Computation; Notes

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1 Computable Functions

§ Basic Concepts A *partial function* is a map f denoted $f : X \nrightarrow Y$ such that there exists a subset $X' \subseteq X$ for which $f : X' \to Y$ is a function. If $X = X'$ then f is said to be a *total function*. The domain of f is denoted by $\mathsf{Dom}\, f$ and defined by the set $\{x : f(x) \text{ is defined}\}$; we say that $f(x)$ is undefined if $x \notin \mathsf{Dom}\, f$. The range of f, denoted by $\mathsf{Ran}\, f$ is the set $\{f(x) : x \in \mathsf{Dom}\, f\}$. Whenever the word "function" is used by itself, assume it to mean "partial function". The usual meaning of the word "function" corresopnds to what we here call "total function."

As an example, consider the partial function

$$\begin{aligned} f : \mathbb{N}_0 &\nrightarrow \mathbb{N}_0 \\ n &\mapsto \sqrt{n}. \end{aligned}$$

If $n \in \mathbb{N}_0$ is not a perfect square, then $f(n)$ is undefined.

1.1 What is a computable function?

§ Informal Discussion An *algorithm* is a sequence of discrete mechanical instructions that terminates. A numerical function is *effectively computable* (or simply *computable*) if an algorithm exists that can be used to calculate the value of the function for any given input from its domain.

§ The Unlimited Register Machine The *unlimited register machine* has an infinite number of *registers* labelled $R_1, R_2, \ldots$, each containing a natural number, if R_i is a register then r_i is the number it contains. It can be represented as follows

R_1	R_2	R_3	R_4	R_5	R_6	R_7	$\cdots$
r_1	r_2	r_3	r_4	r_5	r_6	r_7	$\cdots$

The contents of a register may be altered by the URM in response to certain *instructions* that it can recognize

§ Instruction set

Name of Instruction	Instruction	URM response
Zero	$\mathsf{Z}(n)$	$r_n \leftarrow 0$
Successor	$\mathsf{S}(n)$	$r_n \leftarrow r_n + 1$
Transfer	$\mathsf{T}(m, n)$	$r_n \leftarrow r_m$
Jump	$\mathsf{J}(m, n, q)$	if $r_m = r_n$ then jump to qth instruction; otherwise proceed to next instruction.