Transit and RV fit - Bayesian approach

> We will try to write a model that incorporates photometric and RV observations, and sample the posterior distribution for the parameters using an MCMC algorithm.

Sometimes the fit of non-linear models by least squares minimisation can lead to wrong results due to getting trapped in local minima. In the Bayesian context, the problem becomes that of sampling from the posterior distribution for the model parameters. MCMC is an efficient algorithm to solve this sampling problem.

Almost every Bayesian analysis has at least two ingredients: the likelihood and the prior. They are related by Bayes theorem:

$$\text{posterior} \propto \text{prior} \times \text{likelihood} \tag{1}$$

$$p(\theta|D) \propto p(\theta) \times p(D|\theta) \tag{2}$$

Here, the set of parameters is θ, and the data is D.

Exercise 1: What is the least informative likelihood we can come up with?

In our case we actually have two datasets, D_1 and D_2, coming from transits and RVs.

Exercise 2: What is the combined likelihood for the two datasets?

Let's use a Gaussian likelihood, and include in our model an extra source of noise s.

$$
\begin{aligned}
p(D|\theta) &= \prod_{i=1}^{N} p(D_i|\theta) \\
&= \prod_{i=1}^{N} \mathcal{N}_{\text{pdf}}(d_i|f(x_i), \sigma_i + s) \\
&= \prod_{i=1}^{N} \frac{1}{\sqrt{2\pi\,(\sigma_i^2 + s^2)}} \exp\left\{ -\frac{[d_i - f(x_i)]^2}{2(\sigma_i^2 + s^2)} \right\} \\
&= (2\pi)^{-N/2} \left(\prod_{i=1}^{N} \frac{1}{\sqrt{\sigma_i^2 + s^2}} \right) \exp\left\{ -\frac{1}{2} \sum_{i=1}^{N} \frac{[d_i - f(x_i)]^2}{2(\sigma_i^2 + s^2)} \right\}
\end{aligned}
$$

so the log-likelihood is

$$
\ln p(D|\theta) = -\frac{N}{2} \ln 2\pi - \sum_{i=1}^{N} \left\{ \frac{\ln(\sigma_i^2 + s^2)}{2} + \frac{[d_i - f(x_i)]^2}{2(\sigma_i^2 + s^2)} \right\} \tag{3}
$$

Exercise 3: Write a function that computes the log likelihood for the transit model and for the radial velocity model.

Exercise 4: Using the distributions included in `scipy`, define priors for each parameter and write a function to produce samples from those prior distributions. Also create a function that calculates the value of log-prior.

Exercise 5: Write a function that computes the posterior and use `emcee` to sample from this posterior.

Exercise 6: Make a corner plot to check the correlation between the parameters and the posterior distribution of the fitted parameters. Plot your chains to check if they have converged.

Exercise 7: Warning: don't forget to oversample you light curve. What happens when you don't? Run your code for different priors. Fix the period and see what happens. Assume you know the stellar density and use a gaussian prior on the $\frac{a}{r_*} = 10.51 \pm 0.15$. Calculate mass and radius of the planet.