DECISION TREE

Dr. Hilman F. Pardede

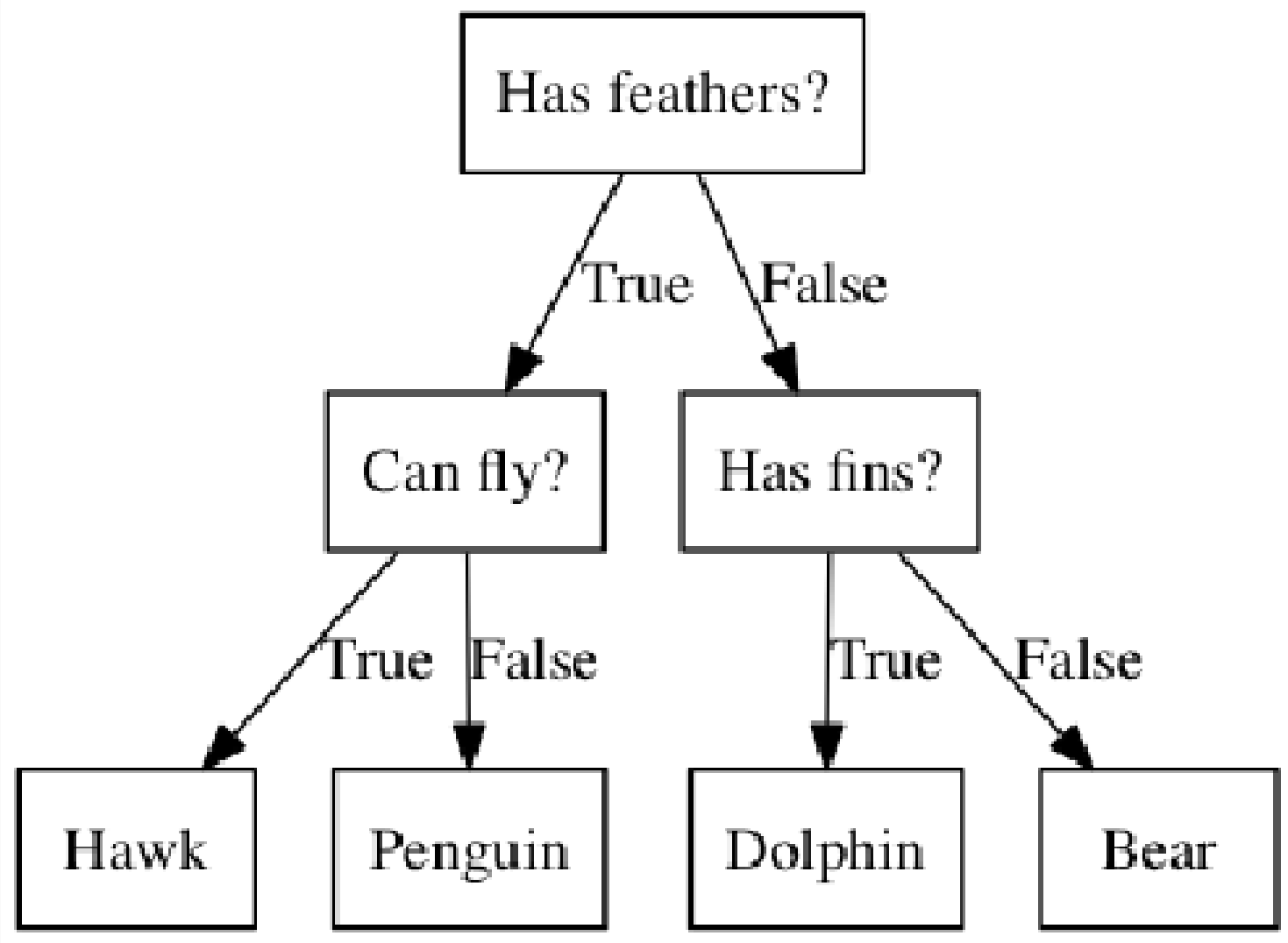
Research Center for Informatics
Indonesian Institute of Sciences

The materials are compiled from the following resources:

