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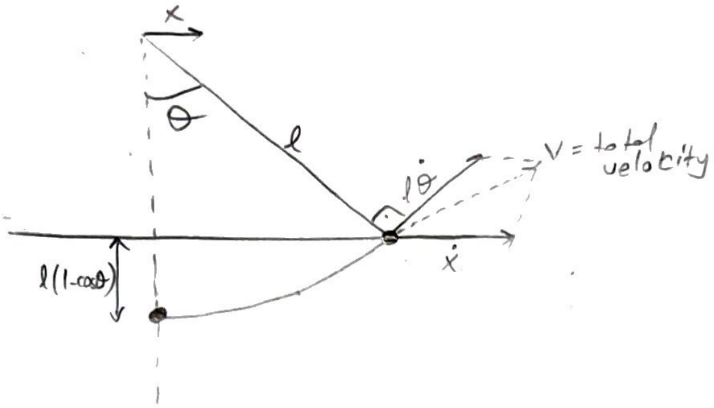

$$V^2 = \dot x^2 + l^2\dot\theta^2 + 2l\dot x\dot\theta\cos\theta$$

$$T = \frac{1}{2}m_1\dot x^2 + \frac{1}{2}m_2\left(\dot x^2 + l^2\dot\theta^2 + 2l\dot x\dot\theta\cos\theta\right)$$

$$V = \frac{1}{2}kx^2 + m_2gl(1-\cos\theta)$$

$$L = T - V$$

$$L = \frac{1}{2}m_1\dot x^2 + \frac{1}{2}m_2\left(\dot x^2 + l^2\dot\theta^2 + 2l\dot x\dot\theta\cos\theta\right) - \frac{1}{2}kx^2 - m_2gl(1-\cos\theta)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot x}\right) - \frac{\partial L}{\partial x} = 0$$

$$\frac{\partial L}{\partial \dot x} = m_1\dot x + m_2\dot x + m_2 l\dot\theta\cos\theta$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot x}\right) = m_1\ddot x + m_2\ddot x + m_2 l\left(\ddot\theta\cos\theta - \dot\theta\sin\theta\,(\dot\theta)\right)$$

$$\frac{\partial L}{\partial x} = -kx$$

$$\ddot x(m_1+m_2) = m_2 l\left(\dot\theta^2\sin\theta - \ddot\theta\cos\theta\right) - kx$$

$$\ddot x = \frac{m_2 l\left(\dot\theta^2\sin\theta - \ddot\theta\cos\theta\right) - kx}{(m_1+m_2)}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot\theta}\right) - \frac{\partial L}{\partial \theta} = 0$$

$$\frac{\partial L}{\partial \theta} = -m_2\dot x\dot\theta l\sin\theta - m_2gl\sin\theta$$

$$\frac{\partial L}{\partial \dot\theta} = m_2l^2\dot\theta + m_2\dot x l\cos\theta$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot\theta}\right) = m_2l^2\ddot\theta + m_2 l\left(\ddot x\cos\theta - \dot x\sin\theta\,\dot\theta\right)$$

$$m_2l^2\ddot\theta + m_2 l\ddot x\cos\theta - m_2 l\dot x\dot\theta\sin\theta = -m_2\dot x\dot\theta l\sin\theta - m_2gl\sin\theta$$

$$l\ddot{\theta} + \ddot{x}\cos\theta = -g\sin\theta$$

$$l\ddot{\theta} = -\ddot{x}\cos\theta - g\sin\theta$$

Let $\dot{x} = u \qquad \dot{\theta} = w$

$\dot{u} = \ddot{x} \qquad \dot{w} = \ddot{\theta}$

$$\dot{u} = \frac{m_2 l\,(w^2\sin\theta - \dot{w}\cos\theta) - kx}{m_1 + m_2}$$

$$\dot{w} = \frac{-\dot{u}\cos\theta - g\sin\theta}{l}$$

$$\dot{u} = \frac{m_2 l\left(w^2\sin\theta + \frac{\dot{u}\cos\theta + g\sin\theta}{l}\cdot\cos\theta\right) - kx}{m_1 + m_2}$$

$$\dot{u} = \frac{m_2 l w^2\sin\theta + m_2\dot{u}\cos\theta + m_2 g\sin\theta\cos\theta - kx}{m_1 + m_2}$$

$$\dot{u}(m_1 + m_2) - m_2\dot{u}\cos^2\theta = m_2 l w^2\sin\theta + m_2 g\sin\theta\cos\theta - kx$$

$$\dot{u}\,(m_1 + m_2 - m_2\cos^2\theta) = m_2 l w^2\sin\theta + m_2 g\sin\theta\cos\theta - kx$$

$$\dot{u} = \frac{m_2 l w^2\sin\theta + m_2 g\sin\theta\cos\theta - kx}{m_1 + m_2 - m_2\cos^2\theta} \quad //$$

$$\dot{w} = \frac{\frac{-m_2 l w^2\sin\theta - m_2 g\sin\theta\cos\theta + kx}{m_1 + m_2 - m_2\cos^2\theta}\cdot\cos\theta - g\sin\theta}{l} \quad //$$

Homework-1

For the tasks 1,2,3,4 ; you can find the solutions in the handwritten paper.

