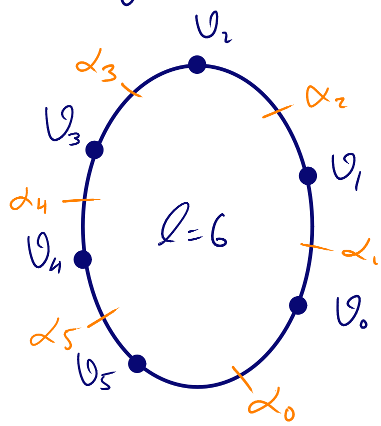
18-Aug-2025



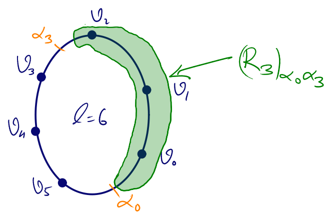
We consider the loop $vs = [v_0, v_1, \dots]$

The α_k denote the internal indices.

Define:

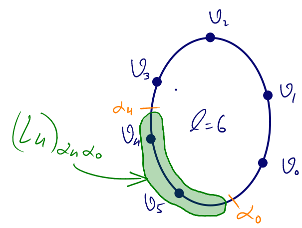
① Right transfer-matrix

$$(R_k)_{\alpha_0\alpha_k} = (T_0)_{\alpha_0\alpha_1}\cdot(T_1)_{\alpha_1\alpha_2}\cdot\ldots\cdot(T_{k-1})_{\alpha_{k-1}\alpha_k}$$



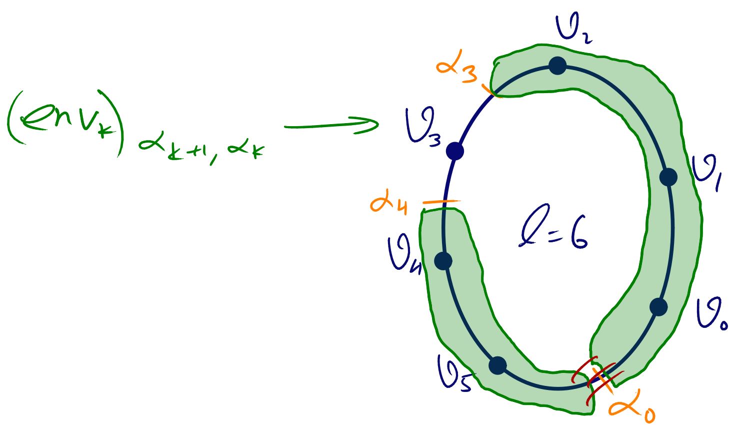
② Left transfer-matrix

$$(L_k)_{\alpha_k\alpha_0} \equiv (T_k)_{\alpha_k\alpha_{k+1}}\cdot(T_{k+1})_{\alpha_{k+1},\alpha_{k+2}}\cdot\ldots(T_{\ell-1})_{\alpha_{\ell-1}\alpha_0}$$



Together we have

$$(env_k)_{\alpha_{k+1},\alpha_k} = \sum_{\alpha_0}(L_{k+1})_{\alpha_{k+1}\alpha_0}\cdot(R_k)_{\alpha_0\alpha_k}$$



Sanity check: $(L_0)_{\alpha\beta} = (T_0\cdot\ldots\cdot T_{\ell-1})_{\alpha\beta}$

$(R_{\ell-1})_{\alpha\beta} = (T_0\cdot\ldots\cdot T_{\ell-1})_{\alpha\beta} = (L_0)_{\beta\alpha}$