Notes on Complex Analysis

September 18, 2020

Contents

1 Power Series in One Variable

1.1 The Field of Complex Numbers

1.2 Formal Power Series

Let $\mathbb{K}$ be a field. The set of formal polynomial in indeterminate X with coefficients in $\mathbb{K}$ is denoted by $\mathbb{K}[X]$. This set, equipped with the usual polynomial addition and scalar multiplication, is an infinite dimensional vector space, spanned by the basis:

$$1, X, \ldots, X^n, \ldots$$

Multiplication of polynomials defines a bilinear product. This additional operation turns $\mathbb{K}[X]$ into an *algebra*. If we drop the "finitely many non-zero coefficients" requirement in the definition of polynomial we arrive at the definition of a *formal power series*. The set of all formal power series with coefficients in field $\mathbb{K}$ is denoted by $\mathbb{K}[[X]]$. The algebra $\mathbb{K}[X]$ is identified with a subalgebra of $\mathbb{K}[[X]]$.

Let $S = \sum_{n \geq 0} a_n X^n \neq 0$ be a formal power series. The *order* of S, denoted by $\omega(S)$, is the smallest k such that $a_k \neq 0$. The order of $0 \in \mathbb{K}[[X]]$ is ∞.

A family of power series $(S_i \in \mathbb{K}[[X]] : i \in I)$ is said to be *summable* if for any $k \in \mathbb{N}$, $\omega(S_i) \geq k$ for all but a finite number number of indices I.