R Notebook

## Fourier Series

The Fourier series for a periodic function of any period is given by:

Where and are given by:

#### Example 1

A periodic function is given by:

* Period given is
* Limits for this period are

This periodic function can be plotted as a piecewise periodic function.

Using ***Ft.func*** the periodic function can be coded using ***ifelse*** commands.  
The ***Ft.func*** can then be plotted using the ***plot.piecewise.fnc***.

***plot.piecewise.fnc*** inputs:

* ***FUN = …***  Name given to the peridoic function eg: ***Ft.func***
* ***.piece.range = …***  Limits of the period eg: ***c(-2, 2)***
* ***.pieces = …***  Number of pieces to be included in the piecewise plot eg: ***c(-2,-1,0,1)*** (4 pieces)
* ***.resolution = …*** Number of points plotted eg: ***101***

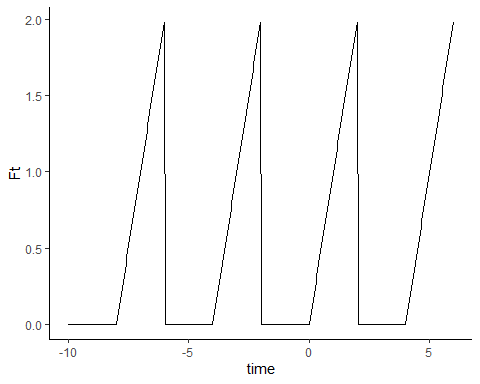
library(tidyverse)

## -- Attaching packages ------------------------------------------------------------------------------ tidyverse 1.2.1 --

## v ggplot2 3.2.0 v purrr 0.3.2  
## v tibble 2.1.3 v dplyr 0.8.3  
## v tidyr 0.8.3 v stringr 1.4.0  
## v readr 1.3.1 v forcats 0.4.0

## -- Conflicts --------------------------------------------------------------------------------- tidyverse\_conflicts() --  
## x dplyr::filter() masks stats::filter()  
## x dplyr::lag() masks stats::lag()

source("scr\_functions.R")  
Ft.func = function(t){  
 ifelse(t<0, 0, t)  
}  
  
plot.piecewise.fnc(.FUN = Ft.func, .piece.range = c(-2, 2), .pieces = c(-2,-1,0,1), .resolution = 101)

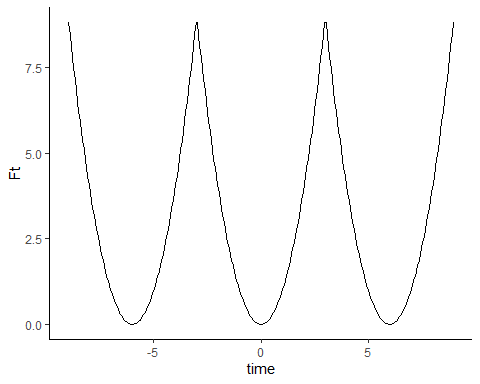


#### Example 2

A periodic function is given by:

* Period given is
* Limits for this period are

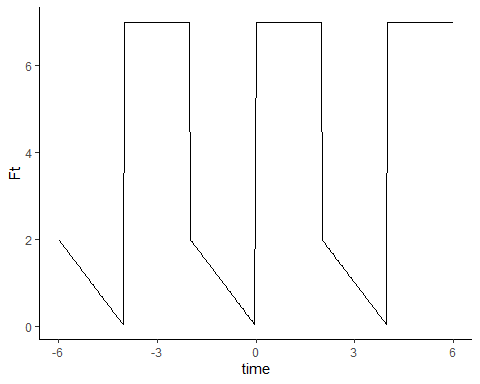
Ft.func2 = function(t){  
 t^2  
}  
  
plot.piecewise.fnc(.FUN = Ft.func2, .piece.range = c(-3, 3))

 \*\*\* #### Example 3

A periodic function is given by:

\* Period given is   
\* Limits for this period are

Ft.func3 = function(t){  
 ifelse(t<0, -t, 7)  
}  
  
plot.piecewise.fnc(.FUN = Ft.func3, .piece.range = c(-2, 2))

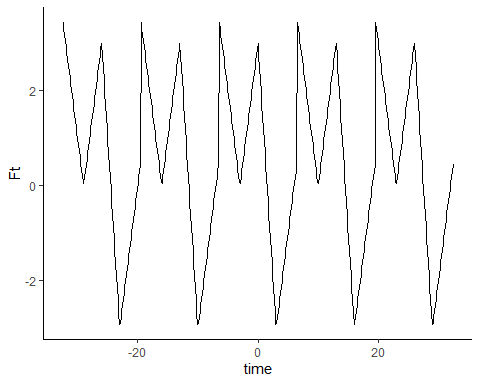
 \*\*\* #### Example 4

A periodic function is given by:

\* Period given is   
\* Limits for this period are

Periodic functions with mulitple definitions for different parts of the domain require more ***ifelse*** commands.

