8.8 $\quad \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x_i-\mu)^2}{2\sigma^2}}$

5) $X \sim N(0, \sigma^2)$ $\quad \ell(x|\sigma^2) = \log(x|\sigma^2) =$

$I(\sigma^2)$

$$-\frac{\log 2\pi}{2} - \frac{1}{2}\log \sigma^2 - \frac{x^2}{2\sigma^2}$$

$$\ell'(x|\sigma^2) = \frac{-1}{2\sigma^2} + \frac{x^2}{2\sigma^4}$$

$\mathrm{Var}[\lambda'(x|\theta)]$. ou $-E[\ell''(x|\sigma^2)]$

$$\ell''(x|\sigma^2) = \frac{1}{2\sigma^4} - \frac{x^2}{2(\sigma^2)^3}$$

$$I(\sigma^2) = -E_{\sigma^2}[\ell''(x|\sigma^2)]$$

$$= \frac{-1}{2(\sigma^2)^2} + \frac{1}{(\sigma^2)^3} E[X^2]$$

$\hookrightarrow \mathrm{Var}(X) - E(X)^2$

$\hookrightarrow \sigma^2 \quad \hookrightarrow 0$

$$\frac{-1}{2\sigma^4} + \frac{\sigma^2}{\cancel{\sigma^6}\ \sigma^4} = \boxed{\frac{1}{2\sigma^4}}$$

7) $X_1, X_2, \ldots, X_n \sim Be(p)$

$\bar{X}_n$ é eficiente de p

$m(\theta) = E(\bar{X}_n) = \theta$

$m'(\theta) = 1$

$$Var(\bar{X}_n) = \frac{[m'(\theta)]^2}{n I(\theta)}$$

$$n I\theta = \frac{n}{\theta(1-\theta)}$$

$$\frac{Var(X_i)}{n}$$

$$= \frac{\theta(1-\theta)}{n}$$

$$\frac{\theta(1-\theta)}{n} = \frac{1}{n}\left(\theta(1-\theta)\right)$$

10) $$f(x|\mu, \sigma^2) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$X_1, \ldots, X_n \sim N(0, \sigma^2)$

$E_\theta[T] = \log \sigma = m(\sigma) \qquad m'(\sigma) = \sigma^{-1}$

$$\lambda(x|\sigma) = \log\left((\sqrt{2\pi}\sigma)^{-1} e^{\frac{x^2}{2\sigma^2}}\right)$$

$$= -\frac{1}{2}\frac{x^2}{\sigma^2} - \frac{1}{2}\left[\log 2\pi + \log \sigma^2\right]$$

$$\lambda'(x|\sigma) = -\frac{1}{2}\left[\frac{-x^2}{\sigma^3} + \frac{2}{\sigma}\right]$$

$$\lambda''(x|\sigma) = -\frac{1}{2}\left[\frac{6x^2}{\sigma^4} - \frac{2}{\sigma^2}\right]$$

$$-E[\lambda''(x|\sigma)] = \frac{1}{2}\left[\frac{6}{\sigma^4}\underbrace{E[x^2]}_{\sigma^2} - \frac{2}{\sigma^2}\right] = \frac{2}{\sigma^2}$$

$$Var \geq \frac{(\sigma^{-1})^2}{\cancel{\sigma^2}}$$

$$\geq \frac{1}{2n}$$

tilibra