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Hierarchical Model of Corruption: Game-Theoretic Approach

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Contents

1 Introduction

Transparency International [1] defines corruption as *"the abuse of entrusted power for private gain"*. It is a problem that is recognized everywhere in the world and, sadly, Russian Federation (as shown in Buckley [2]) is not only no exception but even a "leader" placing 129 of 180 countries in Corruption Perception Index of 2019 [3] (very corrupt), which shows the relevance of the problem.

Corruption occurs in relations between people and companies - agents who, in order to benefit from it, should make strategic decisions. This condition makes it possible to use game-theoretic apparatus to analyze it and there are many scientific works on the topic yet they mostly address the corruption in form of game between two or maximum three players. This research differs in its approach: it broadens the scope and analyzes corrupt officials acting not as isolated agents but as parts of a bigger hierarchical structure in hope to obtain insights that may help to combat corruption in organizations.

Research object is corruption (embezzlement and bribery) inside a hierarchy.

Aim of this study is to analyze corruption in hierarchical context and find conditions under which it is minimal.

Objectives:

1. Study the relevant literature.
2. Create and study the hierarchical model of corruption (both non-cooperative and cooperative cases).
3. Write a code simulation for the model.
4. Solve the particular case of the model.

5. Analyze the solution.

6. Find the conditions for corruption minimization.

2 Main part

2.1 Literature review

Spengler [4] in great depth (analysis, two extensions, three player types, laboratory experiments) studies the extensive-form game between Client, Official and Inspector (Figure 2.1) and improves previous models by making probabilities of actions endogenous, suggests mixed equilibrium as solution and asymmetric penalties (with focus on officials) as anti-corruption measure. The carcass of the game inspired the inspection stage of this research.

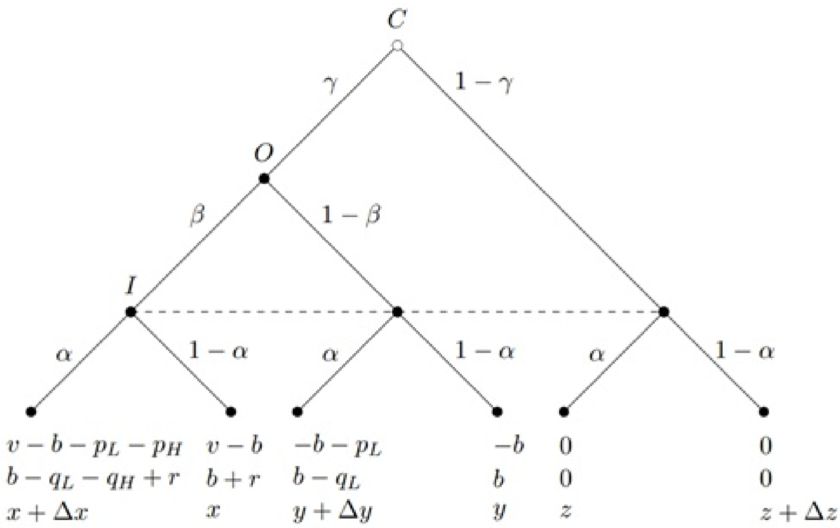

Figure 2.1: Extensive-form game without reporting.

Attanasi et al. [5] focus on the psychological aspect of embezzlement game with player triplet Donor-Intermediary-Recipient. They study what moral presuppositions players have and experimentally derive what their irrational guilt-aversion moves are. Their results showcase that stealing intermediary has guilt towards both the recipient, whose payoffs he affects, and the donor, whose he does not. The study suggests that if the results are true producing high public expectations of morality of intermediaries would reduce embezzlement.

Shenje [6] studies Briber-Bribee (based on Zimbabwean public sector corruption) and comes to the mixed Nash Equilibrium solution based on the values of costs and incomes. The way to affect these values is again top-down and varies from policy recommendations to educating the officials. Song et al. [7] focuses on Committee-Department embezzlement game (based on Chinese corruption) and comes to conclusions similar to Shenje.

Zyglidopoulos et al. [8] studies corruption in multinational companies and outlines tetrad of conditions needed for its success:

1. Existence of opportunity for corrupt action.
2. Small risk of negative repercussions.
3. Willingness to engage in corrupt activity.
4. Capability to act in a corrupt way.

Kumacheva [9] presents a multi-stage hierarchical game, which studies corruption in forms of tax evasion and auditor bribing, which inspired the model of this study. The work considers three-level structure: administration, inspector, taxpayers. Taxpayers declare their level of income and choose the size of bribe, administration chooses probabilities of auditing and reauditing, inspector chooses to accept or reject the suggested bribe. The solution suggests that the administration should choose probabilities of auditing and reauditing that depend on the tax, penalty and fine rates and taxpayers should declare their true level of income. The extension for inspection mistakes is also considered.

Gorbaneva et al. [10] analyze corruption via hierarchical control systems, namely investment-construction projects and electricity theft. Hierarchy is comprised of triplet "principal-supervisor-agent". In the first system the supervised competition for resources is considered and allocations in situations of no bribes and Nash equilibrium in simultaneous bribing game of n agents are suggested with comparison between corrupt and non-corrupt cases considered for $n = 2$. The condition for bribing to be unprofitable for the supervisor is provided. In the second system the electricity provider (principal) sends the inspector (supervisor) to check whether the client company (agent) declares their consumption truthfully (which is akin to tax evasion problem). The condition for the agent to report the actual consumption and the ways for the principal to ensure this condition are given.

Gorbaneva and Ougolnitsky [11] study concordance of public and private interests models with different profit functions of the society and individuals. The main parameter of analysis is price of anarchy (ratio of values of the game in the worst Nash equilibrium to the best situation) and social price of anarchy (the same but the public benefit is used instead of values of the game). The utility of using impulsion (economic) and compulsion (administrative) methods to improve these parameters is examined. The ideas of meta-game synthesis (including corrupt version) are suggested.

Vasin and Panova [12] discuss corruption (taxpayers' evasion and bribing the inspector) in transition economies taking Russia as example. Their model depicts a hierarchical game: homogenous population of taxpayers with income distributed according to some density function, each taxpayer declares a level of income that maximizes their utility and the authority chooses the audit probability that does the same for it. The non-corrupt models of progressive tax and linearly dependent on undeclared income fines are studied. The corrupt model includes homogeneous taxpayers with two possible levels of income (high and low), inspecting auditor which can be bribed and center which tries to maximize its payoff - sum of all taxes and fines minus costs of inspection (auditor checks taxpayer) and reinspection (center checks auditor on a declared low taxpayer). Mathematical solutions based on parameters (size of tax, fine, bribe, costs of inspection and reinspection) is suggested. Authors also describe possible applications of their results to the Russian economy, they give the optimal audit probability for the rates of 1997, the cut-off difference between a priori and declared profit, probabilistic cut-off for enterprises to be audited selection, warning on the irrelevance of the assumptions in case of organized corruption.

Savvateev [13] studies corruption and lobbying in transition economies. The first model includes utility-maximizing manufacturers that compete for a production resource. In the first case there is a possibility of lobbying (which costs some amount of resource) to get subsidies which are collected as taxes from manufacturers; in the second case there is no such possibility and there is a free market of the resource; in the third case there is a mix. With the fixed tax rate the second always Pareto dominates the first, nonetheless there are situations in which the majority of agents will vote against the transition to free market (for example, those who have the bigger amounts resource benefit from subsidies because they can allocate more amounts into lobbying to get it), even though the total production of the latter is lower.
The second model studies "principal-agent" framework of the controlling superior and the working subordinate (subordinates) in a Stackelberg competition. Each subordinate simultaneously chooses the level of corruption knowing what investigation intensities (based on the levels of corruption) the superior allocated. Cut-off strategies that constitute a strong Nash equilibrium (coalitionally or anti-coalitionally stable) are suggested to be the solution of the game. For the one-type subordinates (equal corruption opportunities) the superior can ensure less than absolute level of corruption (the value depends on size of fine and amount of available resources). In

case of two types of subordinates there is a "chain reaction effect": the less corrupt agents choose not to be corrupt at all and the more corrupt agents choose to be corrupt, yet get all the attention of the superior, who does not waste any resources on checking the first type agents, then in second iteration agents of second type reduce their level of corruption, i.e. the choice of less corrupt affects the choice of more corrupt. In case of N types the conditions for "chain reaction effect" to occur. The suggestion similar to "broken windows theory" is given: in case of different capabilities of corruption, the authorities should fight the low-level corruption because it will affect every other level up to the top.

