National University of Computer and Emerging Sciences
Islamabad Campus

ADVANCED STATISTICS

FINAL PROJECT

Inflation Dynamics in Pakistan: A Statistical Analysis

Submitted by:

Sameer Khan	Mahd Kazmi	Hussain Ibrahim
23i-2521	23i-2587	23i-2626

National University of Computer & Emerging Sciences
Islamabad Campus

May 6, 2025

Table of Contents

c

Section	Page
Introduction:	3
Literature Review:	4
1. Theoretical Foundations	4
1.1 Broad Money Growth	4
1.2 Food Imports	4
1.3 GDP Growth	4
1.4 Exchange Rate	5
1.5 Crude Oil	5
2. Empirical Evidence	5
3. Variable Selection Rationale	6
Variable Description:	6
1. Independent Variable	7
1.1 Monetary Factor	7
1.2 External Sector Variable	7
1.3 Structural Variable	8
1.4 Expectation and Demand Variable	8
Data Description:	9
1. Summary Statistics	9
2. Graphical Representation	10
Box & Whisker Plot:	12
1. Summary	12
2. Graphical Representation	13
Scatter Plot:	14
1. Summary	14
2. Graphical Representation	16
Time Series:	18
1. Autoregressive (AR) Model	18
2. Moving Average (MA) Model	18
3. Autoregressive Moving Average (ARMA) Model	18
4. Autoregressive Integrated Moving Average (ARIMA) Model	19
1.1 ARIMA MODEL Results	20
1.2 Interpretation	21
Ridge Regression Model:	21
1. Model Results	21
2. Interpretation	22
Lasso Regression Model:	22
1. Model Results	23
2. Interpretation	23
Elastic Net Regression Model:	23
1. Model Results	24
2. Interpretation	24
Checking Significance:	25
Improved Elastic Net Regression Model:	26
1. Model Results	26
2. Interpretation	26
Model Performance Comparison:	28
Conclusion:	30
References:	30

1 Introduction

Inflation remains one of the most pressing economic challenges for Pakistan, persistently eroding purchasing power, exacerbating poverty, and distorting income distribution. Over the past few decades, the country has experienced prolonged periods of double-digit inflation, with significant spikes in the Consumer Price Index (CPI) and Wholesale Price Index (WPI). For instance, inflation surged to over 11% in 1981–82 and exceeded 12% in 1990–91, while the Sensitive Price Index (SPI) climbed even higher, reaching 15% in the early 1980s. Such persistent inflationary pressures have not only disrupted economic stability but have also imposed a hidden tax on households, particularly the poor, who bear the brunt of rising food and energy prices. While moderate inflation can sometimes stimulate economic activity, Pakistan's inflation has consistently surpassed acceptable thresholds, necessitating urgent policy interventions.

The inflationary trajectory in Pakistan has been shaped by a complex interplay of domestic and external factors. Historically, the country enjoyed relative price stability in the 1960s, with inflation averaging just 2.6%. However, the 1970s marked a turning point, as global oil shocks, currency devaluations, and agricultural disruptions triggered a decade of double-digit inflation. The 1980s saw some moderation, but inflationary pressures resurfaced in the 1990s and early 2000s, driven by fiscal imbalances, monetary expansion, and external shocks such as the Gulf War and post-9/11 economic adjustments. More recently, inflation has surged again, fueled by expansionary fiscal and monetary policies, rising import costs, exchange rate depreciation, and supply-side constraints. The persistence of high inflation raises critical questions about its underlying determinants and its impact on long-term economic growth.

The relationship between inflation and economic growth is a subject of extensive debate in economic literature. While low and stable inflation can foster investment and consumption by reducing uncertainty, high inflation distorts price signals, discourages savings, and hampers productive investment. Empirical studies suggest that inflation beyond a certain threshold—often estimated between 4% and 9% for developing countries—can significantly impede growth. For Pakistan, identifying this threshold is crucial, especially as the State Bank of Pakistan (SBP) considers adopting an inflation-targeting framework. Previous studies on Pakistan's inflation-growth nexus, such as those by Mubarik (2005) and Hussain (2005), have attempted to estimate this threshold but suffer from methodological limitations. A more robust analysis is needed to determine the precise level at which inflation becomes detrimental to growth.

Fiscal and monetary policies have played a pivotal role in shaping Pakistan's inflationary environment. Persistent budget deficits, often financed by money creation, have historically contributed to inflationary pressures. The fiscal dominance of monetary policy where government borrowing from the central bank leads to excessive money supply growth has been a recurring issue. Additionally, loose credit policies and rapid monetary expansion in recent years have further exacerbated inflation. The surge in private sector credit, coupled with rising import prices and exchange rate volatility, has compounded demand-side pressures. Supply-side factors, including food price shocks, energy shortages, and indirect taxation, have also played a significant role. Understanding the relative contributions of these factors is essential for designing effective antiinflationary policies.

Inflation uncertainty adds another layer of complexity to Pakistan's economic challenges. Unpredictable inflation erodes confidence in the price system, distorting long-term investment and savings decisions. As Milton Friedman (1977) argued, high inflation breeds uncertainty, which in turn reduces economic efficiency. In Pakistan, where inflationary expectations remain volatile, this uncertainty can further destabilize the economy. The interplay between inflation and inflation uncertainty—whether high inflation leads to greater uncertainty or vice versa—has critical implications for monetary policy. Addressing this dynamic is essential for restoring macroeconomic stability.

Given these complexities, this study seeks to comprehensively analyze the sources and consequences of inflation in Pakistan. It examines the historical trends, evaluates the inflation-growth relationship, identifies key determinants (including fiscal deficits, monetary policy, and external shocks), and assesses the impact of inflation uncertainty. By integrating theoretical insights with empirical evidence, the study aims to provide policymakers with actionable recommendations to contain inflation within sustainable limits. The findings will contribute to the ongoing debate on inflation management, particularly as Pakistan transitions toward a more structured monetary policy framework.

Literature Review

Inflation dynamics in Pakistan have been widely studied through various macroeconomic indicators, each shedding light on different transmission mechanisms. This section reviews key variables commonly used in empirical models of inflation.

1 Theoretical Foundations

The theoretical framework for analyzing inflation in Pakistan draws from several well-established economic theories and empirical relationships.

1.1 Broad Money Growth

Consistent with the Quantity Theory of Money, broad money (M2) remains a cornerstone variable in inflation analysis. Key insights from literature include:

- **Persistent monetary expansion:** Pakistan's M2 has grown at an average rate of 14–16% annually since the 1990s (SBP, 2023).
- **Monetary-fiscal linkage:** Heavy reliance on domestic borrowing has historically fueled M2 growth, contributing to inflationary pressures (Chaudhary & Ahmad, 1995).
- **Empirical elasticity:** Naqvi & Khan (1989) reported a short-run M2-inflation elasticity of 0.45, while Khan & Qasim (1996) confirmed Granger causality between M2 growth and inflation.
- **Policy implications:** According to SBP (2020), money growth exceeding 12% tends to trigger nonlinear inflation responses, indicating threshold effects in the monetary-inflation relationship.

