Revisiting the Solow Model

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Model Specification (i)

$$\ln\left(\frac{Y_{i,t}}{L_{i,t}}\right) = \gamma + \beta_1 \ln(n_i + g_i + \delta_i) + \beta_2 \ln(s_i) + \epsilon$$

for some country i:

- $\frac{Y_t}{L_t}$, total income per capita for predicted year t
- γ, constant
- n_i, population growth rate
- g_i, technology advancement rate
- δ_i,depreciation rate of capital
- s_i, savings rate
- ϵ_i, error term

Model Specification (i)

$$\ln\left(\frac{Y_{i,t}}{L_{i,t}}\right) = \gamma + \beta_1 \ln(n_i + g_i + \delta_i) + \beta_2 \ln(s_i) + \epsilon_i$$

Restrictions on coefficients:

- $\beta_1 = -\frac{\alpha}{1-\alpha}$ and $\beta_2 = \frac{\alpha}{1-\alpha}$
- α, physical capital's share of income
- From data $\alpha \approx 0.48 \implies \frac{\alpha}{1-\alpha} \approx 0.92$
- $\implies \beta_2 = |\beta_1| \approx 0.92$

Theoretical assumption

- $E(\epsilon_i | s_i, n_i) = 0$

Econometric assumption

- $E(\epsilon_1 | s_i, n_i, g_i, \delta_i) = 0$

Model Specification (ii) and (iii)

$$\ln\left(\frac{Y_{i,t}}{L_{i,t}}\right) = \psi + \phi_1 \ln(n_1 + g_i + \delta_i) + \phi_2 \ln(s_i) + \phi_3 \ln(h_i) + \omega_i$$

for some country i:

- ψ, constant
- h_i, human capital index
- ω_i, error term
- $\frac{Y_{i,t}}{L_{i,t}}$, n_i, g_i, δ_i, s_i defined as in specification (i)

Specification of model (iii):

$$\ln\left(\frac{Y_{i,t}}{L_{i,t}}\right) = \psi + \phi_1 \ln(n_1 + g_i + \delta_i) + \phi_2 \ln(s_i) + \phi_3 \ln(h'_i) + \omega_i$$

- h'_i, education index

Model specifications (ii) and (iii)

$$\ln\left(\frac{Y_{i,t}}{L_{i,t}}\right) = \psi + \phi_1 \ln(n_1 + g_i + \delta_i) + \phi_2 \ln(s_i) + \phi_3 \ln(h_i) + \omega_i$$

Restrictions on coefficients:

- $\phi_1 = -\frac{\alpha}{1-\alpha-\theta}$, $\phi_2 = \frac{\alpha}{1-\alpha-\theta}$ and $\phi_3 = \frac{\theta}{1-\alpha-\theta}$
- θ, human capital's share of income
    - assume like Mankiw et al. (1992) $\theta = 0.33$
    - $\implies |\phi_1| = \phi_2 = \frac{\alpha}{1-\alpha-\theta} \approx 2.53$ and $\phi_3 = \frac{\theta}{1-\alpha-\theta} \approx 1.74$

Theoretical assumption:

- $E(\omega_i | s_i, n_i, h_i) = 0$

Econometric assumption:

- $E(\omega_i | s_i, n_i, g_i, \delta_i, h_i) = 0$

Data sources

- World bank
  - Population Growth (annual %) (n)
  - Gross capital formation (% of GDP) (s)
  - GDP growth (annual %) ($\approx g$)
- Penn World Table version 10.0
  - Labour share income (α)
  - Depreciating rate of capital (δ)
  - Real GDP (Y)
  - Population (L)
  - Human Capital (h)
- United Nations Development Programme
  - Education Index (h')
- All data hosted at `https://github.com/iatrogenic/Solow-TPROG/tree/main/data`