2.2 Model

2.2.1 Description

The corruption is modeled as a hierarchical game consisting of two stages: embezzlement and inspection. The players are supposed to be risk-neutral and utility-maximizing. Only monetary payoffs are considered (although, the monetized value of anything can be used in the formulas).

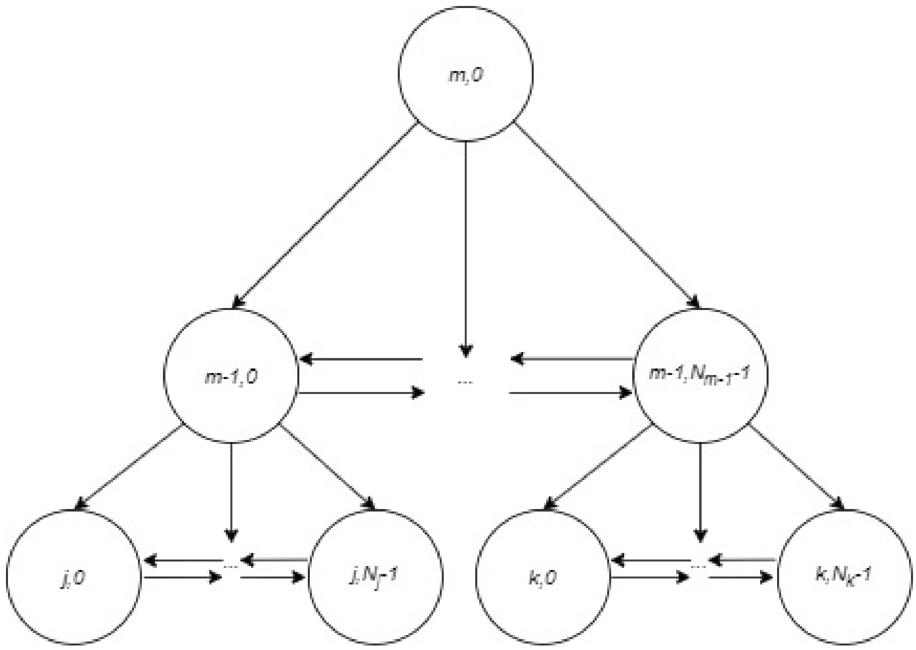

Figure 2.2: Hierarchy of the officials.

Hierarchy is a directed graph with the following meaning of the links:

- ij – i is the superior of j;
- ji – i is the subordinate of j;
- both ij and ji – i and j are colleagues (equals);
- neither ij nor ji – i and j are unrelated (they are on different levels with no superior-subordinate relationships).

In the first stage the company allocates amount of money M_m to solve a problem. This money goes down the hierarchy (Figure 2.2) of officials with each of them having a chance to embezzle some of it before passing it to subordinates. The cut-off value M_n is the minimal amount of money that needs to leave level n in order to create at least semblance of work (before bloating the budget). G_n is the amount of money entering level n. The steal

$$S^*_{n,i} = \frac{G_n - M_n}{N_n}$$

is optimal. Any $S_{n,i} > S^*_{n,i}$ is not optimal since it either breaks the cut-off condition or causes stealing from a colleague on the same level. Any $S_{n,i} < S^*_{n,i}$ is not optimal since it is possible to get more. It is also important to note that $S^*_{n,i}$ is optimal from the risk-neutral and utility-maximizing perspective only in case it is possible to bribe the inspector with the amount of money less than the stealing; otherwise, it is better not to steal at all.

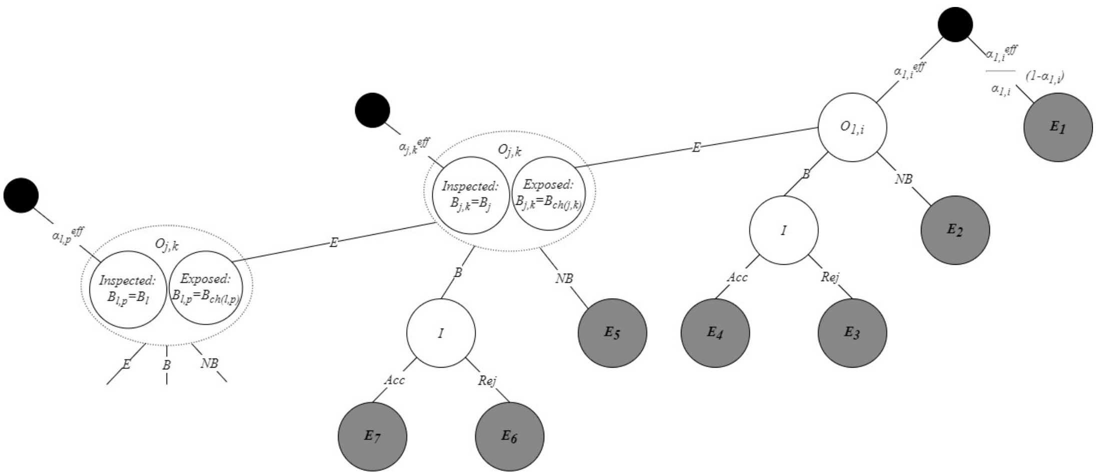

Figure 2.3: Graph of the inspection game.

$$I - inspector; \quad O_{x,y} - official\ (x,y); \quad E_Z - end\ (outcome)\ Z$$

$$boss(1) = (j,k)$$

$$boss(j) = (l,p)$$

In the second stage inspector checks some official $O_{n,i}$ for corruption. The inspector has perfect technology, so, if there has been an embezzlement, it will be revealed. The probability of inspection is proportional to the total amount of stealing up to this level. From the inspector's point of view, all officials on one level are equivalent.

The inspected official has three possible actions (Figure 2.3):

1. B – attempt to bribe the inspector (size is a natural number chosen at will);

2. NB – do not attempt to bribe the inspector;

3. E – expose the stealing of someone who stole more (boss).

$$S_n = \sum_{i=0}^{N_n-1} S_{n,i} \tag{2.1}$$

$$S_n = \sum_{i=0}^{N_n-1} S_{n,i}$$

$$\alpha_n^{eff} = \alpha_n \prod_{k=n+1}^{m} (1 - \alpha_k)$$

$$\alpha_m^{eff} = \alpha_m = 0$$

$$\alpha_{n,i} = \frac{\alpha_n}{N_n}$$

$$\alpha_{n,i}^{eff} = \frac{\alpha_n^{eff}}{N_n}$$

In the first case depending on the size of the bribe inspector either accepts or rejects it. If the bribe is accepted, official $O_{n,i}$ loses it but keeps full stealing. Let $0 \leq \kappa_{n,i} \leq 1$ be the part of stealing that official managed to hide (offshore company, friend or relative). Then in case of rejected bribe the official keeps the amount $\kappa_{n,i} S_{n,i}$, loses the bribe and will have to pay fines for steal $F(W_{n,i}, S_{n,i})$ and bribe $Fb(B_(n,i))$. In the second case the official pays full fine and keeps $\kappa_{n,i} S_{n,i}$. In the third case the exposed official $O_{l,j}$ is making a decision. Let $0 \leq \theta_{n,i} \leq 1$ be the part of fine that official will have to pay because of cooperation (he will be pardoned from paying $(1-\theta_{n,i})F(W_{n,i}, S_{n,i})$). If official $O_{l,j}$ does not bribe the inspector or the bribe is rejected, $O_{n,i}$ will have to pay $\theta_{n,i} F(W_{n,i}, S_{n,i})$. If the bribe is accepted then stealing of both officials will be covered up and no fine will be imposed, stealing will be kept in full.

The inspector decides to accept the bribe and cover the stealing up or to reject the bribe and investigate further. He also bears the inspection cost Ci_n in any case and cover-up cost $Cu(S_{n,i})$ if the bribe is accepted.