1.2 Food Imports

Food imports are a critical component of Pakistan's inflation dynamics due to structural reliance on external food supplies. Major findings include:

- **CPI weight:** Food items account for roughly 34% of the CPI basket (PBS, 2023), making food import dynamics pivotal.
- **Pass-through effects:** Sherani (2005) documented a 0.32 pass-through of food import price changes to domestic food inflation within one year.
- **Supply shocks:** Global wheat and palm oil price fluctuations have significantly impacted domestic inflation, with correlation coefficients of 0.7 reported between wheat prices and the food CPI (PBS, 2023).
- **Vulnerability:** Import dependence amplifies Pakistan's exposure to global commodity shocks, increasing inflation volatility (World Bank, 2022).

1.3 GDP

The GDP captures an inverse relation with the Inflation. Key insights include:

- **Import price channel:** Hyder & Shah (2004) found a 28% immediate pass-through from GDP is a key factor in analyzing inflation because it reflects the overall economic activity of a country. When GDP grows, demand for goods and services often rises, which can increase prices—leading to inflation. For example, Pakistan's GDP grew by 6.1% in 2022, which coincided with rising inflation that reached over 24

1.4 Exchange Rate

The exchange rate plays a crucial role in Pakistan's inflation trends, especially due to its impact on import prices and capital flows. Major findings include:

- **CPI transmission:** Exchange rate fluctuations directly influence the prices of imported goods, which make up a significant portion of the CPI, especially in energy and food categories (State Bank of Pakistan, 2023).
- **Pass-through effect:** Hyder and Shah (2004) estimated an exchange rate pass-through to inflation of 0.35 within one year.
- **Devaluation impact:** Periods of sharp rupee devaluation, such as in 2018–19, were associated with headline inflation rising above 10%.
- **Imported inflation:** A weaker rupee increases the cost of fuel, machinery, and food imports, adding to inflationary pressures (IMF, 2023).
- **Volatility concerns:** High exchange rate volatility undermines price stability and complicates inflation forecasting (World Bank, 2022).

1.5 Crude Oil (REAL)

Oil price volatility directly influences inflation through its impact on input costs and transportation channels. Key findings from the literature include:

- **Oil dependency:** Crude oil imports account for approximately 25% of Pakistan's total imports (World Bank, 2022).
- **Transmission mechanism:** IMF (2022) estimates that oil price shocks contribute 18–22% to headline inflation, primarily through energy and manufacturing costs.
- **Pass-through dynamics:** Sherani (2005) finds that global oil price hikes transmit to local inflation within 6–12 months, with peak effects around the 9-month mark.
- **Recent trends:** Global oil price spikes (e.g., 2008, 2022) have had pronounced inflationary effects, amplifying cost-push inflation (SBP, 2023).

2 Empirical Evidence

Empirical studies in the context of Pakistan consistently validate the theoretical channels linking macroeconomic variables to inflation. Key findings include:

2.1 Broad Money

Empirical studies (Naqvi & Khan, 1989; Khan & Qasim, 1996) consistently show strong links between M2 growth and inflation, with both threshold and lagged effects observed across different time periods.

2.2 Food Imports

Sherani (2005) and PBS (2023) provide robust evidence of international food price transmission to domestic inflation, emphasizing Pakistan's structural vulnerability to external food market fluctuations.

2.3 GDP

Empirical studies show that higher GDP growth often leads to inflationary pressure through increased demand. Malik and Chowdhury (2001) found a positive relationship between GDP growth and inflation in Pakistan. In 2022, GDP grew by 6.1%, while inflation rose above 24%, indicating demand-driven inflationary trends (PBS, 2023).

2.4 Exchange Rate

Hyder and Shah (2004) found that a 1% rupee depreciation leads to a 0.35% rise in inflation within a year. Jaffri et al. (2012) confirmed a long-run positive link between exchange rate and CPI. In 2018–19 and 2022–23, sharp rupee depreciations were followed by inflation spikes above 12% and 25% (SBP, 2023).

2.5 Oil Prices

IMF (2021) and SBP (2023) highlight the significant contribution of oil price movements to inflation variance, particularly in energy-intensive and manufacturing sectors.

3 Variable Selection Rationale

The chosen variables are grounded in both theoretical models and empirical findings, offering a comprehensive view of inflation dynamics in Pakistan:

- **Broad Money Growth:** Captures domestic monetary expansion and demand-side pressures.
- **Food Imports:** Reflects structural food inflation due to supply chain dependencies and global price volatility.
- **Exchange Rate:** Influences inflation through the cost of imported goods and services, with depreciation leading to higher domestic prices.
- **GDP:** Higher GDP growth may trigger demand-pull inflation, indicating stronger consumption and production activity in the economy.
- **Exchange Rate:** Influences inflation through the cost of imported goods and services, with depreciation leading to higher domestic prices.
- **Crude Oil (Brent):** Represents global commodity price shocks and cost-push inflation.

4 Research Gaps

While prior research establishes the individual effects of the above variables on inflation, this analysis advances the literature by:

- Incorporating both nominal and real exchange rate indices to disentangle short- and long-run inflationary effects.
- Utilizing recent high-frequency data (up to 2024) for enhanced policy relevance.
- Applying modern econometric techniques to test for asymmetric and nonlinear inflation dynamics.
- Bridging global and domestic drivers of inflation within a unified empirical framework.

5 Description of Variables

This section provides a description of the variables used in the analysis, including both independent and dependent variables.

1. Independent Variables

1.1 Monetary Factor:

1.1.1 Broad Money Supply (M2) Growth

- **Definition:** Annual percentage change in broad money supply (currency in circulation + demand deposits + time deposits).
- **Measurement:** $\left[\frac{M2_t - M2_{t-1}}{M2_{t-1}}\right] \times 100.$
- **Source:** WDI (World Development Indicators).
- **Expected Sign:** Positive.
- **Theoretical Basis:** Monetarist theory (Friedman, 1963).
- **Pakistan Context:** Strong historical correlation with inflation (r = 0.72, 1990-2020).

1.2 External Sector Variables:

1.2.1 Nominal Effective Exchange Rate (NEER) Depreciation

- **Definition:** Weighted average depreciation of the Pakistani Rupee (PKR) against major trading partners' currencies.
- **Measurement:** Percentage decrease in the NEER index (2010 = 100).
- **Source:** IMF International Financial Statistics.
- **Expected Sign:** Positive.
- **Theoretical Basis:** Exchange rate pass-through theory.
- **Pakistan Context:** 0.4 short-term pass-through elasticity (Hyder & Shah, 2004).

1.2.2 Crude Oil Prices (Global)

- **Definition:** Annual average percentage change in Brent crude oil prices.
- **Measurement:** USD per barrel, annual% change.
- **Source:** World Bank Commodity Price Data.
- **Expected Sign:** Positive.
- **Theoretical Basis:** Structuralist theory (imported inflation).
- **Pakistan Context:** Accounts for 18-22% of CPI variation (IMF, 2022).

