

Optimising T-Shirt distributioun

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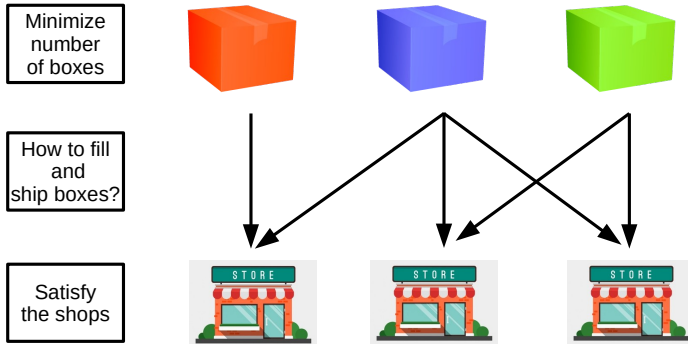
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October 10, 2018

Problem description

T-Shirt distribution problem

You are a T-Shirt distributor and want to optimise your packaging process when sending boxes of T-Shirts to stores.



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T-Shirt distribution problem

You are a T-Shirt distributor and want to optimise your packaging process when sending boxes of T-Shirts to stores.

- It is expensive to train your staff on packing T-Shirts into many different types of boxes.
- Each store requires a certain number of T-Shirts of given sizes.
- Stores will only accept receiving more T-Shirts than ordered in the case of medium and large Shirts in colors black, blue or red.
- Over-stocking is bad, but under-stocking is a lot (say 10 times) worse.
- You are supplied with boxes that fit 4, 6, 8 or 10 T-Shirts and can not ship partially filled boxes.
- You offer a total of 24 different T-Shirts.
(Six different sizes offered in four different colors)

Optimisation problem

Variables in the optimisation problem

Orders: $o_i \in \mathbb{N}_{\geq 0}^{24}$

Boxes: $b_j \in \mathbb{N}_{\geq 0}^{24}$, with $b_j^T \mathbf{1} \in \{4, 6, 8, 10\}$, for all $j \in 1, \dots, M$

Delivery plan: $\alpha_{(i,j)} \in \mathbb{N}_{\geq 0} \forall i, j$

Deliveries: $s_i = \sum_{j=1, \dots, M} \alpha_{(i,j)} b_j$ for all $j \in 1, \dots, 58$

Where $\alpha_{i,j}$ is the number of boxes of type j that store i will receive and M is the total number of different box-types used.

Formulation as an integer optimisation problem

$$\text{minimize: } \underbrace{\beta M}_{\text{Penalise number of boxes}} + \sum_{i=1}^{58} \underbrace{\mathbb{1}_{s_i > o_i} (s_i - o_i)}_{\text{Penalise overstocking}} + \underbrace{10 \cdot \mathbb{1}_{s_i < o_i} (o_i - s_i)}_{\text{Penalise understocking more}}$$

subject to:

$$M \in \mathbb{N}_{\geq 1}$$

$$\mathbf{b}_j \in \mathbb{N}_{\geq 0}^{24} \quad \forall j = 1, \dots, M$$

$$\alpha_{(i,j)} \in \mathbb{N}_{\geq 0}^{24} \quad \forall i = 1, \dots, 58, j = 1, \dots, M$$

$$s_i = \sum_{j=1, \dots, M} \alpha_{(i,j)} \mathbf{b}_j \leq o_i^{\text{limit}} \quad \forall i = 1, \dots, 58$$

- Note: Optimal solution depends on penalty parameter $\beta > 0$.
- In general integer optimisation problems are NP-Hard.

Algorithm 1 Brute-force algorithm

```
1:  $N_{max}$  = maximum possible number of box types
2: for all  $N \in \{1, 2, \dots, N_{max}\}$  do
3:   for all possible combinations of  $N$  box types do
4:     compute penalty given  $N$  and total over- and under-stock
5:   end for
6: end for
```

However, $N_{max} \sim 10^{11}$, so $\sim 2^{10^{11}} - 1$ combinations of box types to consider
 \Rightarrow brute force approach unfeasible

Initial Look at the Data

Our data

58 stores with 4 colour and each has 6 sizes

Overstock by 1 Medium or Large black, blue, or red t-shirt allowed in each store

Black						Blue					
XS	S	M	L	XL	2XL	XS	S	M	L	XL	2XL
1	2	2	3	4	1	1	2	2	5	3	1
1	2	4	3	2	1	1	2	3	3	4	1
1	2	2	5	2	1	1	2	2	5	3	1
1	2	2	3	4	1	1	2	4	3	3	1
1	2	4	3	2	1	1	2	3	3	4	1
1	2	4	3	2	1	1	2	3	4	3	1

Table: Examples from the data for black and blue t-shirt colours. Shops fall into one of three categories in black, and one of four categories in blue.

Initial Look at the Data

- Fundamental problem: Even number for all types of boxes vs some stores have odd sums → overstock is unavoidable
- Try find further structure: min, average, mode, sum subtract the mode subtract the min → small scale first → pattern inside each colour