Payoffs in each end are as follows (in format $E_X : U_{j,k}\ ;\ U_{1,i}\ ;\ U_I$):

$E_1 : W_{j,k} + S_{j,k}\ ;\ W_{1,i} + S_{1,i}\ ;\ W_I$

$E_2 : W_{j,k} + S_{j,k}\ ;\ W_{1,i} + \kappa_{1,i} S_{1,i} - F(W_{1,i}, S_{1,i})\ ;\ W_I + R(S_{1,i}) - Ci_1$

$E_3 : W_{j,k} + S_{j,k}\ ;\ W_{1,i} + \kappa_{1,i} S_{1,i} - (F(W_{1,i}, S_{1,i}) + B_{1,i} + Fb(B_{1,i}))\ ;\ W_I + R(S_{1,i}) - Ci_1$

$E_4 : W_{j,k} + S_{j,k}\ ;\ W_{1,i} + S_{1,i} - B_{1,i}\ ;\ W_I + B_{1,i} - Ci_1 - Cu(S_{1,i})$

$E_5 : W_{j,k} + \kappa_{j,k} S_{j,k} - F(W_{j,k}, S_{j,k})\ ;\ W_{1,i} + \kappa_{1,i} S_{1,i} - \theta_{1,i} F(W_{1,i}, S_{1,i})\ ;\ W_I - (Ci_1 + Ci_j) + R(S_{1,i}) + R(S_{j,k})$

$E_6 : W_{j,k} + \kappa_{j,k} S_{j,k} - (F(W_{j,k}, S_{j,k}) + B_{j,k} + Fb(B_{j,k}))\ ;\ W_{1,i} + \kappa_{1,i} S_{1,i} - \theta_{1,i} F(W_{1,i}, S_{1,i})\ ;\ W_I - (Ci_1 + Ci_j) + R(S_{1,i}) + R(S_{j,k})$

$E_7 : W_{j,k} + S_{j,k} - B_{j,k}\ ;\ W_{1,i} + S_{1,i}\ ;\ W_I + B_{j,k} - (Ci_1 + Ci_j + Cu(S_{1,i}) + Cu(S_{j,k}))$

Table 2.1: Ends' descriptions.

End	Description
1	No inspection.
2	Subordinate is inspected, no bribe.
3	Subordinate is inspected, bribe is rejected.
4	Subordinate is inspected, bribe is accepted.
5	Boss is exposed by the subordinate, no bribe.
6	Boss is exposed by the subordinate, bribe is rejected.
7	Boss is exposed by the subordinate, bribe is accepted.

All subsequent ends are similar to $E_5,\ E_6,\ E_7$ with the difference in the set of the exposed officials.

The official's total utility of is comprised of wage, stealing and expected loss, which depends on his actions and actions of other players:

$$U_{n,i}(S_{n,i}, B_{n,i}, A_{n,i}) = W_{n,i} + S_{n,i} - \alpha^+_{n,i} L(A_{n,i}, A_{-n,i})$$

$$A_{n,i} \in \{B, NB, E\} \quad n \neq m-1;\ n \neq m;\ \ i = 0 \ldots N_n - 1$$

$$A_{m-1,i} \in \{B, NB\} \quad i = 0 \ldots N_{m-1} - 1$$

$$A_{m,0} \in \emptyset$$

$$A_I \in \{Acc, Rej\}$$

$$A_{-n,i} = (A_{k,j}, \ldots, A_I)\ \ \forall n = 1, \ldots, m-1;\ \ k \neq n;\ \ j = 0 \ldots N_k - 1$$

$\alpha^+_{n,i}$ is the chance of inspection (both direct and via being exposed) that is calculated as follows:

$$\alpha_{n,i}^{+} = \alpha_{n,i}^{eff} + \sum_{(l,j)\in SE(n,i)} \alpha_{l,j}^{+},$$

where $SE(n,i) = \{(v,p)\} :\ (v,p) \in subs(n,i)\ \&\ A_{v,p} = E$

The inspector's utility is as follows:

$$U_I(A_I, A_{n,i}, T) = W_I + \alpha_{n,i}^{+} K(A_I, A_{n,i}, T),$$

where

$$K(A_I, A_{n,i}, T) = \begin{cases} K(A_I, A_{boss(n,i)}, T \cup (n,i))\ if A_{n,i} = E \\ B_{n,i} - \sum_{(l,j)\in T}[Cu(S_{l,j}) + Ci_l]\ if A_{n,i} = B\ \&\ A_I = Acc \\ \sum_{(l,j)\in T}[R_I(S_{l,j}) - Ci_l]\ if A_{n,i} \in \{B, NB\}\ \&\ A_I = Rej \end{cases},$$

where W_I – inspector's wage, $T = \{(v,k)\}$ – set of ids of inspected and exposed officials.

The state's utility is calculated as follows:

$$U_s(A_{n,i}, A_I, T) = M_m - \sum_{j=1}^{m-1} S_j - \sum_{X\in\{I\}\cup H} W_X + \alpha_{n,i}^{+} D(A_{n,i}, A_I, T),$$

where

$$D(A_{n,i}, A_I, T) = \begin{cases} F(S_{n,i}, W_{n,i}) + \sum_{(l,j)\in T}[(1-\kappa_{l,j})S_{l,j} - R_I(S_{l,j})] + \\ + \sum_{(v,p)\in T\setminus\{(n,i)\}} \theta_{v,p} F(W_{v,p}, S_{v,p})\ if\ A_{n,i} = NB \\ D(A_{boss(n,i)}, A_I, T \cup \{(n,i)\})\ if\ A_{n,i} = E \\ D(NB, A_I, T) + B_{n,i} + Fb(B_{n,i})\ if\ A_{n,i} = B\ \&\ A_I = Rej \\ 0\ if\ A_{n,i} = B\ \&\ A_I = Acc \end{cases},$$

The level of corruption is

$$LoC = \frac{\sum_{j=1}^{m-1} S_j}{M_m}$$

In this model, conditions from Zyglidopoulos et al. [8] can be seen incorporated in the following way:

1. Opportunity exists because an official has access to the money flow.
2. Risk of negative repercussions is small since the probability of an official being inspected is small, plus he can always try to bribe himself out.
3. Willingness to engage in corruption is provided by monetary utility maximization of an agent.
4. Capability to act in a corrupt way is shown in abilities to embezzle and bribe.

2.2.2 Example

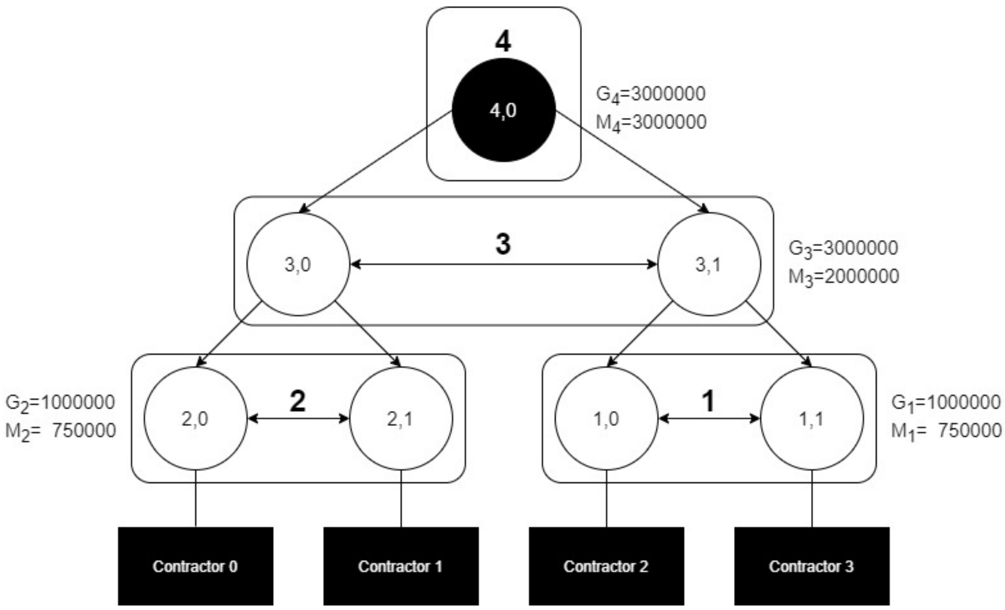

Figure 2.4: The hierarchy of officials in example.

$$boss(n,i) = \begin{cases} (4,0)\ if\ n=3 \\ (3,0)\ if\ n=2 \\ (3,1)\ if\ n=1 \end{cases}$$

For the constructed scheme, a particular example with two levels and six officials (Figure 2.4) is considered. The company (municipality) allocated 3 million to build a high-quality playground but only half of that sum was given to the contractors, the medium-quality playground is built.