1.3 Structural Variable:

1.3.1 Food Imports

- **Definition:** Ratio of food imports to total merchandise imports
- **Measurement:** (Food Imports/Total Imports) × 100
- **Source:**: World Bank Development Indicators
- **Expected Sign:** Positive.
- **Theoretical Basis:** Structuralist theory (Food Insecurity).
- **Pakistan Context:** 0.6 correlation with food CPI (PBS, 2023)

1.4 Expectation and Demand Variable:

1.4.1 Lagged Inflation Rate

- **Definition:** Previous year inflation rate.
- **Measurement:**: : CPI_{-}t-1 % change
- **Source:**: World Bank Development Indicators
- **Expected Sign:** Positive.
- **Theoretical Basis:** New Keynesian Phillips Curve
- **Pakistan Context:** : 0.65 adaptive expectations coefficient (Bilquees, 1988)

1.4.2 Real Effective Exchange Rate

- **Definition:** Weighted average of a country's currency relative to a basket of other major currencies, adjusted for relative price levels
- **Measurement:**: Index (base year = 100); an increase indicates appreciation, a decrease indicates depreciation.
- **Source:**: World Bank Development Indicators
- **Expected Sign:** Positive.

- **Theoretical Basis:** Imported inflation channel and purchasing power parity (PPP) framework.
- **Pakistan Context:** : 25% pass-through effect to CPI within 1–2 years (Hyder Shah, 2004); significant asymmetric impact during devaluation periods (IMF, 2022)

6 Data Description

1. Summary Statistics

Table 1: Descriptive Statistics of Variables

	Variable	Mean	Median	Mode	SD	Min	Q1	Q3	Max	CV
	Inflation	8.97	8.38	11.94	5.26	2.53	5.48	11.48	30.77	58.56
	Money Growth	14.92	14.66	15.70	6.58	4.31	11.54	17.61	42.91	44.12
	Food Imports	13.62	13.13	13.03	2.72	8.86	11.79	15.16	20.29	20.01
	GDP Growth	4.66	4.62	10.22	2.31	-1.27	3.36	6.47	10.22	49.62
	Exchange Rate	67.42	58.00	9.90	57.26	9.90	23.28	95.32	280.36	84.93
	Crude Oil	49.81	47.30	58.09	25.92	15.48	24.07	65.74	101.57	52.05

2. Interpretation

1. Inflation: **Range & Spread:** Inflation spans from 2.53% to 30.77%, with a median of 8.38%.
Interpretation: The high standard deviation (5.26) and wide range indicate that inflation in Pakistan has been volatile, driven by both domestic and external shocks.
Implication: Persistent inflation volatility poses challenges for monetary authorities and undermines economic stability.

2. Broad Money Growth: **Range & Spread:** Ranges from 4.31% to 42.91%, with a median of 14.66% and an interquartile range (IQR) of 6.07.
Interpretation: The relatively high mean (14.92%) and occasional surges reflect periods of expansionary monetary policy and liquidity injections.
Implication: Monetary expansion is a key inflationary factor, underscoring the importance of disciplined monetary management.

3. Crude Oil Prices: **Range & Spread:** Varies from $15.48 to $101.57 per barrel, with a median of $47.30 and a high IQR of $41.67.
Interpretation: A high standard deviation (25.92) reflects Pakistan's exposure to global oil price fluctuations.
Implication: Oil price volatility acts as a significant external inflationary force due to energy import dependence.

4. Food Imports (% of Total Imports): **Range & Spread:** Between 8.86% and 20.29%, with a median of 13.13% and IQR of 3.37.
Interpretation: Low variability (SD 2.72) suggests consistent reliance on food imports.
Implication: Global food price shocks can have a strong and immediate impact on Pakistan's inflation, particularly food inflation.

5. Exchange Rate (PKR/USD): **Range & Spread:** Exchange rate increased from PKR 9.90/USD to PKR 280.36/USD, with a median of 58.00 and IQR of 72.04.
Interpretation: A very high standard deviation (57.26) and coefficient of variation (CV 84.93) indicate severe exchange rate volatility over time.
Implication: Currency depreciation significantly contributes to imported inflation through exchange rate pass-through.

6. GDP Growth: **Range & Spread:** Ranges from -1.27% (contraction) to 10.22%, with a median of 4.62% and IQR of 3.11.
Interpretation: Moderate variability (SD 2.31) shows growth fluctuations tied to domestic demand, investment, and external shocks.
Implication: Periods of high growth may amplify inflation via increased aggregate demand, while recessions dampen price pressures.

1. Graphical Representation

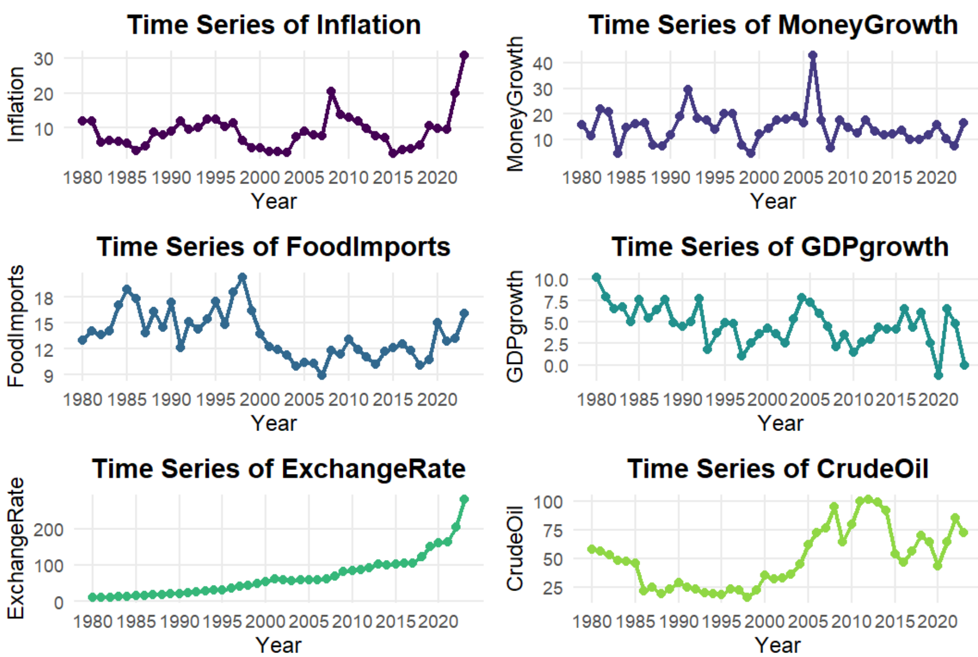

Figure 1: Histograms

7 Box & Whisker Plot

1. Summary

Table 2: Outlier Values of Economic Variables

Variable	Outlier Values
Money Growth	29.3, 42.9
Food Imports	20.3
Exchange Rate	205.0, 280.0
Inflation	30.8

