) = B(\theta, \phi_0)\left(\frac{\omega}{v_m}\right)^2 (r - b)\left(\frac{b}{r}\right)^2 sin\theta$$

$B(\theta, \phi_0)$ represents the field at $r = b$.

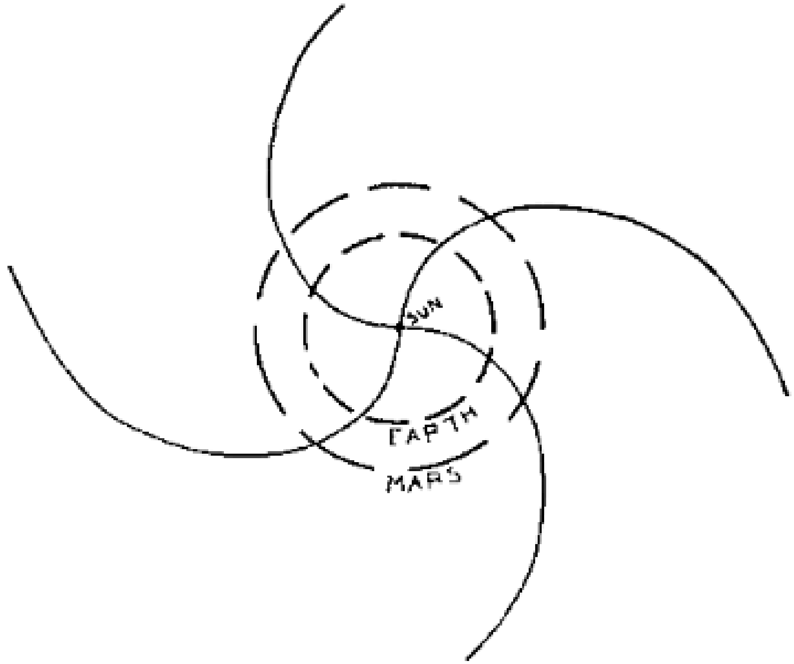
If only the solar dipole field threads the escaping gas:

$$B(\theta, \phi_0) = B_0 cos\theta$$

If there are more complex field of the equatorial regions participate:

$$B(\theta, \phi_0) = \sum A_{nm} P_n^m(cos\theta) cosm(\phi_0 - \phi_m)$$

However, their lines of force all follow the streamline equation.



$b = 5 \times 10^{11} cm, v_m = 1000 km/sec$

Streamline:

$$\frac{r}{b} - 1 - \ln\left(\frac{r}{b}\right) = \frac{v_m}{b\omega}(\phi - \phi_0)$$

The lines of force spiral more and more with increasing r.

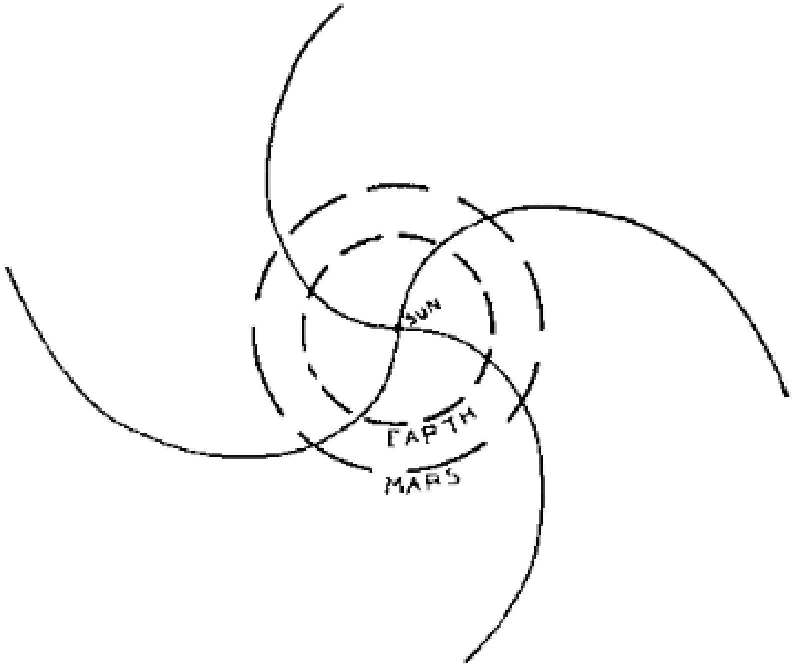
For small r:

$$B_\phi \cong 0$$

For large r:

$$\frac{B_\phi}{B_r} \sim \frac{\omega r sin\theta}{v_m}$$

which increases without limit.



The surface on which $B_\phi = B_r$ and each line of force makes an angle $\frac{\pi}{4}$ with the radius vector is :

$$r - b = \frac{v_m}{\omega} sin\theta$$

Since $v_m \gg \omega b$, it is the circular cylinder:

$$r \cong \frac{v_m}{\omega} sin\theta$$

Interplanetary Magnetic Field and Retardation of Solar Rotation

Plasma Instability and the Planetary Magnetic Shell