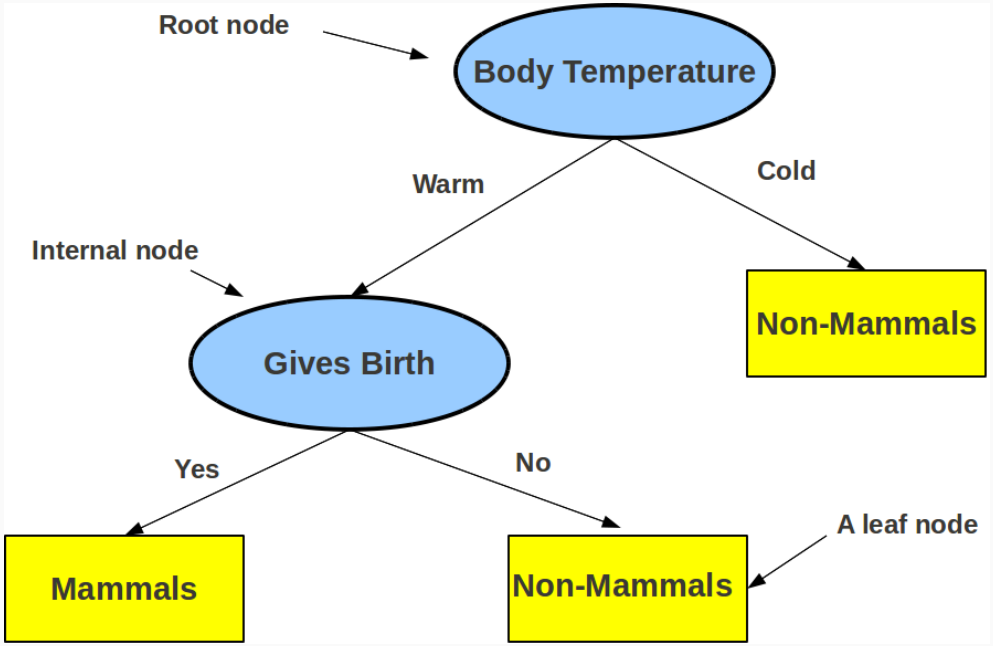
- https://github.com/joaquinvanschoren/ML-course
- https://www.cse.iitk.ac.in/users/piyush/courses/ml_autumn16/ML.html
- http://sli.ics.uci.edu/Classes/2015W-273a

DECISION TREE BASICS

DECISION TREE



- Defined by a hierarchy of rules (in form of a tree)



- Rules form the internal nodes of the tree (topmost internal node = root)
- Each internal node tests the value of some feature and "splits" data across the outgoing branches
- Note: The tree need not be a binary tree
- (Labeled) Training data is used to construct the Decision Tree (DT)
- The DT can then be used to predict label y of a test example x

BUILDING DECISION TREES

- Split the data in two (or more) parts
- Search over all possible splits and choose the one that is most informative
- Repeat recursive partitioning until stopping conditions are satisfied

DECISION TREE: AN EXAMPLE

day	outlook	temperature	humidity	wind	play
1	sunny	hot	high	weak	no
2	sunny	hot	high	strong	no
3	overcast	hot	high	weak	yes
4	rain	mild	high	weak	yes
5	rain	cool	normal	weak	yes
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- Deciding whether to play or not to play Tennis on a Saturday

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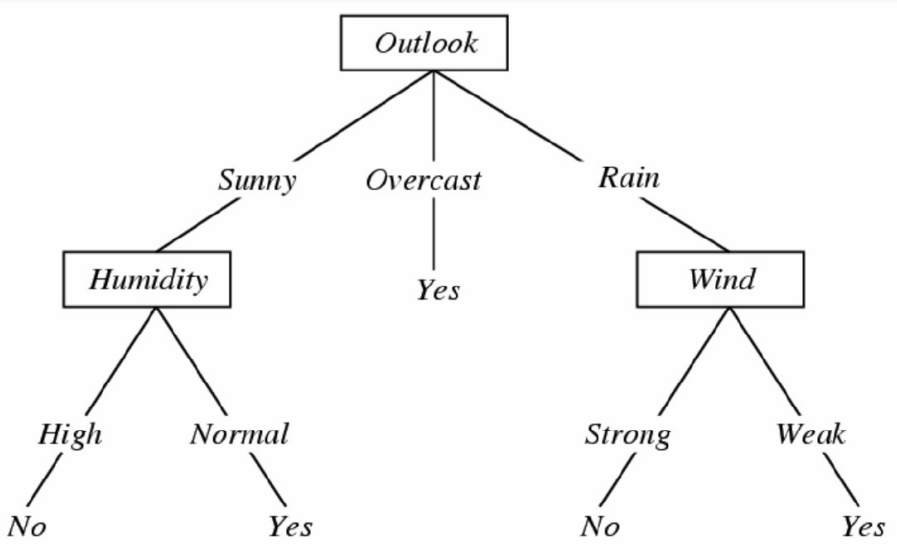

