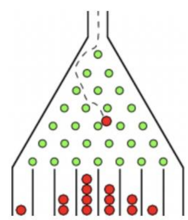
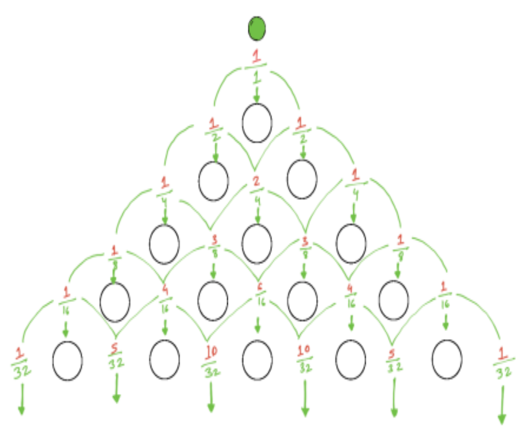
Quantum Walks and Monte Carlo

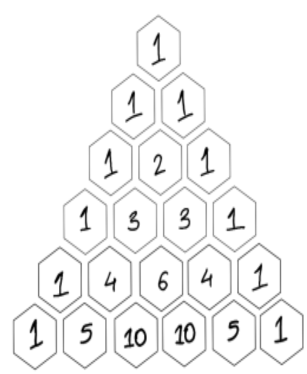
The paper introduces a Quantum Galton Board (QGB) — a quantum version of a classical device used to demonstrate randomness and probability called the Galton Board. To understand the Quantum Galton Board, it is essential to grasp the concept of the Galton Board first.

What is a Galton Board?



A Galton board is a triangular board with rows of pegs depicted in the figure above. At each peg, the ball has an equal probability of bouncing to the left or the right. When balls are dropped from the top, they randomly bounce left or right at each peg. Over time, the balls accumulate in bins at the bottom, forming a bell-shaped curve.



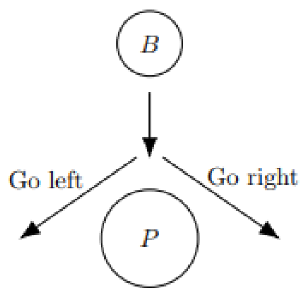


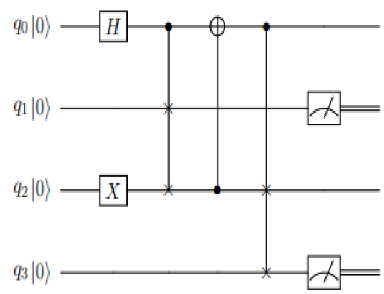
The number of paths a ball can take to reach any given peg on the Galton board can be represented by the numbers in Pascal's triangle. Each number in the triangle shows the total number of unique routes to a corresponding peg. Using those numbers, we can calculate the probability of a ball landing in each bin. The highest numbers in the middle of Pascal's triangle correspond to the greatest number of paths, meaning a ball has a higher chance of landing in the central bins. This probability decreases towards the edges, creating the bell-shaped curve that represents a normal distribution.

Quantum Galton Board

The approach to building a Quantum Galton Board (QGB) is to model the behavior of an individual classical board's peg. In a classical Galton board, a ball hits a peg and has a 50% chance of going left or right. A quantum circuit is designed to model the action of a single peg, and this basic unit is then replicated to build the entire system of the Galton board.

So, let's understand the design of the quantum circuit for a single peg.





In a single peg on a Galton board, a ball is dropped, hits a peg, and with a 50/50 chance, goes either left or right before landing in one of the two bins below. Let's see how this is mimicked on a quantum circuit.

We use three working qubits (q1, q2, and q3) and one control qubit (q0), as can be seen in the figure. q0 decides whether to push the ball left or right, and thanks to quantum mechanics, it allows both outcomes to happen at once**.** The control qubit (q_0) is put into a superposition with a Hadamard gate, mimicking the 50/50 chance of a ball bouncing left or right. An X-gate places a "ball" on the middle qubit (q_2), marking its starting position. Controlled SWAP gates, with q_0 as the controller, then enable the ball to move left or right to qubits q_1 and q_3 in a superposition—simulating both potential outcomes at once. To ensure consistent operation, an inverted CNOT gate is used to stabilize the control qubit. The circuit concludes with measurements on qubits q_1 and q_3, revealing the ball's final position in one of two bins, just as a classical ball would. This entire unit can be duplicated to construct a full-scale quantum Galton board.

In a Quantum Galton Board (QGB), you can create different distributions by introducing a bias, which is analogous to slanting the pegs on a classical board. On a QGB, this bias is introduced by replacing the Hadamard gate with a rotation gate.

The Hadamard gate creates a perfect 50/50 superposition, which represents an unbiased choice of bouncing left or right. A rotation gate, on the other hand, allows you to set a specific probability for each outcome. By carefully choosing the angle of rotation, you can make the "ball" (the qubit) more likely to move left or right, thereby biasing its path.By adjusting the rotation angles for each "peg" in the quantum circuit, you can fine-tune the final probability distribution of the ball's position. This allows the QGB to generate a wide variety of distributions beyond just the normal distribution, giving you precise control over the output.

Note: The figures are from the paper "Universal Statistical Simulator"

Arxiv link: https://arxiv.org/abs/2202.01735