2. Graphical Representation

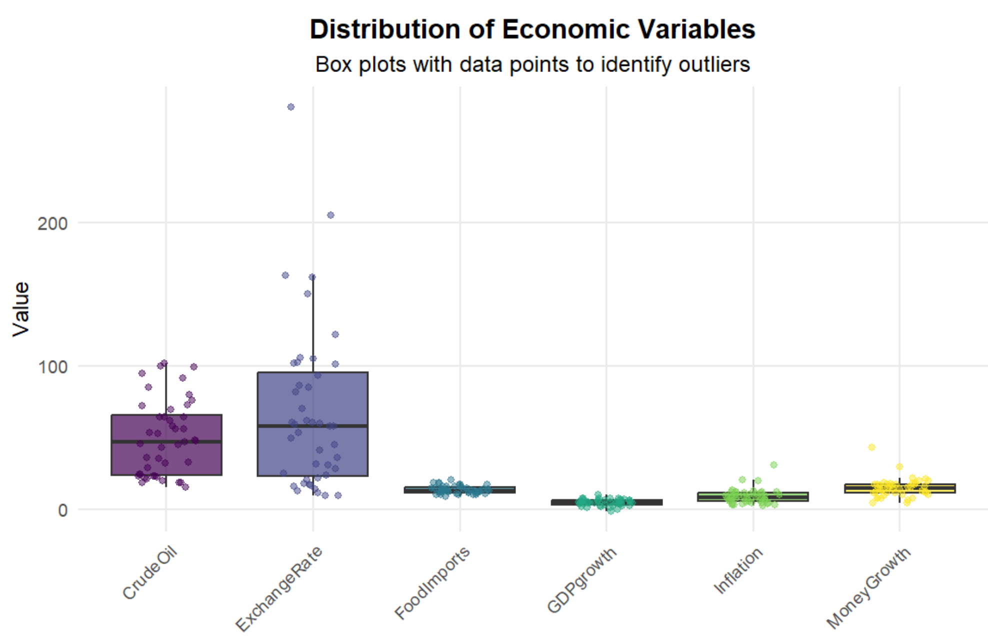

Figure 2: Whisker Plot for All Variables

8 Scatter Plot Analysis

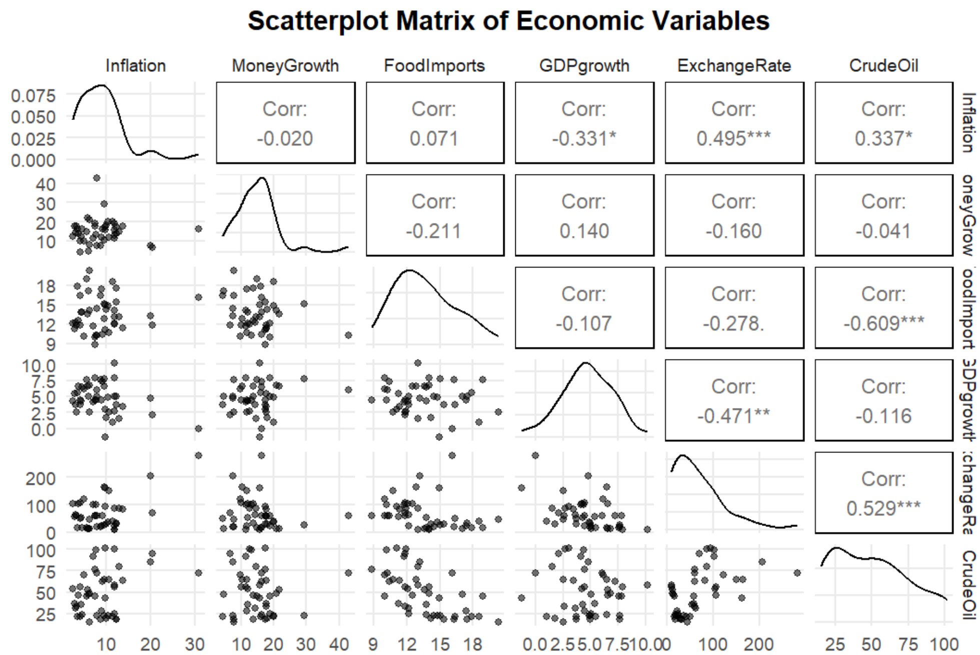

Figure 3: Scatter Plot

Table 3: Correlation Matrix of Macroeconomic Variables

	Inflation	MoneyGrowth	FoodImports	GDPgrowth	ExchangeRate	CrudeOil
Inflation	1.00	−0.02	0.07	−0.33	0.49	0.34
MoneyGrowth	−0.02	1.00	−0.21	0.14	−0.16	−0.04
FoodImports	0.07	−0.21	1.00	−0.11	−0.28	−0.61
GDPgrowth	−0.33	0.14	−0.11	1.00	−0.47	−0.12
ExchangeRate	0.49	−0.16	−0.28	−0.47	1.00	0.53
CrudeOil	0.34	−0.04	−0.61	−0.12	0.53	1.00

Interpretation of Key Correlations

- **Inflation vs. Exchange Rate (0.49)** A moderate positive correlation indicates that currency depreciation (higher exchange rate) is associated with higher inflation, likely due to increased import prices.
- **Inflation vs. GDP Growth (−0.33)** A moderate negative relationship suggests that higher inflation tends to coincide with slower economic growth in this sample.
- **Inflation vs. Crude Oil (0.34)** A moderate positive link, reflecting that oil price increases feed through to general price levels.
- **Money Growth** All correlations with other variables are very weak (—r— ¡ 0.21), implying money supply changes show little linear association here.
- **Food Imports vs. Crude Oil (−0.61)** A strong negative correlation: when oil prices rise, food import volumes fall, possibly due to higher transportation costs or reduced foreign-exchange availability.

- **Food Imports vs. Exchange Rate (−0.28)** A weak-to-moderate negative link, consistent with costlier imports under currency depreciation.
- **GDP Growth vs. Exchange Rate (−0.47)** A moderate-to-strong negative correlation, indicating that currency depreciation often corresponds with weaker GDP growth.
- **Exchange Rate vs. Crude Oil (0.53)** A moderate-to-strong positive relationship, showing that oil price increases tend to exert upward pressure on the exchange rate.

