

Phase kickback

We Start with,
 $|0\rangle$, $|1\rangle$

known
as
Up
qubit

Down
qubit

Apply Hadamard,

$$\left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right), \quad \left(\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle\right)$$

Composite System,

$$\begin{aligned} & \left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right) \otimes \left(\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle\right) \\ &= \frac{1}{\sqrt{2}}|0\rangle \otimes \left(\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle\right) + \frac{1}{\sqrt{2}}|1\rangle \otimes \left(\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle\right) \end{aligned}$$

This

Applying CNOT,

$$\begin{aligned} & \frac{1}{\sqrt{2}}|0\rangle \otimes \left(\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle\right) + \frac{1}{\sqrt{2}}|1\rangle \otimes \left(\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle\right) \\ &= \frac{1}{\sqrt{2}}|0\rangle \otimes \left(\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle\right) - \frac{1}{\sqrt{2}}|1\rangle \otimes \left(\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle\right) \end{aligned}$$

$$= \left(\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle\right) \otimes \left(\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle\right)$$

This

Before CNOT, the sign of $|1\rangle$ in up qubit is positive
 After CNOT, the sign of $|1\rangle$ in up qubit is negative

This is called phase kickback.

i.e. the phase (negative sign) from $|1\rangle$ of the down qubit has been kicked back to the phase of $|1\rangle$ of the up qubit.

In short,

$|0\rangle \otimes |1\rangle$
 Apply H on both

$|+\rangle \otimes |-\rangle$

Apply CNOT(first qubit, second qubit)

$|-\rangle \otimes |+\rangle$