

Phase Kickback

We start with, $|0\rangle$, $|1\rangle$
known as Up qubit Down qubit

Apply Hadamard,

$$\left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right), \left(\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle\right)$$

Composite System,

$$\left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right) \otimes \left(\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle\right)$$

← This

$$= \frac{1}{\sqrt{2}}|0\rangle \otimes \left(\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle\right) + \frac{1}{\sqrt{2}}|1\rangle \otimes \left(\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle\right)$$

Applying CNOT,

$$\frac{1}{\sqrt{2}}|0\rangle \otimes \left(\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle\right) + \frac{1}{\sqrt{2}}|1\rangle \otimes \left(\frac{1}{\sqrt{2}}|1\rangle - \frac{1}{\sqrt{2}}|0\rangle\right)$$

$$= \frac{1}{\sqrt{2}}|0\rangle \otimes \left(\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle\right) - \frac{1}{\sqrt{2}}|1\rangle \otimes \left(\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle\right)$$

$$= \left(\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle\right) \otimes \left(\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle\right)$$

← This

Before CNOT, the sign of $|1\rangle$ in up qubit is positive

After CNOT, the sign of $|1\rangle$ in up qubit is negative

This is called phase kickback.

i.e. the phase (negative sign) from $|1\rangle$ of the down qubit has been kicked back to the phase of $|1\rangle$ of the up qubit.

In short,

$$|0\rangle \otimes |1\rangle$$

Apply H on both

$$|+\rangle \otimes |-\rangle$$

Apply CNOT (first qubit, second qubit)

$$|-\rangle \otimes |-\rangle$$