The values for characteristics of players are in Tables 2.2 and 2.3.

Table 2.2: Values of officials' characteristics.

$O_{n,i}$	$W_{n,i}$	$S_{n,i}$	$\kappa_{n,i}$	$\theta_{n,i}$	$\alpha_{n,i}$	$B_{n,i}$	$F(S_{n,i})$	$Fb(B_{n,i})$
$3, i$	90,000	500,000	0.6	–	0.167	150,000	1,620,000	5,625,000
$2, i$	40,000	125,000	0.3	$1/100$	0.208	62,500	720,000	2,812,500
$1, i$	40,000	125,000	0.3	$1/100$	0.250	62,500	720,000	2,812,500

Table 2.3: Values of Inspector's characteristics.

W_I	$Ci_{\{1,2\}}$	Ci_3	$R(S_{\{1,2\},i})$	$R(S_{3,i})$	$Cu(S_{\{1,2\},i})$	$Cu(S_{3,i})$
70,000	10,000	25,000	40,000	75,000	5,000	12,500

2.2.3 Solution

The game cannot be solved via backward induction, since official does not know characteristics and utilities of boss and inspector for sure. In order to solve it, the simulation code in Python (the listing is in Appendix A) was written and executed.

The analysis of results yields the stable outcome via the following processes (assumption is that all officials are self-interested, utility maximizing and incapable of communicating with each other):

Table 2.4: Results of simulation for the initial settings.

	OptOpt_EB	OptOpt_BB	NoneOpt_NBB	OptNone_BNB	NoneNone_NBNB
$(3,0)$	523,136	564,934	565,055	90,000	90,000
$(3,1)$	535,972	565,004	564,835	90,000	90,000
$(2,0)$	165,000	156,277	40,000	162,407	40,000
$(2,1)$	165,000	156,294	40,000	162,405	40,000
$(1,0)$	165,000	158,935	40,000	160,187	40,000
$(1,1)$	165,000	158,975	40,000	160,240	40,000
I	156,602	131,233	105,137	81,219	70,000
State	1,090,000	1,090,000	1,590,000	2,090,000	2,590,000
LoC	0.500	0.500	0.333	0.167	0.000

1. Find the action yielding maximal utility for bosses.
2. Find the best response of subordinates to the 1.
3. Find the best response of bosses to the 2.
4. Repeat until there are no deviations.

$OptOpt\_BB \rightarrow OptOpt\_EB \rightarrow OptOpt\_EB$ in case of Table 2.4.

Or:

1. Find the action yielding maximal utility for subordinates.
2. Find the best response of bosses to the 1.
3. Find the best response of subordinates to the 2.
4. Repeat until there are no deviations.

$OptOpt\_EB \rightarrow OptOpt\_EB$ in case of Table 2.4.

The stable outcome is when all officials steal optimally, subordinates expose, bosses bribe and inspector accepts the bribe.

Proposition. The obtained equilibrium cannot be called Nash because due to the lack of information about Inspector's payoffs official cannot choose the optimal bribe. We suggest the notion of *Nash-like* equilibrium:

$$(S^*_{n,i}, B^*_{n,i}, A^*_{n,i}) = argmax\{U_{n,i}(S_{n,i}, B_{n,i}, A_{n,i}) \mid minB_{n,i} = B^v_{n,i}\}.$$

In that equilibrium officials maximize their utility within confines of not knowing three important things: the utility functions of Inspector, the bribe size to be suggested by their boss (or whoever ends up paying it), and the optimal bribe size. They have only hypothesis $B^v_{n,i}$ of the minimal sufficient bribe – they are not able to suggest the lesser bribe (because they believe it will be rejected).

2.2.4 Sensitivity Analysis and Corruption Minimization

In order to minimize corruption the bribe must be rejected. That will cause a loss of not hidden stealing and a fine to the official, which are supposed discourage him from stealing in the first place. The ultimate decision (to accept or reject the bribe) is made by the Inspector. Since Inspector maximizes his utility, it depends on which action yields the most profit, i.e. the sign of the inequality

$$U_I(Acc) \gtreqless U_I(Rej) \rightarrow B_{n,i} - \sum_{(l,j)\in T} Cu(S_{l,j}) \gtreqless \sum_{(l,j)\in T} R_I(S_{l,j})$$

The corruption is minimized when

$$B_{n,i} - \sum_{(l,j)\in T} Cu(S_{l,j}) \leq \sum_{(l,j)\in T} R_I(S_{l,j})$$

$$\sum_{(l,j)\in T} [R_I(S_{l,j}) + Cu(S_{l,j})] \geq B_{n,i}$$

At the same time the size of bribe is chosen by the official and since they do not know what bribe is sufficient, they chooses somewhat blind. Yet in order to remain corrupt he must get more from stealing and bribing than from not doing so:

$$U_{n,i}(S^*_{n,i}, B^*_{n,i}, B) - U_{n,i}(0, 0, NB) = S^*_{n,i} - \alpha^+_{n,i} B^*_{n,i}$$

In case when $\alpha^+_{n,i} B^*_{n,i} \geq S^*_{n,i} \rightarrow B^*_{n,i} \geq \frac{S^*_{n,i}}{\alpha^+_{n,i}}$ inspected official prefers not to steal in the first place. Thus in order to minimize the corruption the reward and cover up function values should satisfy

$$\sum_{(l,j)\in T} [R_I(S_{l,j}) + Cu(S_{l,j})] \geq \frac{S^*_{n,i}}{\alpha^+_{n,i}} \; \forall T,$$

in the best case $T = \{O_{n,i}\}$, in the worst case –

$$T = \{O_{n,i}, O_{j,k}, O_{l,p}, \dots\} \; O_{j,k} \in SE(n,i); \; O_{l,p} \in SE(j,k)$$

In order to be accepted, the bribe for inspected chain T must be:

$$B_{optT} > \sum_{(l,j)\in T} [R_I(S_{l,j}) + Cu(S_{l,j})] \geq \frac{S^*_{n,i}}{\alpha^+_{n,i}}$$

$$B_{optT}(\zeta) > \sum_{(l,j)\in T} [R_I(S_{l,j}) + Cu(S_{l,j})] + \zeta \; \zeta > 0$$

For the corruption minimization, it must hold that

$$B_{optT}(\zeta) \geq \frac{S^*_{n,i}}{\alpha^+_{n,i}} \rightarrow \alpha^+_{n,i} B_{optT}(\zeta) \geq S^*_{n,i}$$

All conclusions valid for $\zeta = x > 0$ are valid for any $\zeta > x$. Obviously, the more cautious (or afraid) the official is, the higher ζ is and the easier it is to deter them from corruption.

Let us provide the example. There are three possible types of chains in the studied hierarchy:

$$T_s = \{O_{2,i}\}; \{O_{1,i}\} \quad T_b = \{O_{3,i}\} \quad T_{ch} = \{O_{2,i}, O_{3,0}\}; \{O_{1,i}, O_{3,1}\} \quad i = 0, 1$$

For simplicity, since levels 1 and 2 are alike (and officials within them are identical), suppose

$$B_{1,i} = B_{2,i} = B_s$$

Since it has already been established that it is optimal for the subordinates to expose their bosses, fighting corruption in chains T_s is senseless: no matter how big the needed bribe is, they will not pay it. It is more useful to fight corruption in chain T_{ch} (make being exposed unprofitable for bosses), then T_b (make being directly inspected unprofitable for bosses) and then T_s under the circumstances of $S_3 = 0$ while following the logic of bigger bribe for bigger stealing. It is possible to formulate three settings, each stricter than the previous.