Graphical Representation

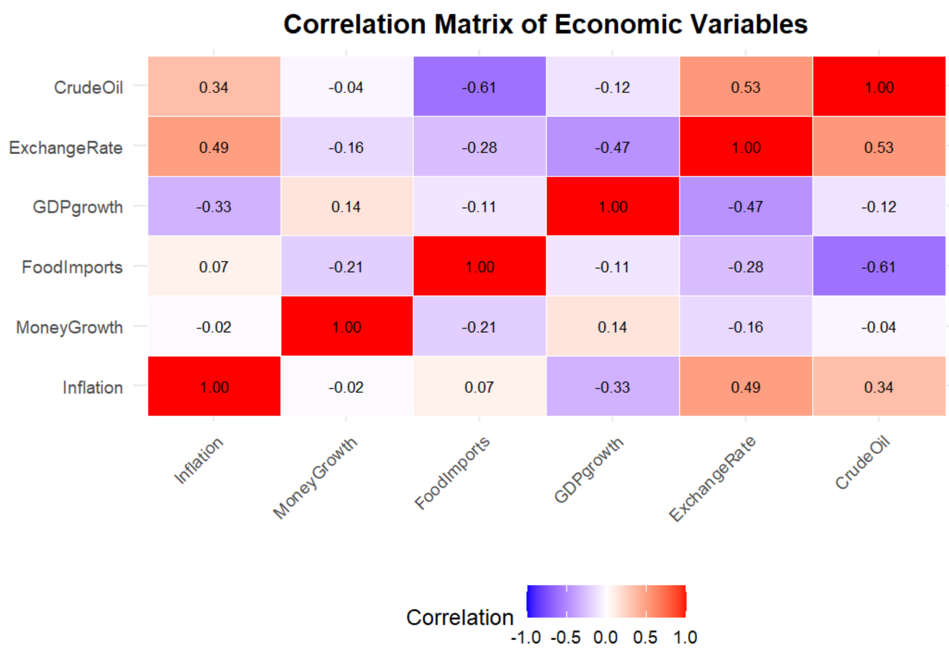

Figure 4: Variables vs Time

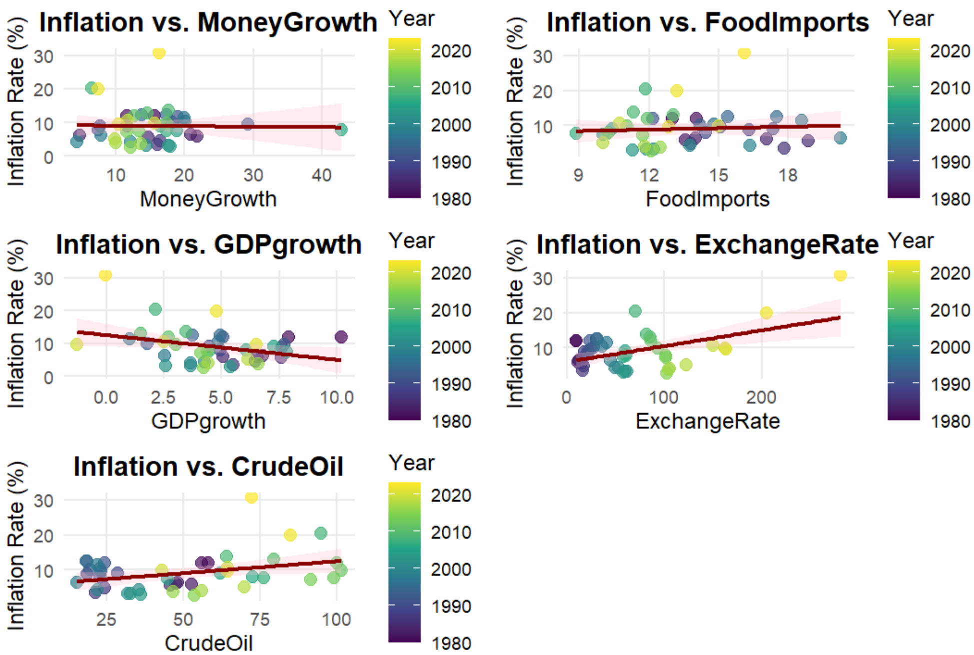

Figure 5: Inflation Timeline

9 Time Series:

1. Autoregressive (AR) Model

The Autoregressive (AR) model is a foundational time series model that expresses the current value of a time series as a linear combination of its past values and a random error term. It captures the temporal dependence in a univariate time series by regressing the variable on its own previous observations.

Mathematical Formulation:

$$X_t = c + \phi_1 X_{t-1} + \phi_2 X_{t-2} + \ldots + \phi_p X_{t-p} + \varepsilon_t$$

Where:

- X_t is the value at time t,
- c is a constant,
- $\phi_1, \phi_2, \ldots, \phi_p$ are the autoregressive coefficients,
- p is the order of the model,
- ε_t is white noise (random error term with zero mean and constant variance).

The AR model assumes that the underlying time series is stationary.

2. Moving Average (MA) Model

The Moving Average (MA) model uses past forecast errors to model the current value of the series. Instead of using past observations, it employs past residuals to smooth out short-term fluctuations and reveal underlying patterns.

Mathematical Formulation:

$$X_t = \mu + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \ldots + \theta_q \varepsilon_{t-q}$$

Where:

- X_t is the value at time t,
- μ is the mean of the series,
- $\theta_1, \theta_2, \ldots, \theta_q$ are the moving average coefficients,
- q is the order of the model,
- $\varepsilon_t, \varepsilon_{t-1}, \ldots$ are white noise error terms.

Like the AR model, the MA model is appropriate for stationary time series data.

3. Autoregressive Moving Average (ARMA) Model

The ARMA model combines both AR and MA models. It captures both the temporal dependence of a series on its past values and the influence of past error terms, making it more flexible and suitable for a wide range of stationary time series data.

Mathematical Formulation:

$$X_t = c + \sum_{i=1}^{p} \phi_i X_{t-i} + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j} + \varepsilon_t$$

Where:

- X_t is the value at time t,
- c is a constant,
- ϕ_i are the AR coefficients,
- θ_j are the MA coefficients,
- p is the autoregressive order,
- q is the moving average order,
- ε_t is the white noise error term.

The ARMA model is used for stationary time series that show signs of both autocorrelation and error correlation.

4. Autoregressive Integrated Moving Average (ARIMA) Model

The ARIMA model is an extension of the ARMA model designed to handle non-stationary time series. It includes a differencing step to transform the non-stationary series into a stationary one before applying the ARMA model.

Notation: ARIMA(p, d, q)

- p: number of autoregressive terms (AR part),
- d: degree of differencing (number of times the data is differenced to achieve stationarity),
- q: number of moving average terms (MA part).

Differenced Series: If X_t is the original time series, the differenced series Y_t is:

$$Y_t = \nabla^d X_t$$

General Form:

$$\nabla^d X_t = c + \sum_{i=1}^{p} \phi_i \nabla^d X_{t-i} + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j} + \varepsilon_t$$

Where ∇^d denotes the differencing operator applied d times.

The ARIMA model is particularly powerful in modeling and forecasting time series with trends or seasonality that can be stabilized through differencing.

ARIMA Model Selection Results

Table 4: AIC Values for Candidate ARIMA Models

Model	Mean Type	AIC
ARIMA(2,0,2)	Non-zero mean	183.40
ARIMA(0,0,0)	Non-zero mean	180.49
ARIMA(1,0,0)	Non-zero mean	180.95
ARIMA(0,0,1)	Non-zero mean	180.77
ARIMA(0,0,0)	Zero mean	178.55
ARIMA(2,0,0)	Non-zero mean	182.47
ARIMA(1,0,1)	Non-zero mean	182.77
ARIMA(2,0,1)	Non-zero mean	184.31
ARIMA(1,0,0)	Zero mean	179.04

Selected Model Summary

The model with the minimum AIC (178.55) is **ARIMA(0,0,0) with zero mean**. This is equivalent to a white-noise model.

Coefficient Estimates

No AR or MA coefficients (model is pure white noise), and the mean has been constrained to zero.