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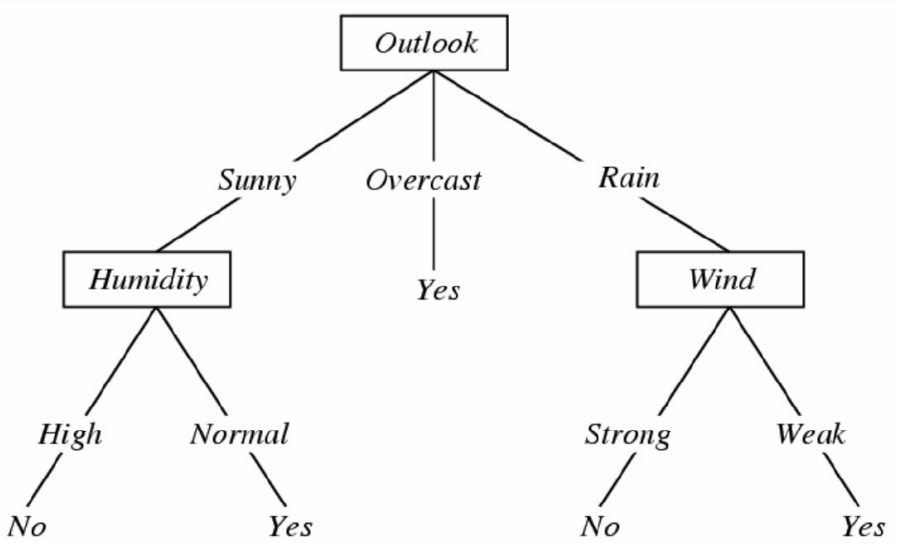

- Deciding whether to play or not to play Tennis on a Saturday
- If we would like to make a rule, which attribute to be the first we split? Why
- How to quantify "the informativeness"

LEARNING IN DT

- Entropy is a measure of randomness/uncertainty of a set

$$E(X) = -\sum_{k=1}^{K} p_k(x) \log_2 p_k(x) \tag{1}$$

with p_k = the relative frequency of class k in the leaf node

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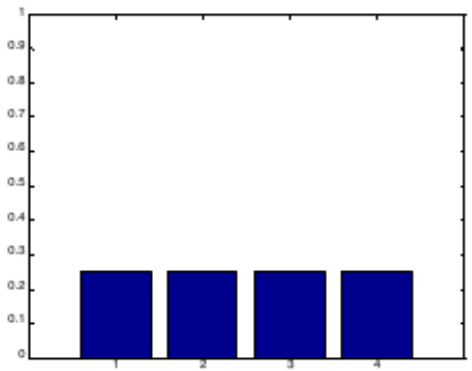

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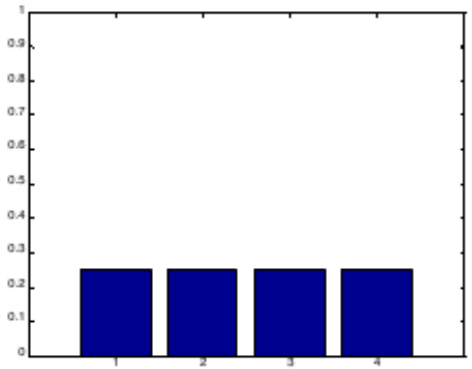

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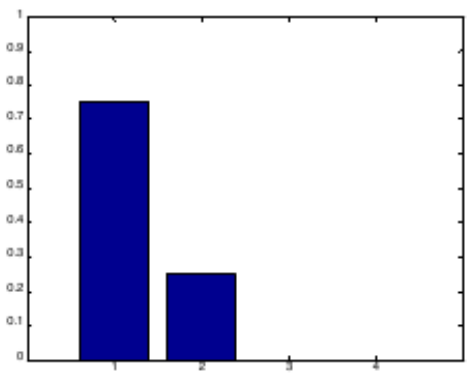