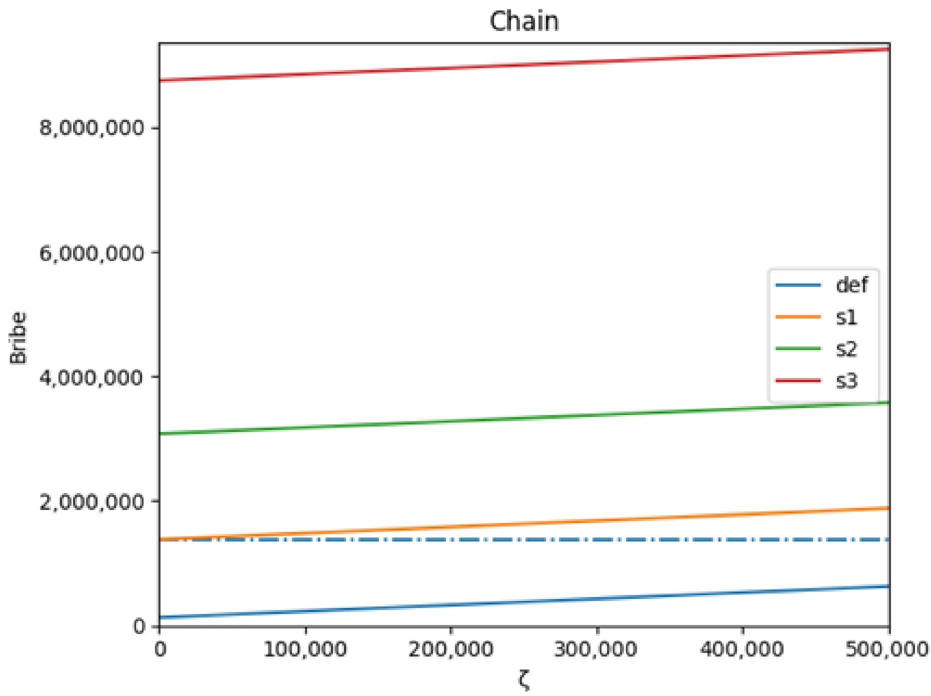

Figure 2.5: The graph of $B_{optT}(\zeta)$ for boss and T_{ch}.

The height of the dash-dot line is

$$\frac{S^*_{3,i}}{\alpha^+_{3,i}} = \frac{500,000}{\frac{\alpha_3}{2} + min[\alpha^+_{2,0} + \alpha^+_{2,1}; \alpha^+_{1,0} + \alpha^+_{1,1}]} = \frac{500,000}{0.36111111093055556} \approx 1,384,615$$

The minimum in the denominator is used to make sure stealing and bribing is not profitable for both bosses.

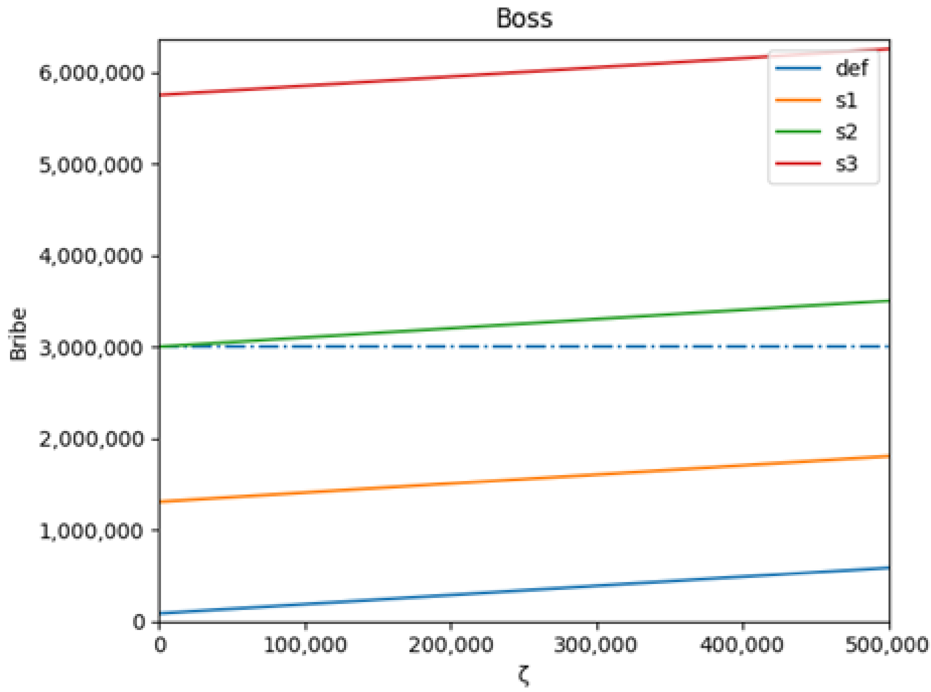

Figure 2.6: The graph of $B_{optT}(\zeta)$ for boss and T_b.

The height of the dash-dot line is

$$\frac{S^*_{3,i}}{\alpha_{3,i}} = \frac{S^*_{3,i}}{\frac{\alpha_3}{2}} = \frac{2 \cdot 500,000}{0.333} \approx 3,000,000$$

This setting is harsher since the probability in denominator is lower.

The height of the dash-dot line is

$$\frac{S_s}{min[\alpha^0_{2,i}; \alpha^0_{1,i}]} = \frac{125,000}{0.041666667} \approx 3,000,000$$

As can be seen from the figures, all possible bribes are above the dash-dot lines of a setting with the point with $\zeta = 1$ being the closest ones to

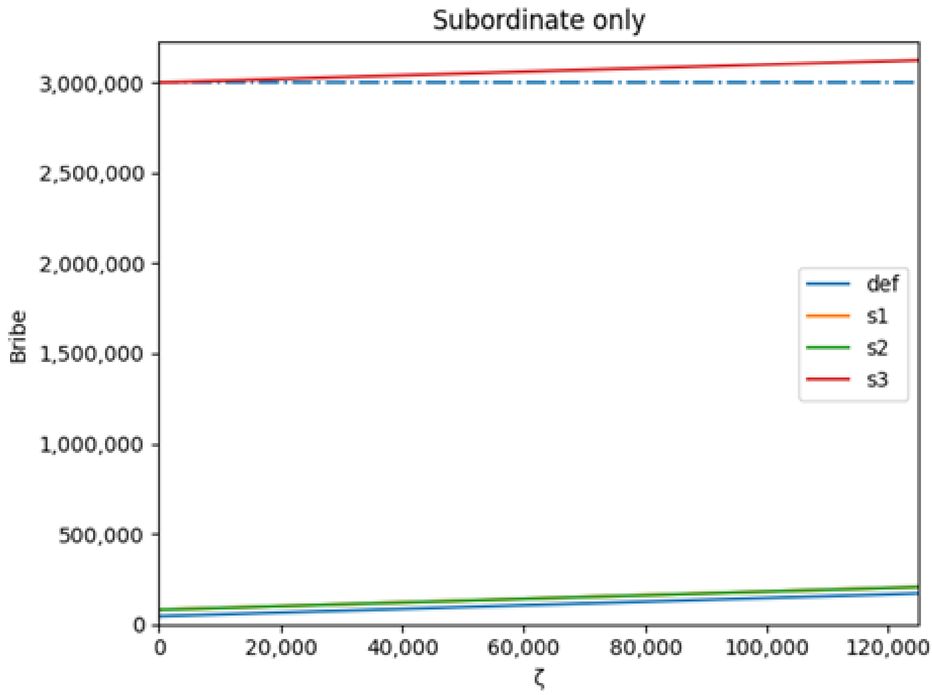

Figure 2.7: The graph of $B_{optT}(\zeta)$ for subordinate and T_s.

them. The officials of type $O_{n,i}$ with $B^{v}_{n,i} = B_{optT}(1)$ is the hardest to discourage from corruption so the corruption minimization should target them. **NB**: they are the ones playing Nash equilibrium strategies: optimal steals, minimal possible bribes.

All settings are simulated 500,000 times with utilities being averaged. The code execution results are presented in Table 2.6 and Figure 2.9 via charts of "corrupt utility" calculated as

$$CU_X = U_X - W_X$$

Due to the assumptions of officials not being able to communicate and not knowing the characteristics of each other and inspector, the averages from the stable solutions are chosen to represent the settings.

The settings changes reduce corruption and it is possible to eliminate the corruption in the model, but the means are extreme.

Table 2.5: Corruption minimization settings.