For the tasks 5 and 6; you can find the solutions in below. (Code, plots and values of x and θ at 10th second)

The code below has been submitted on Moodle. In order to run it and see the results follow these steps:

1- ***Run main.py file.***
2- ***A new window pop up and x and theta values at 10th second will appear in the console as well as first plot which you can see in Figure 1.***
3- ***After close Figure 1., you will see second Figure, Figure 2 respectively. Continue until you see all the figures.***

Python Code

```
import matplotlib
import numpy as np
import math
import matplotlib.pyplot as plt

l = 0.5
g = 9.8
m1 = 100
m2 = 10
timeTotal = 15
dt = 0.0015
k = 1000
theta0 = np.pi / 4
thetadot = 0
x = 0.1
xdot = 0


def fn(theta, w, x, u, g, l, m1, m2, k):
    return np.array([w, (((-m2 * l * w * w * np.sin(theta) - m2 * g *
np.sin(theta)* np.cos(theta) + k * x) / (m1 + m2 - m2 *
np.cos(theta)*np.cos(theta))) * np.cos(theta) - g * np.sin(theta)) / l,
                     u,(m2 * l * w * w * math.sin(theta) + m2 * g *
np.sin(theta)* np.cos(theta) - k * x) / ( m1 + m2 - m2 * np.cos(theta) *
np.cos(theta))])


yn = np.array([theta0, thetadot, x, xdot])
yExplicit = yn.copy()

time = 0
while time < timeTotal:
    y =yn+dt*fn(yn[0], yn[1], yn[2], yn[3], g, l, m1, m2, k)
    yn = y.copy()
    time = time + dt
    # if int(time) == 10:
    #     print("After 10 seconds value of θ is :" +str(round(yn[0], 3)),
"\nAfter 10 seconds value of x is :"+str(round(yn[2], 3)))
```

```
    #     continue
    yExplicit = np.vstack((yExplicit, y))
t = np.linspace(0, timeTotal, num=len(yExplicit))
plt.rcParams["figure.figsize"] = 15, 5
a = plt.plot(t, yExplicit[:, 0], label='Theta', color='red')
b = plt.plot(t, yExplicit[:, 2], label='Dispx', color='blue')
plt.axhline(0, color='black')
plt.legend(loc='best')
plt.show()


plt.rcParams["figure.figsize"] = 15, 5
d = plt.plot(yExplicit[:, 0], yExplicit[:, 2], label='Theta and x',
color='blue')
plt.axhline(0, color='black')
plt.legend(loc='best')
plt.show()


plt.rcParams["figure.figsize"] = 15, 5
c = plt.plot(yExplicit[:, 1], yExplicit[:, 3], label='thetadot and xdot',
color='red')
plt.axhline(0, color='black')
plt.legend(loc='best')
plt.show()


plt.rcParams["figure.figsize"] = 15, 5
d = plt.plot(yExplicit[:, 2], yExplicit[:, 3], label='x and xdot',
color='blue')
plt.axhline(0, color='black')
plt.legend(loc='best')
plt.show()


plt.rcParams["figure.figsize"] = 15, 5
c = plt.plot(yExplicit[:, 0], yExplicit[:, 1], label='Theta and thetadot',
color='red')
plt.axhline(0, color='black')
plt.legend(loc='best')
plt.show()
```