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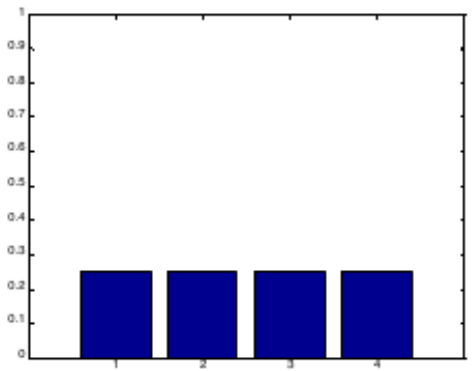

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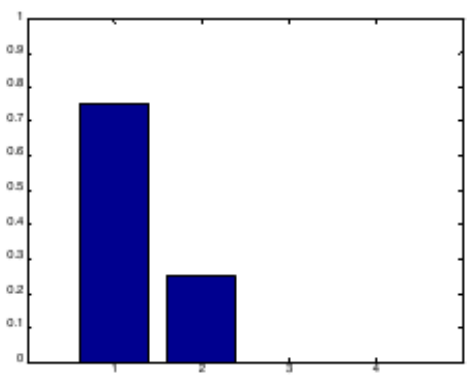

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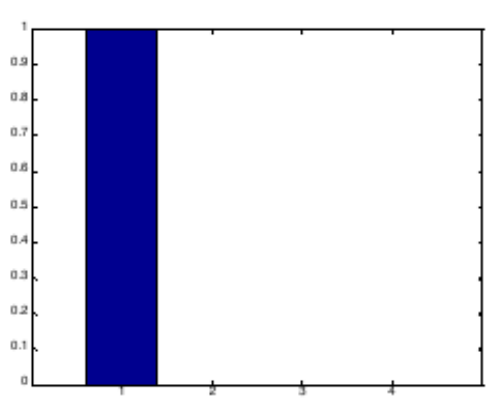

H(x) = 1 log 1 = 0 bits

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- We can assess informativeness of each feature by looking at how much it reduces the entropy of the class distribution
- Information Gain (IG) on knowing the value of some feature V

$$G(X, V) = E(X) - \sum_{v=1}^{V} \frac{|X_v|}{|X|} E(X_v) \tag{2}$$

with p_k = the relative frequency of class k in the leaf node, X = the training set, X_v denotes the subset of elements of X for which feature V has value v

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- Coming back to playing tennis..
- Let's begin with the root node of the DT and compute IG of each feature
- Consider feature "wind" $\in$ {weak,strong} and its IG w.r.t. the root note:

- Root node: S = [9+, 5−] (all training data: 9 play, 5 no-play)
- Entropy: $E(S) = (9/14)\log_2(9/14)(5/14)\log_2(5/14) = 0.94$
- $S_{weak} = [6+, 2] \Rightarrow E(S_weak) = 0.811$
- $S_{strong} = [3+, 3] \Rightarrow E(S_{strong}) = 1$

$$IG(S, wind) = E(S) - \frac{|S_{weak}|}{|S|}E(S_weak) - \frac{|S_{strong}|}{|S|}E(S_{strong})$$
$$= 0.948/140.8116/141$$
$$= 0.048$$

- At the root node, the information gains are:
- $IG(S, wind) = 0.048$ (we already saw)
- $IG(S, outlook) = 0.246$
- $IG(S, humidity) = 0.151$
- $IG(S, temperature) = 0.029$
- "outlook" has the maximum $IG \Rightarrow$ chosen as the root node
- Iterate - for each child node, select the feature with the highest IG
- Other criteria for judging feature informativeness: Gini-index, misclassification rate

AVOIDING OVERFITTING IN DT

OVERFITTING: CONTROLLING COMPLEXITY OF DECISION TREES

- Decision trees can very easily overfit the data. Regularization strategies:
- Pre-pruning: stop creation of new leafs at some point
  - Limiting the depth of the tree, or the number of leafs
  - Requiring a minimal leaf size (number of instances)
- Post-pruning: build full tree, then prune (join) leafs
  - Reduced error pruning: evaluate against held-out data
  - Many other strategies exist.
  - scikit-learn supports none of them (yet)

Some key strengths:

- Simple and each to interpret
- Do not make any assumption about distribution of data
- Easily handle different types of features (real, categorical/nominal, etc.)
- Very fast at test time (just need to check the features, starting the root node and following the DT until you reach a leaf node)
- Multiple DTs can be combined via ensemble methods (e.g., Decision/random Forest)
- Each DT can be constructed using a (random) small subset of features

Some key weaknesses:

- Can be unstable if some labeled examples are noisy
- Can sometimes become very complex unless some pruning is applied