Setting	$R(S_{\{1,2\},i})$	$Cu(S_{\{1,2\},i})$	$R(S_{3,i})$	$Cu(S_{3,i})$	$B_{suff-ch}$	B_{suff-b}	B_{suff-s}	T	B_{optT}
Default	40,000.0	5,000.0	75,000.0	11,250.0	131,251.0	86,251.0	45,001.0	-	-
1	60,000.0	20,000.0	875,000.0	429,615.4	1,384,616.4	1,304,615.4	80,000.0	ch	1,384,615.4
2	60,000.0	20,000.0	2,000,000.0	1,000,000.0	3,080,000.0	3,000,000.0	80,000.0	b	3,000,000.0
3	2,000,000.0	1,000,000.0	3,250,000.0	2,500,000.0	8,750,000.0	5,750,000.0	3,000,000.0	s	3,000,000.0

Table 2.6: Change in corrupt utility after corruption minimization.

AVG	def	$s1$	$s2$	$s3$		$def \to s1$	$def \to s2$	$def \to s3$
(3, 0)	143,336.69	0.00	0.00	0.00		-100.00 %	-100.00 %	-100.00 %
(3, 1)	147,691.36	0.00	0.00	0.00		-100.00 %	-100.00 %	-100.00 %
(2, 0)	109,345.65	80,560.62	80,554.08	0.00		-26.32 %	-26.33 %	-100.00 %
(2, 1)	109,236.93	80,548.62	80,554.81	0.00		-26.26 %	-26.26 %	-100.00 %
(1, 0)	96,252.46	78,231.37	78,253.14	0.00		-18.72 %	-18.70 %	-100.00 %
(1, 1)	96,099.14	78,242.55	78,230.66	0.00		-18.58 %	-18.59 %	-100.00 %
Inspector	36,663.69	11,026.42	11,018.90	0.00		-69.93 %	-69.95 %	-100.00 %

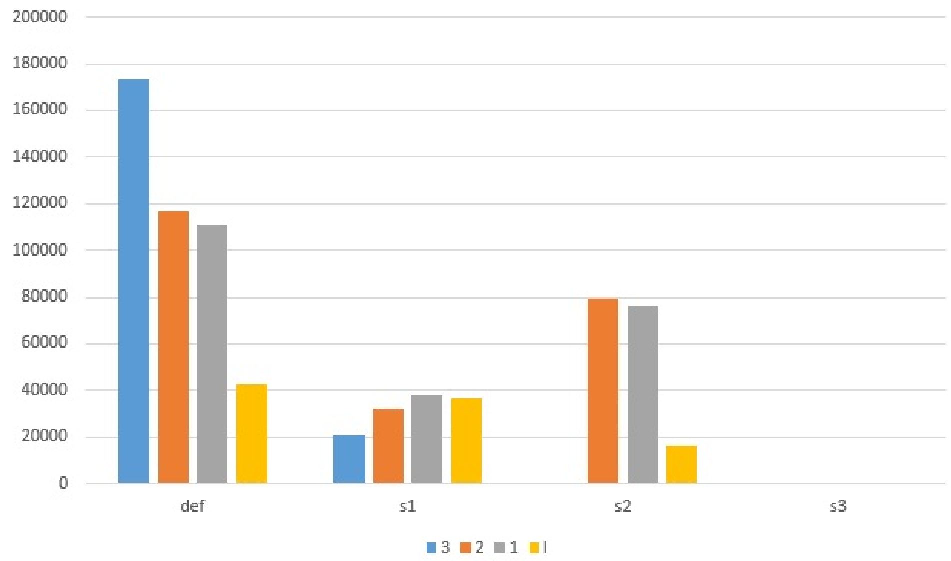

Figure 2.8: Average "corrupt utility" of players after corruption minimization.

2.2.5 Mild Corruption Minimization

The values of settings in Table 2.5 might be considered extreme or impossible to implement in real life, so let us limit the optimal bribe size:

$$B_{optT} \leq S^*_{n,i}$$

With that limitation, we have four possible settings (including default), which we will name $'zettings'$ to avoid confusion (and to amuse ourselves):

Table 2.7: Mild corruption minimization zettings.

Zetting	$R(S_{\{1,2\},i})$	$Cu(S_{\{1,2\},i})$	$R(S_{3,i})$	$Cu(S_{3,i})$	$B_{suff-ch}$	B_{suff-b}	B_{suff-s}	T	B_{optT}
Default	40000	5000	75000	11250	131251	86251	45001	-	-
1	70000	35000	270000	124999	500000	395000	105001	ch	500000
2	0	0	300000	199999	500000	500000	1	b	500000
3	85000	39999	250000	125000	500000	375001	125000	s	125000

Table 2.8: Change in utilities after mild corruption minimization.

AVG	def	$z1$	$z3$		$def \to z1$	$def \to z3$	$z1 \to z3$
(3, 0)	143,336.69	69,432.38	69,307.13		-51.56 %	-51.65 %	-0.18 %
(3, 1)	147,691.36	79,857.00	79,864.25		-45.93 %	-45.92 %	0.01 %
(2, 0)	109,345.65	76,485.57	76,168.38		-30.05 %	-30.34 %	-0.41 %
(2, 1)	109,236.93	76,497.06	76,163.28		-29.97 %	-30.28 %	-0.44 %
(1, 0)	96,252.46	75,106.62	74,542.06		-21.97 %	-22.56 %	-0.75 %
(1, 1)	96,099.14	75,127.30	74,531.44		-21.82 %	-22.44 %	-0.79 %
Inspector	36,663.69	70,989.22	71,822.14		93.62 %	95.89 %	1.17 %

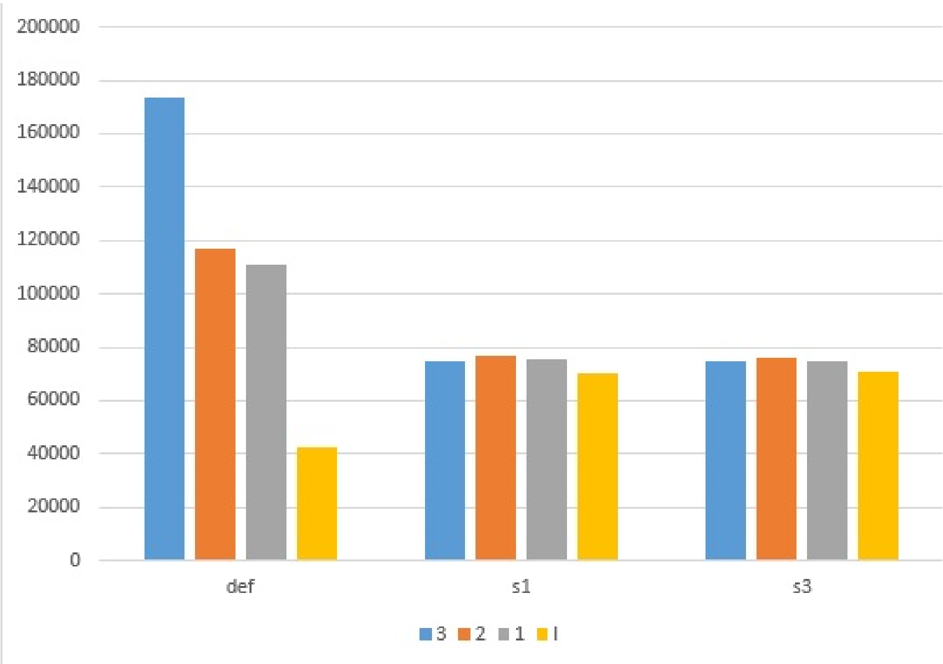

Figure 2.9: Average "corrupt utility" of players after mild corruption minimization.

The settings changes reduce corruption, decrease revenue for $O_{n,i}$ and increase for I, which might also be beneficial since focusing the corrupt money in one place simplifies control. Mild Corruption Minimization is less extreme, effective, but less so than Corruption Minimization.

2.3 Cooperative Extension of the Model

2.3.1 Description

Bosses need some way of protecting themselves from subordinate. One of the ways is to form a coalition. Two or more officials can form a coalition, in which: members cannot expose each other; members' steals are divided among them according to the stated allocation rule; bribe (in the case when one of the members is inspected) is compiled collectively.

