7.3

(2) Mostre que $V \leq 0.01$ após 22 itens serem selecionados e $V > 0.01$ até serem selecionados 7 itens.

~~$\theta \sim Unif[0, 1]$~~
~~Tome N amostra~~

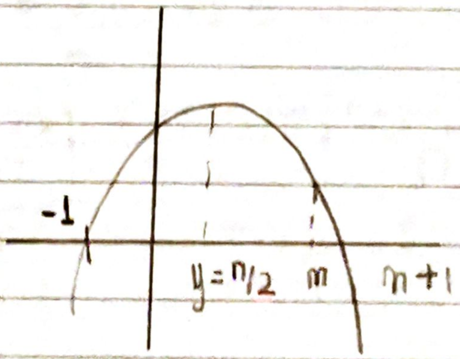
$$V = \frac{(Y+1)(Z+1)}{(Y+Z+2)^2\,(Y+Z+3)}$$

Y = defeituosas
Z = sem defeito $\Rightarrow n = y + 3$
n = nº de amostras $\Rightarrow z = n - 4$

$$V = \frac{(y+1)(n-4+1)}{(n+2)^2\,(n+3)} = \frac{-(y-(-1))\,(y-(n+1))}{(n+2)^2\,(n+3)}$$

$$V = \frac{f(y)}{(n+2)^2(n+3)}$$

$$f(y) = -(y-(-1))\cdot(y-(n+1))$$



$$\frac{f(0)}{(n+2)^2(n+3)} \leq V \leq \frac{f(n/2)}{(n+2)^2(n+3)}$$

$$\frac{n+1}{(n+2)^2(n+3)} \leq V \leq \frac{\left(\frac{n}{2}+1\right)\left(\frac{n}{2}+1\right)}{(n+2)^2\,(n+3)}$$

7.3 17)

O número de min que uma pessoa espera é uniforme $([0, \theta])$

$$\xi(\theta) = \begin{cases} \frac{192}{\theta^4}, & \theta \geq 4 \\ 0, & c.c \end{cases}$$

Observamos $x_1 = 5$, $x_2 = 3$ e $x_3 = 8$

Encontre a posteriori

$$\underbrace{\xi(\theta|x)}_{\text{posteriori}} \propto \underbrace{\xi(\theta)}_{\text{priori}} \underbrace{f_n(x|\theta)}$$

$X \sim Unif[0, \theta]$, $f(x|\theta) = \frac{1}{\theta}$, $x \in [0, \theta]$

$$f_n(\underset{\sim}{x}) = f(x_1|\theta) \cdot f(x_2|\theta) \cdot f(x_3|\theta) = \frac{1}{\theta^3}, \quad \theta \geq 8$$

$x_1 \in [0, \theta]$ $\quad$ $5 \in [0, \theta]$

$x_2 \in [0, \theta]$ $\quad$ $3 \in [0, \theta]$

$x_3 \in [0, \theta]$ $\quad$ $8 \in [0, \theta]$

0 3 5 8 θ

$$\xi(\theta|x) \propto \frac{1}{\theta^4} \cdot \frac{1}{\theta^3}, \quad \theta \geq 8$$

$$\xi(\theta|x) \propto \frac{1}{\theta^7}, \quad \theta \geq 8$$

$$\int_8^{+\infty} \xi(\theta|x) = 1 \Rightarrow c \int_8^{+\infty} \frac{1}{\theta^7} = 1 \qquad \Rightarrow c = 6 \cdot 8^6$$

$$c \left[\frac{1}{-6\theta^6}\right]_8^{+\infty} = 1 \Rightarrow c \cdot \frac{1}{6 \cdot 8^6} = 1$$

(19) Suponha $X_1, \ldots X_n$ amostras com

$$f(x|\theta) = \begin{cases} \theta x^{\theta-1}, & 1 > x > 0 \\ 0, & c.c \end{cases}$$

Como θ desconhecido e com dist priori Gamma(α, β)
Determine a méd e var a posteriori de θ.

$$\xi(\theta|x) \propto \xi(\theta) \cdot f_n(\underline{x}|\theta)$$

$$f_n(x|\theta) = \prod_{j=1}^{n} \theta x_j^{\theta-1} = \theta^n \left(\prod x_j\right)^{\theta-1}, \quad \prod_{j=1}^{n} x_j = P$$

$$\rightarrow \theta^n P^{\theta-1}$$

$$\xi(\theta|x) \propto \theta^{\alpha-1} e^{-\beta\theta} \cdot \theta^n \cdot P^{\theta-1}$$

$$= \theta^{(\alpha+1)-1} e^{-\beta\theta} P^{\theta-1}$$

$$= \theta^{(\alpha+n)-1} e^{-\beta\theta} (\underbrace{e^{\ln p}}_{P})^{\theta-1} = \theta^{(\alpha+n)-1} e^{\theta(\ln p-\beta)-\ln p}$$

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21) $x_1, x_2, \ldots x_n \sim \exp(\theta)$

$$\xi(\theta) = \frac{1}{\theta}, \ \theta > 0$$

Posterori = ?

$$\xi(\theta/x) \propto \xi(\theta)\, f_n(\underset{\sim}{x}/\theta)$$

$x \sim \exp(\theta)$

$$f(x_i/\theta) \propto \theta e^{-x_i\theta}$$

$$f_n(\underset{\sim}{x}/\theta) \propto \Pi\, \theta\, e^{-x_i\theta} = e^{-0.s} \cdot \theta^{n}$$

$$\xi(\theta/x) = \frac{1}{\theta} \cdot \theta^{n} e^{-0.s} = \theta^{n-1} e^{-0.s}$$

$\theta/x \sim$ Gamma (n, s)