Optimising T-Shirt distributiuon

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Problem description

T-Shirt distribution problem

You are a T-Shirt distributor and want to optimise your packaging process when sending boxes of T-Shirts to stores.

- Each store requires a certain number of T-Shirts of given sizes.
- It is expensive to train your staff on packing T-Shirts into many different types of boxes. You are supplied with boxes that fit 4, 6, 8 or 10 T-Shirts and can not ship partially filled boxes.
- You offer a total of 24 different T-Shirts.
  (Six different sizes offered in four different colors)
- Stores will only accept receiving more T-Shirts than ordered in the case of medium and large Shirts in colors black, blue or red.
- Over-stocking is bad, but under-stocking is a lot (say 10 times) worse.

Optimisation problem

Variables in the optimisation problem

$$
\begin{aligned}
\text{Orders: } & \mathrm{o}_i \in \mathbb{N}_{\geqslant 0}^{24} \\
\text{Boxes: } & \mathrm{b}_j \in \mathbb{N}_{\geqslant 0}^{24}, \text{ with } \mathrm{b}_j^T 1 \in \{4, 6, 8, 10\}, \text{ for all } j \in 1, ..., M \\
\text{Delivery plan: } & \alpha_{(i,j)} \in \mathbb{N}_{\geqslant 0} \ \forall i, j \\
\text{Deliveries: } & \mathrm{s}_i = \sum_{j=1,\dots,M} \alpha_{(i,j)} \mathrm{b}_j \ \text{ for all } j \in 1, \dots, 58
\end{aligned}
$$

Where $\alpha_{i,j}$ is the number of boxes of type j that store i will receive and M is the total number of different box-types used.

Formulation as an integer optimisation problem

$$\text{minimize:} \quad \underbrace{\beta M}_{\text{Penalise number of boxes}} + \sum_{i=1}^{58} \underbrace{\mathbb{1}_{s_i > o_i}(s_i - o_i)}_{\text{Penalise overstocking}} + \underbrace{10 \cdot \mathbb{1}_{s_i < o_i}(o_i - s_i)}_{\text{Penalise understocking more}}$$

subject to:

$$M \in \mathbb{N}_{\geqslant 1}$$

$$b_j \in \mathbb{N}_{\geqslant 0}^{24} \ \forall \ j = 1, \ldots, M$$

$$\alpha_{(i,j)} \in \mathbb{N}_{\geqslant 0}^{24} \ \forall \ i = 1, \ldots, 58, \ j = 1, \ldots, M$$

$$s_i = \sum_{j=1,\ldots,M} \alpha_{(i,j)} b_j \leqslant o_i^{\text{limit}} \ \forall \ i = 1, \ldots, 58$$

- Note: Optimal solution depends on penalty parameter $\beta > 0$.
- In general integer optimisation problems are NP-Hard.

Algorithm 1 Brute-force algorithm

```
N_max = maximum possible number of box types
for all N ∈ {1, 2, ..., N_max} do
    for all possible combinations of N box types do
        compute penalty given N and total over- and under-stock
    end for
end for
```

However, $N_{max} \sim 10^{11}$, so $\sim 2^{10^{11}} - 1$ combinations of box types to consider
$\Rightarrow$ brute force approach unfeasible

Initial Look at the Data

Our data

58 stores with 4 colour and each has 6 sizes
Overstock by 1 Medium or Large black, blue, or red t-shirt allowed in each store

Black						Blue					
XS	S	M	L	XL	2XL	XS	S	M	L	XL	2XL
1	2	2	3	4	1	1	2	2	5	3	1
1	2	4	3	2	1	1	2	3	3	4	1
1	2	2	5	2	1	1	2	2	5	3	1
1	2	2	3	4	1	1	2	4	3	3	1
1	2	4	3	2	1	1	2	3	3	4	1
1	2	4	3	2	1	1	2	3	4	3	1

Table: Examples from the data for black and blue t-shirt colours. Shops fall into one of three categories in black, and one of four categories in blue.

Initial Look at the Data

- Fundamental problem: Even number for all types of boxes vs some stores have odd sums $\rightarrow$ overstock is unavoidable
- Try find further structure: min, average, mode, sum $\rightarrow$ subtract the mode $\rightarrow$ subtract the min $\rightarrow$ small scale first $\rightarrow$ pattern inside each colour