Joining a coalition brings advantages and disadvantages. Advantages are insurance against being exposed; better coordination in terms of stealing amounts (irrelevant in the model, but might be important in real life); more certainty in terms of the sufficient bribe (grand coalition knows exactly how the inspection happened); bigger bribe (thus less chance of being rejected) with less problems conjuring up one for each of the members вЂ“ at least, potentially. Disadvantages are higher chances of being inspected; higher fines for "organized group felonies"; allocation might not be favourable for some members.

Not any group of officials can and/or should form a coalition. For example, take a pair $\{(1,0),(2,0)\}$. They do not "know" each other вЂ“ there are no ties connecting them directly, so it must be hard for them to communicate, the former cannot expose the latter because they are not in "superior-subordinate" relationships, forming this coalition is senseless and should not be possible.

We suggest the rule *"any official with direct or indirect connection (path in the hierarchy graph) to another can be in the coalition with him"*. In other words, no disconnected components are allowed in the coalition. For example, coalition $\{(2,0),(3,0),(3,1)\}$ is possible, but $\{(2,0),(3,0),(1,0)\}$

is not. It is possible to build twenty-four different coalitions according to this rule plus there are six single-player coalitions. Coalitions are characterized by:

- set of coalition members, its subsets and their sizes:

$$C = \bigcup_{(n,i)\in C} \{(n,i)\} = \bigcup_{n\in C} C_n \quad N_C = |C|$$

$$C_j = \bigcup_{(j,i)\in C} \{(j,i)\} \quad N_{C,j} = \sum_{(j,i)\in C} 1 = |C_j| \leq N_j$$

- partial utility of a member (the part official gets from stealing and potentially bribing in coalition)

$$RU^C_{n,i} = U_{n,i}(S_{n,i}, 0, BC) - W_{n,i}$$

- coalitional actions: members of coalition never expose, always bribe jointly and cannot refrain from stealing (if they do not want it is better for them not to join coalition in the first place)

$$S_{n,i} > 0 \;\&\; A_{n,i} = BC \quad \forall (n,i) \in C,$$

- coalitional stealing

$$S_C = \sum_{(n,i)\in C} S_{n,i},$$

- coalitional bribe is

$$B_C = S_C \cdot b_C,$$

- the chance of inspection

$$\alpha_C = \bigcup_{(n,i)\in C} \alpha^+_{n,i}.$$

This chance can also be portrayed as the vector of different probabilities $\alpha_C = (\alpha_{ch}; \alpha_b; \alpha_s)$ since any official but the ultimate subordinate is unsure about the source of inspection (and there are more than one official in coalition). The same applies to the coalitional bribe: $B_C = (B_{ch}; B_b; B_s)^T$. From that we get: $\alpha_C B_C = \alpha_{ch} B_{ch} + \alpha_b B_b + \alpha_s B_s$.
In the non-cooperative case for boss every term goes into α_{ch}, since they, the coalition of one, cannot know the source of inspection and every term goes into α_s for subordinates since there is no other way for them to be inspected but the direct. With increase in size of the coalition the source of inspection gets clearer.

If inspector accepts the bribe, coalition loses only it, if he does not, coalition loses the bribe and every coalition member suffers the fine for organized stealing:

$$U^C_{n,i}(A_I) = RU^C_{n,i} - \begin{cases} 0 \; if \; A_I = Acc \\ Fsc(S_C) + Fcb(B_C) \; if \; A_I = Rej \end{cases}$$

where $Fc(S_C)$ and $Fb(B_C)$ are fines for coalitional stealing and bribing respectively.

2.3.2 Allocation Rules

2.3.3 Stability

The payoff is called *individually stable* if it is in the Imputation set
$I(v) = \{X \in R^{N_C} \mid X(C) = v(C), \quad X_{n,i} \geq v(\{(n,i)\}) \; \forall (n,i) \in C\}$,
i.e. it is not worse for individual to join the coalition, than to be alone.
The payoff is called *coalitionally stable* if it is in the Core
$C(v) = \{X \in R^{N_C} \mid X(C) = v(C), \quad X(S) \geq v(S) \; \forall S \subset C\}$,
i.e. no subgroup of players has an incentive to deviate.

For the discussed example we have already derived 30 coalitions. Since officials on one level have the same characteristics, we can simplify the analysis by reducing the coalitions to check by categorizing fifteen coalition types:

Single-player

$$1P = \{\{(1,0)\},\{(1,1)\},\{(2,0)\},\{(2,1)\},\{(3,0)\},\{(3,1)\}\}.$$

Subordinate-subordinate left

$$SSL = \{\{(2,0),(2,1)\}\}.$$

Subordinate-subordinate right

$$SSR = \{\{(1,0),(1,1)\}\}.$$

Boss-boss $BB = \{\{(3,0),(3,1)\}\}$.

Boss-subordinate left

$$1B1SL = \{\{(3,0),(2,0)\},\{(3,0),(2,1)\}\}.$$

Boss-subordinate right

$$1B1SR = \{\{(1,0),(3,1)\},\{(1,1),(3,1)\}\}.$$

Boss-boss-subordinate left

$$BB1SL = \{\{(2,0),(3,0),(3,1)\},\{(2,1),(3,0),(3,1)\}\}.$$

Boss-boss-subordinate right

$$BB1SR = \{\{(1,0),(3,1),(3,0)\},\{(1,1),(3,1),(3,0)\}\}.$$

Boss-2-subordinates left

$$1B2SL = \{\{(3,0),(2,0),(2,1)\}\}.$$

Boss-2-subordinates right

$$1B2SR = \{\{(3,1),(1,0),(1,1)\}\}.$$

2-subordinates-boss-boss left

$$2SBBL = \{\{(2,0),(2,1),(3,0),(3,1)\}\}.$$

2-subordinates-boss-boss right

$$2SBBR = \{\{(1,0),(1,1),(3,1),(3,0)\}\}.$$

Subordinate-boss-boss-subordinate

$$1SBB1S = \{(\{(2,0),(3,0),(3,1),(1,0)\},\{(2,0),(3,0),(3,1),(1,1)\},$$
$$\{(2,1),(3,0),(3,1),(1,0)\},\{(2,1),(3,0),(3,1),(1,1)\})\}.$$

2-subordinates-boss-boss-subordinate left

$$2SBB1SL =$$
$$\{\{(2,0),(2,1),(3,0),(3,1),(1,0)\},\{(2,0),(2,1),(3,0),(3,1),(1,1)\}\}.$$

2-subordinates-boss-boss-subordinate right

$$2SBB1SR =$$
$$\{\{(2,0),(3,0),(3,1),(1,0),(1,1)\},\{(2,1),(3,0),(3,1),(1,0),(1,1)\}\}.$$

Grand coalition

$$GC = \{(2,0),(2,1),(3,0),(3,1),(1,0),(1,1)\}.$$

2.3.4 Analysis of the Rules

Single-player coalitions are not discussable since they are impervious to the rules.

Hypothesis 1. No coalition-rule pair can be individually stable if it does not provide officials better payoffs than they have on their own. If we take stealing scenario, it is the payoff of exposing the boss (if you can) because it is better to expose boss than to bribe inspector yourself: if they accept boss's bribe you are scot-free, and if they reject, they would have probably rejected yours – at least that behavior was modeled the most efficient so far.

Hypothesis 2. Coalition-rule pair might not be coalitionally stable if it does not include simultaneously boss and both of his subordinates: there is a possibility of excluding members of not full branch. For example, in coalitions $\{(2,0),(2,1),(3,0),(3,1),(1,0)\}$,$\{(2,0),(2,1),(3,0),(3,1),(1,1)\}$ and $\{(2,0),(2,1),(3,0),(3,1)\}$ official $(3,1)$ can be exposed by his subordinates $(1,1)$, $(1,0)$ and $\{(1,0),(1,1)\}$ respectively) and (depending on his stealings and sufficient bribe after exposure) it might be more profitable to expel them from the coalition so that $(3, 1)$'s problem is no longer problem of the coalition.

