Computation; Notes

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1 Computable Functions

§ Basic Concepts A *partial function* is a map f denoted $f : X \nrightarrow Y$ such that there exists a subset $X' \subseteq X$ for which $f : X' \to Y$ is a function. If $X = X'$ then f is said to be a *total function*. The domain of f is denoted by $\mathsf{Dom}\, f$ and defined by the set $\{x : f(x) \text{ is defined}\}$; we say that $f(x)$ is undefined if $x \notin \mathsf{Dom}\, f$. The range of f, denoted by $\mathsf{Ran}\, f$ is the set $\{f(x) : x \in \mathsf{Dom}\, f\}$. Partial functions generalize the concept of function which here corresponds to the definition of total function.

1.1 What is a computable function?

Informally, an *algorithm* is a sequence of mechanical instructions. A numerical function is *effectively computable* if an algorithm exists that can be used to calculate the value of the function for any given input from its domain.