– LECTURE 0 –

A SICKNESS UNTO DEATH

OR, AN ENQUIRY INTO THE THEORY OF MEASURE AS IT CONCERNS THOSE PROCESSES OF A STOCHASTIC KIND

UNDERGROUND RESEARCH DIVISION

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Nihil – $\emptyset$

"It is clear that I can only deliver to you, to each of you, what you are already on the verge of absorbing."

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1. The Doctrine of Chances

The general problem of measure is our starting point. Given $A \subseteq X$ we want to be able to assign a quantity $m(A)$ to this set, which can be said, in some sense, to be its "measure". Take, for example, $X = \mathbb{R}$ and $A = [a, b]$, for which we can set $m(A) = b - a$. Some questions immediately arise: what properties must m obey? and what is its proper domain, i.e. which sets can be said to be *measurable*? The answer to this last question requires the introduction of the elementary structure of measure theory– the σ-algebra.

Definition 1. A σ-algebra over a set X is a set $\mathcal{F} \subseteq \mathcal{P}(X)$ satisfying

(1) $\emptyset \in \mathcal{F}$,
(2) $A \in \mathcal{F} \implies X \setminus A \in \mathcal{F}$,
(3) If $A_1, A_2, \ldots \in \mathcal{F}$ then

$$\bigcup_{i \in \mathbb{N}} A_i \in \mathcal{F}.$$

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