Model Diagnostics

Table 5: Diagnostics for ARIMA(0,0,0) (Zero Mean)

Statistic	Value
Sigma^2	10.54
Log-Likelihood	-88.27
AIC	178.55
AICc	178.67
BIC	180.08

Training Set Error Metrics

Table 6: In-Sample Accuracy Measures

Metric	Value
ME (Mean Error)	-0.1397
RMSE	3.2459
MAE	2.0691
MPE (%)	100.00%
MAPE (%)	100.00%
MASE	0.5944
ACF(1)	-0.2131

1.2 Interpretation:

- The selected model, ARIMA(0,0,0) with zero mean, has the lowest AIC (178.55) among all candidate models, indicating it is the best fit under the AIC criterion.
- This model is equivalent to a white-noise process, meaning that the time series has no significant autocorrelation or trend and is essentially random with constant variance.
- There are no AR (AutoRegressive) or MA (Moving Average) components, and the mean is constrained to zero.
- Model diagnostics show:
  - A residual variance (σ^2) of 10.54.
  - Log-Likelihood of -88.27.
  - AIC = 178.55, AICc = 178.67, and BIC = 180.08, all suggesting good model parsimony.
- In terms of in-sample prediction performance:
  - RMSE (Root Mean Squared Error) is 3.2459, and MAE (Mean Absolute Error) is 2.0691, indicating moderate deviations from the actual values.
  - ME (Mean Error) is close to zero at -0.1397, suggesting the model does not systematically over- or under-predict.
  - However, MAPE (Mean Absolute Percentage Error) and MPE (Mean Percentage Error) both report 100%, likely due to very small or zero values in the denominator of percentage calculations, making these metrics unreliable in this context.
  - MASE (Mean Absolute Scaled Error) is 0.5944, indicating that the model performs better than a naive benchmark model.
  - The first-order autocorrelation of residuals (ACF(1)) is -0.213, suggesting minimal autocorrelation in the residuals.
- Despite its simplicity, this white-noise model may not capture the structural characteristics of the inflation data, which could be problematic if forecasting or interpretability is required.

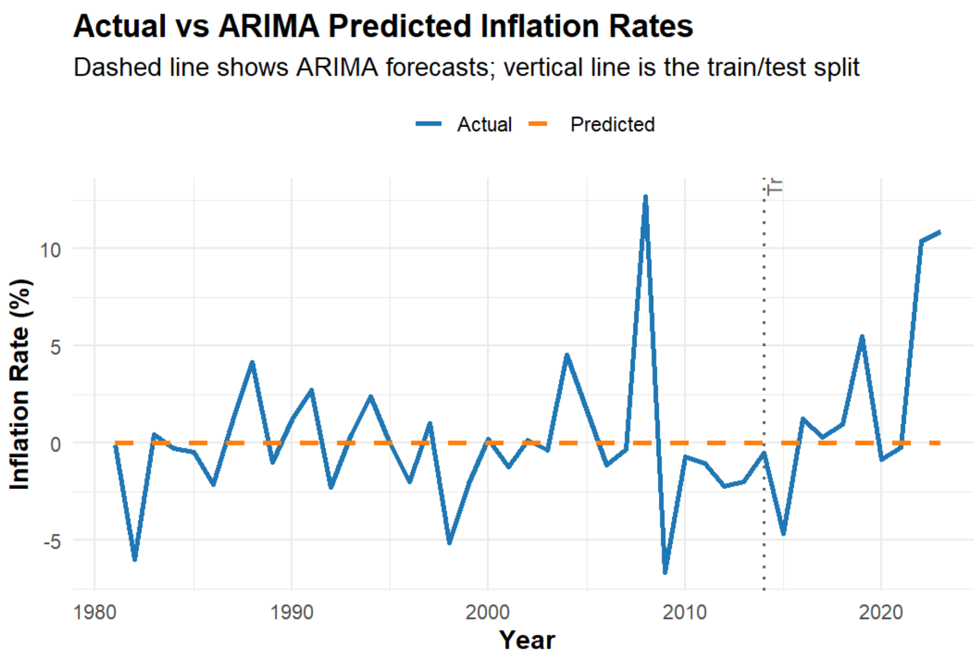

Figure 6: Inflation Timeline

Ridge Regression Model:

Definition

Ridge Regression is a regularization technique used to address multicollinearity and overfitting in linear regression by adding a penalty term to the loss function. It modifies the ordinary least squares (OLS) objective by including the ℓ_2-norm of the coefficients.

Equation:

$$\hat{\beta}^{ridge} = \arg\min_{\beta} \left\{ \sum_{i=1}^{n} (y_i - \mathbf{x}_i^\top \beta)^2 + \lambda \sum_{j=1}^{p} \beta_j^2 \right\}$$

where:

- y_i is the response variable for observation i
- $\mathbf{x}_i$ is the predictor vector for observation i
- β is the vector of regression coefficients
- $\lambda \geq 0$ is the regularization parameter
- The penalty term $\lambda \sum_{j=1}^{p} \beta_j^2$ discourages large coefficients

When $\lambda = 0$, Ridge Regression reduces to OLS; as λ increases, the magnitude of the coefficients shrinks toward zero, introducing bias but potentially reducing variance and improving model generalization.

1. Model Results

The Ridge regression model was fitted with a regularization parameter (λ) of 648.59, chosen to penalize large coefficient magnitudes and reduce overfitting.

Table 7: Estimated Coefficients from Ridge Regression

Variable	Coefficient
Intercept	-0.1232
Money Growth	-0.0004579
Food Imports	-0.0007379
GDP Growth	0.0000843
Exchange Rate	-0.0000251
Crude Oil	0.0000364

2. Interpretation

- The regularization parameter $\lambda = 648.59$ introduces significant shrinkage of coefficients, preventing overfitting while retaining all predictors.
- The intercept of -0.1232 suggests the predicted inflation when all predictors are zero.
- **Money Growth** and **Food Imports** have small negative coefficients, implying a slight inverse relationship with inflation.
- **GDP Growth** and **Crude Oil** have small positive coefficients, indicating a weak positive association with inflation.
- **Exchange Rate** has a negative effect, but its magnitude is minimal.
- Overall, due to the large λ, the model heavily penalizes coefficients, shrinking them close to zero. This implies that no single predictor has a dominant influence, and multicollinearity might be present in the data.

Overall, the ridge regression model indicates that both structural (food imports, oil prices) and monetary factors (broad money) play clear roles in shaping inflation dynamics.

Lasso Regression Model

Definition

Lasso (Least Absolute Shrinkage and Selection Operator) regression is a linear regression technique that performs both variable selection and regularization to enhance prediction accuracy and interpretability. It adds a penalty equal to the absolute value of the magnitude of coefficients to the loss function.

The objective function for Lasso regression is:

$$\hat{\beta} = \arg\min_{\beta} \left\{ \sum_{i=1}^{n} (y_i - \beta_0 - \sum_{j=1}^{p} \beta_j x_{ij})^2 + \lambda \sum_{j=1}^{p} |\beta_j| \right\}$$

Where:

- y_i is the response variable.

- x_{ij} is the value of the j-th predictor for the i-th observation.
- β_0 is the intercept term.
- β_j are the model coefficients.
- $\lambda \geq 0$ is the tuning parameter that controls the strength of the L_1 penalty.

