XVA notes

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1 Introduction

Definition 1. ***Counterparty credit exposure*** *is the amount a company could potentially lose in the event of one of its counterparties defaulting.*

Associated to the measurement of counterparty credit exposure is the price of its hedging. This price we call the *Credit Valuation Adjustment* (CVA).

The management of this exposure can be done through a CSA (Credit Support Annex), which stipulates the terms regulating the exchange of collateral. Therefore, it is key for an XVA engine to model the dynamics of assets and the collateral.[1]

1.1 Modeling Counterparty Credit Exposure

Definition 2. *Denote the value of a portfolio by V_t, the associated* ***PFE*** *is defined by*

$$PFE_{\alpha,t} = \inf\{x : \mathbb{P}(V_t \leq x) \geq \alpha\}.$$

Definition 3.

$$EPE_t = \mathbb{E}[\max(V_t, 0)]$$

Example 1.1. *Suppose we purchase a call option on a given stock S with strike K and maturity T. The dynamics of S are given by*

$$dS_t = (r - d)S_t dt + \sigma S_t dW_t$$

where d are the associated dividends and r is the (assumed constant) risk-free rate. Let's compute the PFE at maturity at a confidence of α. Note that

[1]Counterparty credit exposure can also be hedged by buying Credit Default Swaps (CDS).

$$
\begin{aligned}
\mathbb{P}(S_T - K \le x) &= \mathbb{P}(\ln S_T \le \ln(x-K)) \\
&= \mathbb{P}\left(\ln(S_0) + (r - d - \frac{\sigma^2}{2})T + \sigma W_T \le \ln(x-K)\right) \\
&= \mathbb{P}\left(W_T \le \ln(x-K) - \ln(S_0) - (r - d - \frac{\sigma^2}{2})T\right) \\
&= N\left(\frac{\ln(x-K) - \ln(S_0) - (r - d - \frac{\sigma^2}{2})T}{\sigma\sqrt{T}})\right).
\end{aligned}
$$

If we assume that at maturity the option is in the money, using the above we get:

$$
\begin{aligned}
PFE_{\alpha,T} &= \inf\{x : S_0 \exp\left((r - d - \frac{\sigma^2}{2})T + \sigma\sqrt{T}N^{-1}(\alpha)\right) - K \le x\} \\
&= S_0 \exp\left((r - d - \frac{\sigma^2}{2})T + \sigma\sqrt{T}N^{-1}(\alpha)\right) - K.
\end{aligned}
$$