World Bank data

country_code	year	gdp_growth	pop_growth	savings
AFG	1990.0	NaN	4.476954	NaN
AFG	1991.0	NaN	6.900124	NaN
AFG	1992.0	NaN	8.546107	NaN
AFG	1993.0	NaN	8.790897	NaN
AFG	1994.0	NaN	7.659796	NaN

- Downloaded from `https://databank.worldbank.org/reports.aspx?source=world-development-indicators`

Penn World Table data

	country_code	country	years	labour_share_income	delta	real gdp	pop	hc
40	ABW	Aruba	1990	NaN	0.036517	1693.878906	0.062149	NaN
41	ABW	Aruba	1991	0.648357	0.036903	1828.760254	0.064622	NaN
42	ABW	Aruba	1992	0.648357	0.036488	1936.334473	0.068235	NaN
43	ABW	Aruba	1993	0.648357	0.035478	2077.835938	0.072504	NaN
44	ABW	Aruba	1994	0.648357	0.034092	2248.299561	0.076700	NaN

- Downloaded from `https://www.rug.nl/ggdc/productivity/pwt/`

Education Index

- Is a component of the Human Development Index (HDI)
- Dataset was not formatted in a way that pandas' `read_csv` could parse.
- We used the lower-level `csv` library to construct a list that is then converted into a pandas `DataFrame` object.
- Available at `http://hdr.undp.org/en/indicators/103706`

```python
import csv, urllib
ei_url = 'https://raw.githubusercontent.com/iatrogenic/Solow-TPROG/main/data/Education_index.csv'

with urllib.request.urlopen(ei_url) as rsp:
  # decode latin1 since some lines contain non-UTF8 characters.
  csvfile = [l.decode('latin1') for l in rsp.readlines()]
  ei = csv.reader(csvfile)
  # remove metadata
  raw = [row[1:] for row in list(ei)[6:-1]]
  # remove all even columns
  formatted_rows = []
  for row in raw:
    formatted_rows.append([row[0]] + row[1::2])

cols = ['country'] + list(range(1990, 2020))
education = pd.DataFrame(formatted_rows, columns=cols)
```

Listing 1: Processing the Education Index dataset

Education Index DataFrame

	country	1990	1991	1992	1993	1994	...	2015	2016	2017	2018	2019
1	Albania	0.583	0.588	0.557	0.542	0.541	...	0.753	0.745	0.747	0.743	0.746
2	Algeria	0.385	0.395	0.405	0.414	0.424	...	0.659	0.660	0.665	0.668	0.672
3	Andorra	..	..	..	..	..	...	0.718	0.722	0.713	0.720	0.720
...	...	...	...	...	...	...	...	...	...	...	...	...
205	World	0.450	0.456	0.456	0.469	0.474	...	0.626	0.631	0.633	0.633	0.637

Further reshaping

- Map country names into country codes
- List time vertically, i.e. in panel data format

country code	year	Education Index
AFG	1990.0	0.122
AFG	1991.0	0.133
AFG	1992.0	0.145
AFG	1993.0	0.156
AFG	1994.0	0.168

```
df['capital_share_income'] = round(1 - df['labour_share_income'
  ], 2)

# adjust scales and decimals
df['delta'] = df['delta'] * 100
df['real_gdp'] = df['real_gdp'] /10

# create: real gdp per capita and decimals
df['real_gdp_per_capita'] = df['real_gdp'] / df['pop']

# final DataFrame
df_regression = df.groupby(['country_code'])['savings', '(n+g+
  delta)', 'pop', 'hc', 'Education Index'].agg('mean')

```

Final DataFrame

country code	savings	$n+g+\delta$	pop	hc	Education Index
ARG	17.515432	7.064752	38.496126	2.762353	0.749379
ARM	26.622005	5.725578	3.048512	3.082355	0.690714
AUS	25.985479	7.377094	20.442071	3.479699	0.897931
AUT	24.880684	6.400530	8.249491	3.178440	0.769207
BDI	11.208659	7.051932	7.618328	1.251119	0.275517
...	...	...	...	...	...
USA	21.313520	7.357352	291.347278	3.615634	0.871862
VEN	22.705581	8.675406	25.006332	2.253260	0.566480
ZAF	18.725002	8.536923	47.420789	2.298774	0.654655
ZMB	36.004405	12.728065	15.429866	2.440716	0.536556
ZWE	12.537147	6.781775	12.271189	2.227122	0.498862

- We also removed three outlier countries.