Ft.func4 = function(t){  
 ifelse(t<=-3, -t-3,  
 ifelse(t>-3 & t<=0, t+3,  
 ifelse(t>0 & t<=3, -2\*t+3,  
 t-6)))  
}  
  
plot.piecewise.fnc(.FUN = Ft.func4, .piece.range = c(-5, 8), .pieces = seq(-2,2))

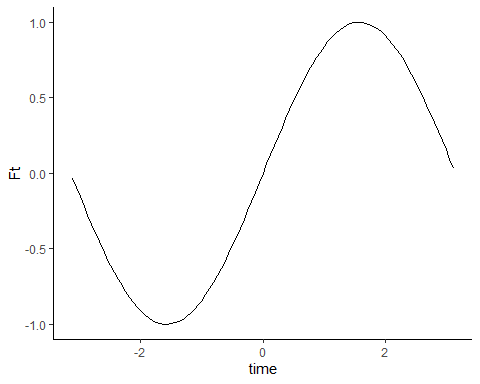


#### Example 5: Functions of period

The Fourier series for a periodic function of period has the general form:

A piecewise plot for the function can be seen:

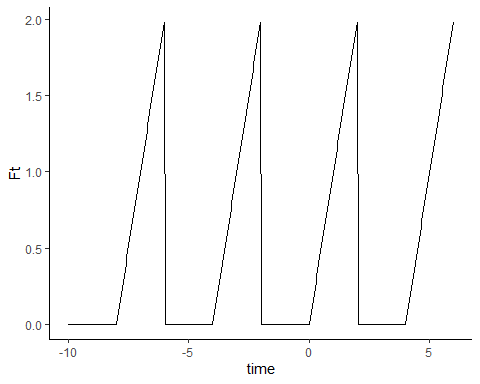
Ft.func5 = function(t){  
 sin(t)  
}  
  
plot.piecewise.fnc(.FUN = Ft.func5, .piece.range = c(0, 2\*pi), .pieces = 0)



#### Example 6

A periodic function (***example 1***) is given by:

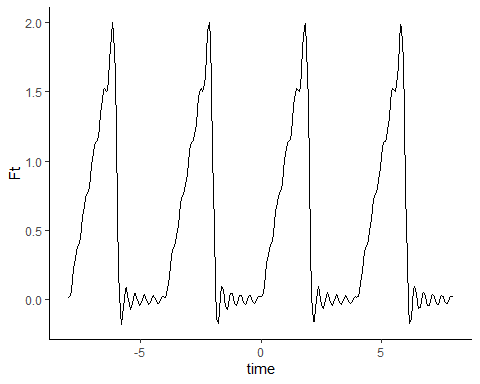
This periodic function is plotted as a piecewise periodic function:



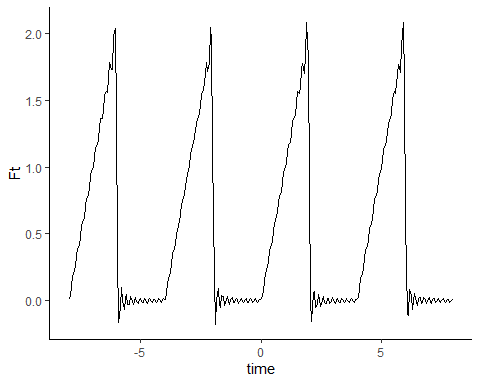
The Fourier Series including constants , and are calculated.

The constants are then coded using using ***ifelse*** commands.  
The Fourier series can then be plotted using the ***plot.fourier.series*** function.

a0 = 1  
an.fn = function(n) {  
 ifelse(as.logical(n%%2), -4/(n^2\*pi^2), 0)  
}  
  
bn.fn = function(n) {  
 2/(n\*pi) \* ifelse(as.logical(n%%2), 1, -1)  
}  
  
plot.fourier.series(.T = 4, .a0 = a0, .an.fun = an.fn, .bn.fun = bn.fn,  
 .t.range = c(-10,6), .n.max = 10, .resolution = 300)



Increasing ***.n.max*** (number of terms in Fourier Series) gives a closer approximation of

***.n.max = 20*** 

***.n.max = 30*** 