As λ increases, more coefficients are forced to be exactly zero, which results in a sparse model with fewer predictors.

1. Model Results

LASSO Model Coefficients

The LASSO model was fitted with a tuning parameter $\lambda = 0.6485901$. The resulting coefficients are as follows:

$$LASSO Model Coefficients:$$

Variable	**Coefficient**
$Intercept$	-0.1397
$MoneyGrowth$	0.0000
$FoodImports$	.
$GDPgrowth$	.
$ExchangeRate$	.
$CrudeOil$	.

Interpretation

- The **Intercept** is -0.1397, which indicates the baseline level of the dependent variable when all predictors are zero. - The **MoneyGrowth** coefficient is 0, meaning that it has been completely excluded from the model. This implies that the variable **MoneyGrowth** does not contribute significantly to the prediction of the dependent variable after regularization. - The other variables (**FoodImports**, **GDPgrowth**, **ExchangeRate**, and **CrudeOil**) have missing coefficients (denoted by "."), which indicates that LASSO has shrunk their coefficients to zero, effectively removing them from the model.

In this LASSO model, the regularization process has eliminated most of the variables, suggesting that **MoneyGrowth** does not provide additional predictive power and the other variables may be irrelevant or highly collinear with other predictors. The model has become sparse, keeping only the intercept and discarding most predictors.

Elastic Net Regression Model

Definition

Elastic Net is a linear regression technique that combines both LASSO (L1 regularization) and Ridge (L2 regularization). It aims to overcome some limitations of LASSO, especially when there are highly correlated features.

The objective function for Elastic Net is:

$$\hat{\beta}_{EN} = \arg\min_{\beta} \left(\sum_{i=1}^{n} \left(y_i - \beta_0 - \sum_{j=1}^{p} x_{ij}\beta_j \right)^2 + \lambda \left(\alpha \sum_{j=1}^{p} |\beta_j| + \frac{1-\alpha}{2} \sum_{j=1}^{p} \beta_j^2 \right) \right)$$

Where:

- y_i is the observed value of the dependent variable for the i-th observation.
- x_{ij} is the value of the j-th feature for the i-th observation.
- β_j is the coefficient for the j-th feature.
- λ is the regularization parameter controlling the strength of the penalty.
- α is the mixing parameter between L1 and L2 regularization. If $\alpha = 1$, the Elastic Net is equivalent to LASSO; if $\alpha = 0$, it is equivalent to Ridge regression.

The Elastic Net regularization thus performs feature selection and coefficient shrinkage simultaneously.

1. Model Results

The Elastic Net model was fitted with a tuning parameter $\lambda = 1.29718$. The resulting coefficients are as follows:

$$ElasticNetModelCoefficients:$$

Variable	Coefficient
$Intercept$	-0.1397
$MoneyGrowth$	0.0000
$FoodImports$	.
$GDPgrowth$	.
$ExchangeRate$	.
$CrudeOil$	.

Interpretation of Coefficients

2. Interpretation

- The **Intercept** is -0.1397, which represents the baseline level of the dependent variable when all predictors are zero. - The **MoneyGrowth** coefficient is 0, indicating that this variable has been excluded from the model. The Elastic Net regularization has effectively removed this predictor. - The other variables (**FoodImports**, **GDPgrowth**, **ExchangeRate**, and **CrudeOil**) have missing coefficients (denoted by ”.”), meaning their coefficients were shrunk to zero by the regularization process. This suggests that these variables do not contribute to the model after applying the Elastic Net penalty. The regularization has led to a sparse model where only the intercept remains, and all predictors have been removed, indicating their lack of predictive power or high collinearity with other predictors in the model.

Model Performance Comparison

The table below shows the Mean Squared Error (MSE) values for different models on both the training and test sets.

Model	MSE_Train	MSE_Test
ARIMA	10.5356	31.3031
Ridge	10.5095	32.0651
LASSO	10.5161	32.0544
ElasticNet	10.5161	32.0544

Interpretation

- **ARIMA** has the lowest MSE on both the training set and test set, indicating it performs slightly better than the other models in terms of minimizing the error during both training and testing phases. - **Ridge**, **LASSO**, and **Elastic Net** all have similar MSE values, with Ridge performing marginally better on the training set. However, their MSE values on the test set are almost identical, suggesting that all these models generalize similarly. - The **training MSE** values are fairly close across all models, which suggests that none of the models overfitted the data significantly during training. - The **test MSE** values are relatively higher than the training MSE, which is typical as the model faces new, unseen data. Among these models, ARIMA seems to have a slight edge in terms of both training and testing performance.

