Operating With Rational Expressions

Example Problems

April 10, 2020

Problem 1: Simplification

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$$\frac{x^2 - 16}{x^3 + 3x^2 - 16x - 48}$$

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$$\frac{x^2 - 16}{x^3 + 3x^2 - 16x - 48}$$

$$= \frac{\cancel{(x-4)}\cancel{(x+4)}}{(x+3)\cancel{(x-4)}\cancel{(x+4)}}$$

$$= \frac{1}{(x+3)}$$

Problem 2: Addition

Write the following sum as a single fraction

$$\frac{2}{x+3} + \frac{1}{2x^2 - 18}$$

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Write the following sum as a single fraction

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$$= \frac{2}{x+3} + \frac{1}{2(x-3)(x+3)}$$

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Write the following sum as a single fraction

$$\begin{aligned} & \frac{2}{x+3} + \frac{1}{2x^2-18} \\ &= \frac{2}{x+3} + \frac{1}{2(x-3)(x+3)} \\ &= \frac{2 \cdot 2 \cdot (x-3)}{(x+3) \cdot 2 \cdot (x-3)} + \frac{1}{2(x-3)(x+3)} \end{aligned}$$

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Problem 3: Multiplication

Multiply and simply as much as possible

$$\frac{12x^2 - 156x + 432}{6x^2 - 64x + 270} \cdot \frac{4x^2 - 32x + 60}{8x^2 - 8x - 96}$$

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&\frac{12x^2 - 156x + 432}{6x^2 - 64x + 270} \cdot \frac{4x^2 - 32x + 60}{8x^2 - 8x - 96} \\
&= \frac{(x^2 - 13x + 36)}{(x^2 - 14x + 45)} \cdot \frac{(x^2 - 8x + 15)}{(x^2 - x - 12)} \\
&= \frac{(x-4)(x-9)}{(x-5)(x-9)} \cdot \frac{(x-5)(x-3)}{(x-4)(x+3)}
\end{aligned}$$

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$$= \frac{(x^2 - 13x + 36)}{(x^2 - 14x + 45)} \cdot \frac{(x^2 - 8x + 15)}{(x^2 - x - 12)}$$
$$= \frac{\cancel{(x-4)}\cancel{(x-9)}}{\cancel{(x-5)}\cancel{(x-9)}} \cdot \frac{\cancel{(x-5)}(x-3)}{\cancel{(x-4)}(x+3)}$$

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$$= \frac{(x^2 - 13x + 36)}{(x^2 - 14x + 45)} \cdot \frac{(x^2 - 8x + 15)}{(x^2 - x - 12)}$$
$$= \frac{\cancel{(x-4)}\cancel{(x-9)}}{\cancel{(x-5)}\cancel{(x-9)}} \cdot \frac{\cancel{(x-5)}(x-3)}{\cancel{(x-4)}(x+3)}$$
$$= \frac{x-3}{x+3}$$