Plots

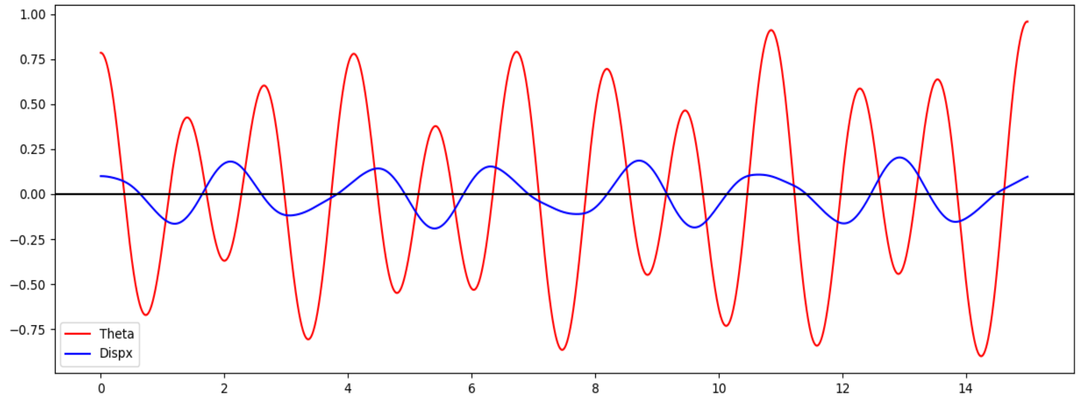

Figure 1. Plot Theta and x with respect to time

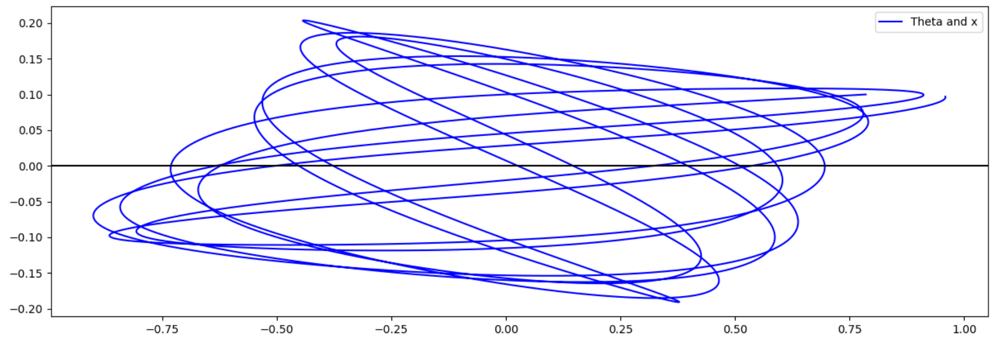

Figure 2. Plot Theta and x

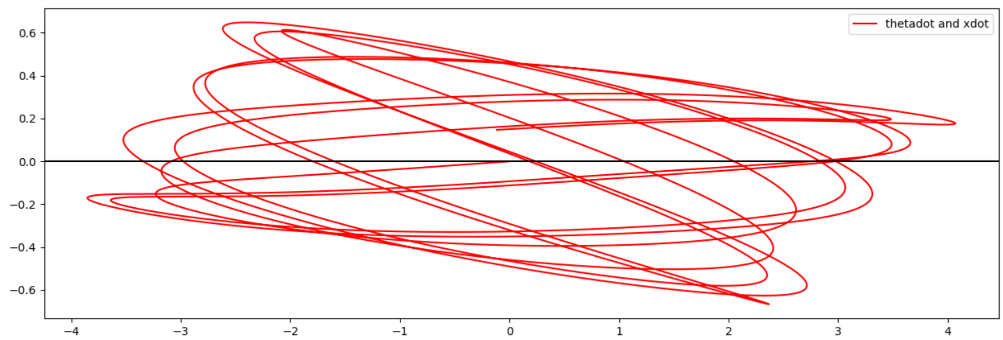

Figure 3. Plot Thetadot and xdot

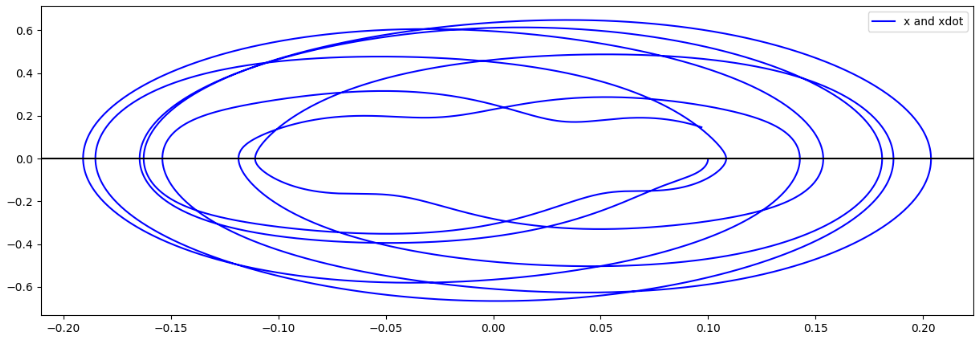

Figure 4. Plot x and xdot

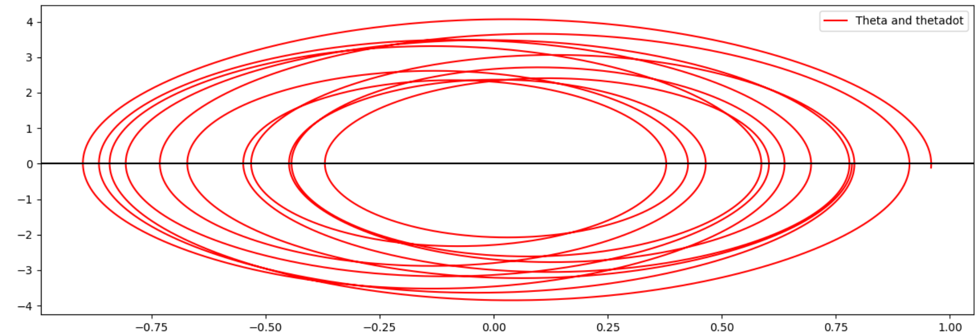

Figure 5. Plot Theta and Thetadot

The values of x and θ after 10s are -0.062 and -0.636.