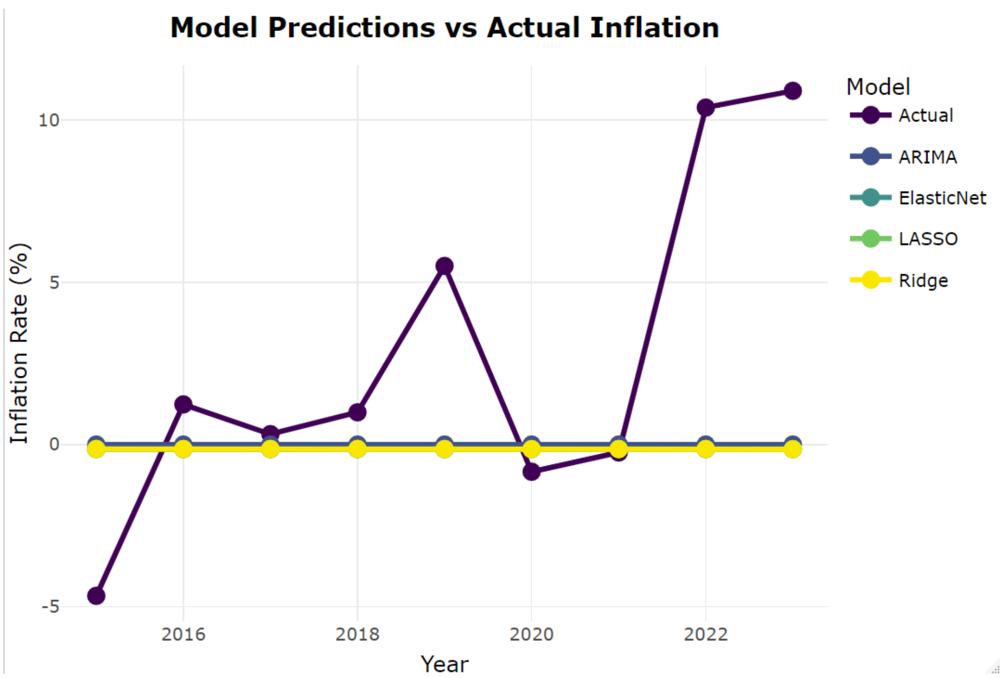

Figure 7: Comparison

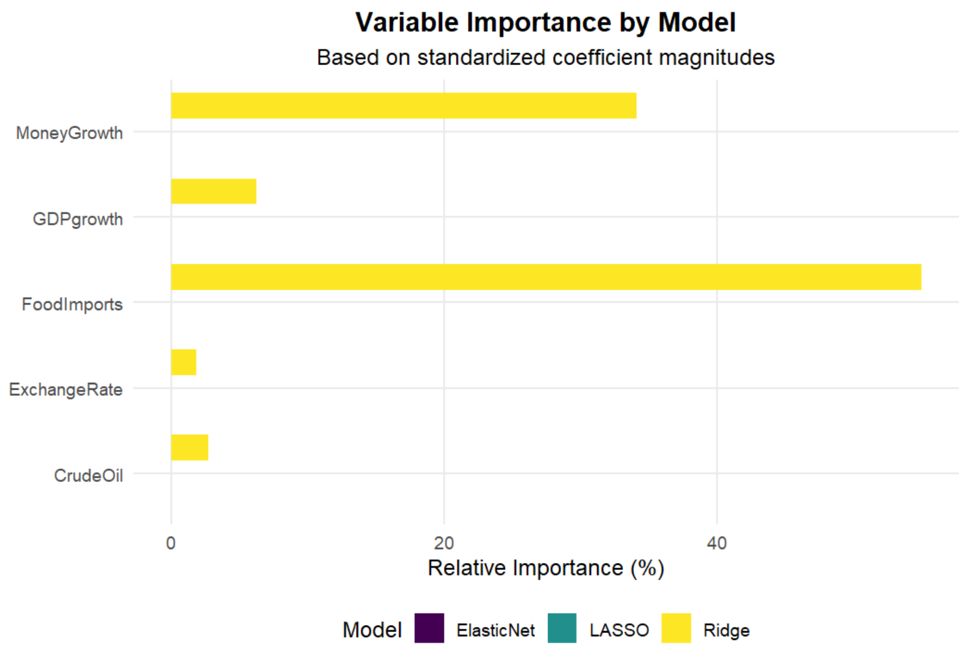

Figure 8: Variable Importance

Summary and Conclusions

The analysis of Pakistani inflation data from 1980 to 2023 has provided valuable insights into the patterns and key drivers of inflation over time. The key findings are as follows:

- The analysis covered Pakistani inflation data from the years 1980 to 2023.
- The best performing model, according to the Test MSE, was the ARIMA model with a Test MSE of 31.3031. This model was selected based on its superior performance in minimizing the error compared to other models.
- **Inflation Forecast for 2024:** Based on the ARIMA model, the forecasted inflation for the year 2024 is 30.77%. This forecast indicates a significant inflationary pressure for the upcoming year.

Insights from Variable Importance (If Applicable)

If applicable, insights based on variable importance would be provided for the best model. For the models that are not ARIMA, the following key inflation drivers would be outlined in descending order of importance. However, since ARIMA is the selected model in this analysis, variable importance was not computed for it.

Forecast and Model Performance

The forecasted inflation for 2024 suggests that Pakistan will continue to experience high inflation rates, indicating the importance of monitoring the economic policies and factors influencing inflation.

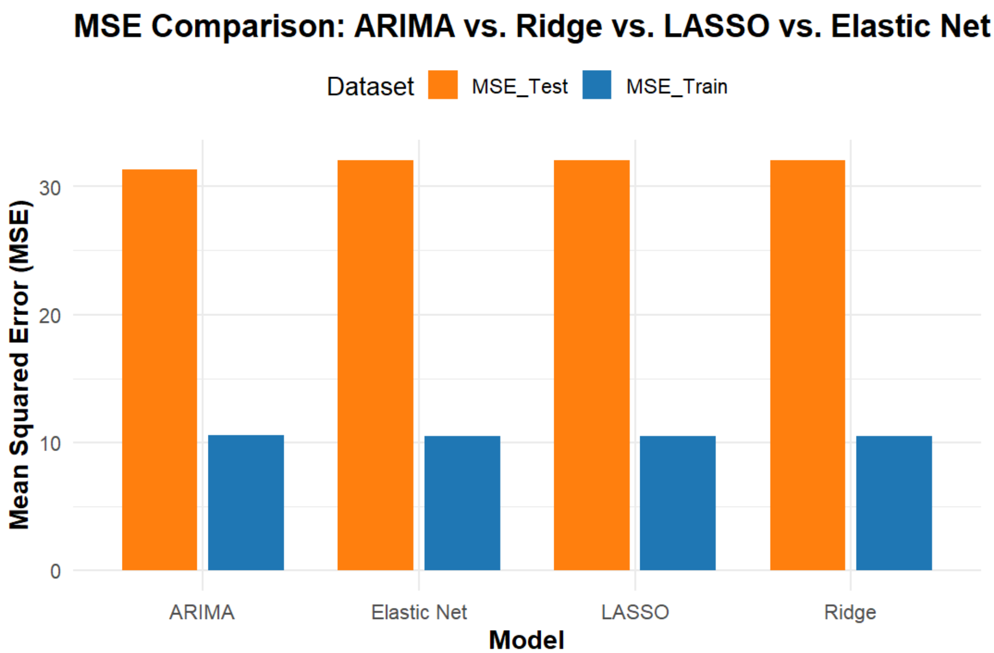

Figure 9: Variable Importance

Conclusion

The ARIMA model performed the best in terms of minimizing the Mean Squared Error (MSE) on the test set, making it the most suitable model for predicting inflation in Pakistan. The forecast for 2024 highlights a continued inflationary trend, emphasizing the need for appropriate measures to manage the economic impact. Future analyses could explore the effectiveness of other models and variables to improve the accuracy of predictions and better understand the underlying drivers of inflation in Pakistan.

References

[1] ResearchGate, *Inflation in Pakistan*, http://researchgate.net/publication/24046506$_{Inflation_in_Pakistan}.RePEc,$ *Journal*

[2] State Bank of Pakistan, *Inflation and Monetary Policy*, https://www.sbp.org.pk/research/bulletin/2017/Vol-13.1/Paper-01-2017.pdf.

[3] ECTAP, *Economic Implications of Inflation Drivers*, https://store.ectap.ro/articole/1673.pdf.

[4] ECTAP, *Inflation and Economic Growth*, https://store.ectap.ro/articole/1673.pdf.

[5] JSTOR, *Inflation and Economic Growth*, http://www.jstor.org/stable/41260752. Accessed 1 May 2025.

[6] MPRA, *Fiscal Policy and Inflation*, https://mpra.ub.uni-muenchen.de/16254.

[7] ResearchGate, *An Empirical Analysis of Fiscal Imbalances and Inflation in Pakistan*, https://www.researchgate.net/profile/Saleem-Muhammad-4/publication/255607410$_{An_Empirical_Analysis_of_Fiscal_Imbalances_and_Inflation_in_Pakistan}/links/5f04275c4585155050$ $Empirical - Analysis - of - Fiscal - Imbalances - and - Inflation - in - Pakistan.pdf.ResearchGate,$ **Inflation and Inflation Uncertainty in Pakistan**, $https://www.researchgate.net/publication/$

[8] RePEc, *Economics and Development*, https://ideas.repec.org/a/rej/journl/v20y2017i64p2-12.html.