(7 8)

3) Uma distribuição de Pareto
X_0 e α desconhecido

$T_1 = \min\{X_1, \dots, X_n\}$

$$f(x \mid x_0, \alpha) = \frac{\alpha x_0^{\alpha} \mathbb{1}\{x \geq x_m\}}{x_1^{\alpha+1}} \begin{cases} 1, & x \geq x_m \\ 0, & x < x_m \end{cases} \qquad T_2 = \prod_{i=1}^{m} x_i$$

$$f_m(\bar{x} \mid x_0, \alpha) = \left(\prod_{i=1}^{m} x_i\right)^{-(\alpha+1)} \cdot \alpha^m X_0^{\alpha m}$$

$$= t_2^{-(\alpha+1)} \cdot \alpha^m X_0^{\alpha m}$$

$$v[\overset{T_1}{r_1(x)}, \overset{T_2}{r_2(x)}; x_0, \alpha] = t_2^{-(\alpha+1)} \alpha^m X_0^{\alpha m} \; g(x_0, t_1),$$

$u(x) = 1$

$$\hookrightarrow \begin{cases} 1, & x_0 \leq t_1 \\ 0, & x_0 > t_1 \end{cases}$$

8) $x_1, x_2, \dots x_n$ formam uma amostra de uma distrib exponencial β desconhecido.
EMV é uma estatística suficiente.

$$f(x \mid \theta) = \theta e^{-\theta x}$$
$$L(\theta) = f_n(\bar{x} \mid \theta) = \theta^m e^{-\theta y}$$

$$\log L(\theta) = m \log \theta - \theta y$$
$$(\log L(\theta))' = 0$$
$$\frac{m}{\theta} - y = 0 \leftrightarrow \theta = \frac{m}{y} = \frac{1}{\bar{X}_m}$$

$T = 1/\bar{X}_m \qquad f(x;\beta) = \beta e^{-\beta x} \rightarrow f_m(x;\beta) = \beta^m e^{-\beta_m m \bar{X}_m}$

$$f_m(x \mid \beta) = \underbrace{\beta^m e^{-\beta m / T}} \quad v(r(x), \beta) \quad \text{e} \quad u(x) = 1$$

7.8.4?

(12) $X_1, X_2, \ldots, X_n$, com pdp: θ desconhecido

$$f(x|\theta) = \begin{cases} 2x/\theta^2, & 0 \leq x \leq \theta \\ 0, & c.c \end{cases}$$

MLE da mediana é mínimo seguinte

$$f_n(x|\theta) = \prod_{i=1}^{n} \frac{2x_i}{\theta^2} \cdot \mathbb{1}\{x \in [0,\theta]\}$$

$$= \frac{2^n \prod_{i=1}^{n} x_i \cdot \mathbb{1}\{x_1, \ldots, x_n \in [0,\theta]\}}{\theta^{2n}} \rightarrow \hat{\theta} = \max\{x_1, \ldots, x_n\}$$

$$P(X \leq m) = 1/2$$

$$\int_{-\infty}^{m} f(x|\theta)\,dx, \quad x_i > 0 \rightarrow \int_0^m \frac{2x}{\theta^2}dx = \frac{1}{2}$$

$$\left.\frac{x^2}{\theta^2}\right|_0^m = \frac{1}{2} \Leftrightarrow \frac{m^2}{\theta^2} = \frac{1}{2}$$

$$2m^2 = \theta^2$$

$$|m| = \left|\frac{1}{\sqrt{2}}\theta\right|$$

$$\boxed{\hat{m} = \frac{1}{\sqrt{2}}\theta}$$

$$f_n(x|\theta) = \underbrace{2^n\left(\prod_{i=1}^{n} x_i\right)}_{u(x)} \cdot \underbrace{\frac{1}{\theta^{2n}} \mathbb{1}\{\hat{\theta}_{MLE} < \theta\}}_{v(\hat{\theta}_{MLE}, \theta)}$$

(16) $X_1, X_2, \dots, X_n$ com distrib de Poisson média $= \lambda$ desconhecida

posteriori

$\alpha_1 = \alpha + \sum x_i$ $\beta_1 = \beta + n$

$\frac{\theta^x e^{-\theta}}{x!}$

$$E[\lambda | X] = \frac{\alpha + \overbrace{\sum x_i}^{T}}{\beta + n}$$

$$f(x|\lambda) = \frac{\lambda^x e^{-\lambda}}{x!} \rightarrow f_n(\bar{x}|\lambda) = \frac{\lambda^{\sum x_i} e^{-n\lambda}}{\prod_{i=1}^{n} x_i!} \quad \cap \{x_1, x_2 \dots x_n > 0\}$$

$$= \underbrace{\frac{1}{\prod_{i=1}^{n} x_i!}}_{u(x)} \cdot \underbrace{\lambda^{\overset{T}{\sum x_i}} e^{-n\lambda}}_{v(T, \lambda)}$$ É suficiente

O estimador de Bayes (função injetiva de T) também é suficiente.