Correlation Matrix

	Savings	$n+g+\delta$	Population	Human Capital	Education Index
Savings	1.000000	0.262102	0.337697	0.211706	0.189340
$n+g+\delta$	0.262102	1.000000	0.254284	-0.542124	-0.547514
Population	0.337697	0.254284	1.000000	-0.061961	-0.079928
Human Capital	0.211706	-0.542124	-0.061961	1.000000	0.961376
Education Index	0.189340	-0.547514	-0.079928	0.961376	1.000000

Performing the regressions

- We perform three regressions using the ordinary least squares method:
    - Baseline, Spec (i)
    - Baseline + Human Capital (PWT), Spec (ii)
    - Baseline + Education Index, Spec (iii)
- We use the `statsmodel` library.
- Both regressors and regressands are taken as logarithms of the corresponding variables in the `DataFrame`.

Dependent variable: $\ln\left(\frac{Y_{2018}}{L_{2018}}\right)$

Model specifications			
	(i)	(ii)	(iii)
Constant	4.8457 ***	3.4926 ***	7.8708
	(2.938)	(3.243)	(8.608)
$\ln(n_i + g_i + \delta_i)$	-1.6198 ***	0.2240	0.1980
	(-5.166)	(0.759)	(0.743)
$\ln(s_i)$	1.8834 ***	0.1435	0.1501
	(3.382)	(0.442)	(0.462)
$\ln(h_i)$		3.2191 ***	
		(12.136)	
$\ln(h'_i)$			2.7584 ***
			(9.008)
N	102	102	102
R^2	0.290	0.716	0.766
Adj. R^2	0.276	0.707	0.758
F-stat	19.07 ***	128.7 ***	59.16 ***

Results (ceteris paribus analysis)

(i)

$$\uparrow 1\%\,(n_i + g_i + \delta_i) \quad \Rightarrow \downarrow 1.62\% \quad \frac{Y_{i,2018}}{L_{i,2018}}{***},\ R^2 = 0.29,\ \bar{R}^2 = 0.28$$

$$\uparrow 1\%\,(s_i) \quad \Rightarrow \uparrow 1.88\% \quad \frac{Y_{i,2018}}{L_{i,2018}}{***}$$

(ii)

$$\uparrow 1\%\,(n_i + g_i + \delta_i) \quad \Rightarrow \uparrow 0.22\% \quad \frac{Y_{i,2018}}{L_{i,2018}},\ R^2 = 0.72,\ \bar{R}^2 = 0.71$$

$$\uparrow 1\%\,(s_i) \quad \Rightarrow \uparrow 0.14\% \quad \frac{Y_{i,2018}}{L_{i,2018}}$$

$$\uparrow 1\%\,(h_i) \quad \Rightarrow \uparrow 3.22\% \quad \frac{Y_{i,2018}}{L_{i,2018}}{***}$$

(iii)

$$\uparrow 1\%\,(n_i + g_i + \delta_i) \quad \Rightarrow \uparrow 0.20\% \quad \frac{Y_{i,2018}}{L_{i,2018}},\ R^2 = 0.77,\ \bar{R}^2 = 0.76$$

$$\uparrow 1\%\,(s_i) \quad \Rightarrow \uparrow 0.15\% \quad \frac{Y_{i,2018}}{L_{i,2018}}$$

$$\uparrow 1\%\,(h_i') \quad \Rightarrow \uparrow 2.76\% \quad \frac{Y_{i,2018}}{L_{i,2018}}{***}$$

Q&A