Assumptions:

1. a

2.3.5 Simulation Results

The champions for the both models will be the setting with the minimal necessary bribe given: we will compare only the best possible cases because in the case of not sufficient bribe the non-cooperative model officials have an advantage of defaulting to the "not stealing and not bribing strategy (None_NB)" while members of coalition do not. Not to mention it is quite computation-heavy and thus is hard to run. The simulation was run $500,000$ times for each coalition-rule pair for the default and all anti-corruption settings (normal and mild) with $\xi = 1$.

Conclusions from the analysis:

1. 1B1SL, 1B1SR, BB1SL and BB1SR types of coalitions do not provide stable divisions under any setting (yellow fill).

2. The 2SBBL, 2SBB1SL, 2SBB1SR and GC with SS rule coalition-rule pairs are coalitionally stable in the default setting (green fill).

3. Under setting s1 all pairs from point 2 plus 2SBB1SL and GC with EQ rule are coalitionally stable (red fill).

4. No rule provides a stable division under settings s2 and s3 (underlined blue font) – the corruption minimization settings work even in case of cooperation.

Setting	def		s1		s2		s3		z1		z3	
Coalition \ Rule	**EQ**	**SS**	**EQ**	**SS**	**EQ**	**SS**	**EQ**	**SS**	**EQ**	**SS**	**EQ**	**SS**
{(3,0),(2,0)}	N	N	N	N	N	N	N	N	N	N	N	N
{(3,0),(2,1)}	N	N	N	N	N	N	N	N	N	N	N	N
{(3,1),(1,0)}	N	N	N	N	N	N	N	N	N	N	N	N
{(3,1),(1,1)}	N	N	N	N	N	N	N	N	N	N	N	N
{(3,0),(2,0),(3,1)}	N	N	N	N	N	N	N	N	N	N	N	N
{(3,0),(2,1),(3,1)}	N	N	N	N	N	N	N	N	N	N	N	N
{(3,1),(1,0),(3,0)}	N	N	N	N	N	N	N	N	N	N	N	N
{(3,1),(1,1),(3,0)}	N	N	N	N	N	N	N	N	N	N	N	N
{(3,0),(2,0),(2,1)}	N	N	N	N	N	N	N	N	N	C	N	C
{(3,1),(1,0),(1,1)}	N	N	N	N	N	N	N	N	N	N	N	N
{(3,0),(2,0),(2,1),(3,1)}	N	C	N	C	N	N	N	N	N	C	N	C
{(3,1),(1,0),(1,1),(3,0)}	N	N	N	N	N	N	N	N	N	N	N	N
{(2,0),(3,0),(3,1),(1,0)}	N	N	N	N	N	N	N	N	N	N	N	N
{(2,0),(3,0),(3,1),(1,1)}	N	N	N	N	N	N	N	N	N	N	N	N
{(2,1),(3,0),(3,1),(1,0)}	N	N	N	N	N	N	N	N	N	N	N	N
{(2,1),(3,0),(3,1),(1,1)}	N	N	N	N	N	N	N	N	N	N	N	N
{(3,0),(2,0),(2,1),(3,1),(1,0)}	N	C	C	C	N	N	N	N	N	C	N	C
{(3,0),(2,0),(2,1),(3,1),(1,1)}	N	C	C	C	N	N	N	N	N	C	N	C
{(3,1),(1,0),(1,1),(3,0),(2,0)}	N	C	N	C	N	N	N	N	N	C	N	C
{(3,1),(1,0),(1,1),(3,0),(2,1)}	N	C	N	C	N	N	N	N	N	C	N	C
{(2,0),(2,1),(3,0),(3,1),(1,0),(1,1)}	N	C	C	C	N	N	N	N	N	C	N	C

Figure 2.10: Simulation results analysis.

5. Stability results under similar zettings z1 and z2 are similar: only SS provides stable outcomes in 1B2SL, 2SBBL, 2SBB1SL, 2SBB1SR and GC (blue fill).

2.3.6 Myerson Value

Myerson value is an adaptation of Shapley value to restricted communication graph. It is stated in Caulier et al. [14] as

$$v^g(S) = \sum_{C \in S|_g} v(C)$$

$S|_g$ denotes the set of connected coalitions of g, i.e., those sets C which are maximal subcoalitions of S such that all pairs of players in C are connected. If S is connected, then its players can communicate and therefore they obtain their initial payoff $v(S)$. Otherwise, players in coalition S can only commu-

nicate among members of the same connected component. As there is no possible communication between different components, players in S can only get the sum of payoffs obtained by each component independently.

$$M_i(v,g) = \sum_{S \subseteq N \setminus \{i\}} \frac{|S|!(n-1-|S|)!}{n!}(v^g(S \cup \{i\}) - v^g(S))$$

For this model the changed and simplified (since there is only one g studied) notation is

$$M_{n,i}(v) = \sum_{S \subseteq H \setminus \{n,i\}} \frac{|S|!(|H|-1-|S|)!}{|H|!}(v(S \cup \{n,i\}) - v(S)), \qquad (2.2)$$

where H – hierarchy, set of all officials.

Two types are studied: classical Myerson (Equation (2.2)) and its modified version Theirson. In the latter there is assumption of "playing nice" is made: subordinates choose to bribe instead of exposing turning $v(C)$ into $v^*(C)$ (Equation (2.3)).

$$T_{n,i}(v) = \sum_{S \subseteq H \setminus \{n,i\}} \frac{|S|!(|H|-1-|S|)!}{|H|!}(v^*(S \cup \{n,i\}) - v^*(S)), \qquad (2.3)$$

To calculate it, we need to write values of all "whole" (fully formable) coalitions:

$$R = \{(1,0)\}; \{(1,1)\}$$

$$\begin{aligned} v(R) &= S_s \\ v^*(R) &= S_s - (1-\alpha_3)(1-\alpha_2)\frac{\alpha_1}{2}B_s = \\ &= S_s - \frac{\alpha_1^{eff}}{2}B_s \end{aligned} \qquad (2.4)$$

$$L = \{(2,0)\};\{(2,1)\}$$

$$\begin{aligned} v(L) &= S_s \\ v^*(L) &= S_s - (1-\alpha_3)\frac{\alpha_2}{2}B_s = \\ &= S_s - \frac{\alpha_2^{eff}}{2}B_s \end{aligned} \tag{2.5}$$

$$\begin{aligned} v(\{(3,0)\}) &= S_b - (\frac{\alpha_3}{2} + \alpha_2^{eff})B_{ch} \\ v^*(\{(3,0)\}) &= S_b - \frac{\alpha_3}{2}B_{ch} \end{aligned} \tag{2.6}$$

$$\begin{aligned} v(\{(3,1)\}) &= S_b - (\frac{\alpha_3}{2} + \alpha_1^{eff})B_{ch} \\ v^*(\{(3,1)\}) &= S_b - \frac{\alpha_3}{2}B_{ch} \end{aligned} \tag{2.7}$$

$$\begin{aligned} v(SSR) &= 2S_s - \alpha_1^{eff} \\ v^*(SSR) &= v(SSR) \end{aligned} \tag{2.8}$$

$$\begin{aligned} v(SSL) &= 2S_s - \alpha_2^{eff} \\ v^*(SSL) &= v(SSL) \end{aligned} \tag{2.9}$$

$$\begin{aligned} v(BB) &= 2S_b - [\alpha_3 + \alpha_2^{eff} + \alpha_1^{eff}]B_{ch} \\ v^*(BB) &= 2S_b - \alpha_3 B_b \end{aligned} \tag{2.10}$$

$$
\begin{aligned}
v(1B1SL) &= S_b + S_s - [(\frac{\alpha_3}{2} + \frac{\alpha_2^{eff}}{2})B_{ch} + \frac{\alpha_2^{eff}}{2}B_s] \\
v^*(1B1SL) &= S_b + S_s - [\frac{\alpha_3}{2}B_b + \frac{\alpha_2^{eff}}{2}B_s]
\end{aligned} \tag{2.11}
$$

$$
\begin{aligned}
v(1B1SR) &= S_b + S_s - [(\frac{\alpha_3}{2} + \frac{\alpha_1^{eff}}{2})B_{ch} + \frac{\alpha_1^{eff}}{2}B_s] \\
v^*(1B1SR) &= S_b + S_s - [\frac{\alpha_3}{2}B_b + \frac{\alpha_1^{eff}}{2}B_s]
\end{aligned} \tag{2.12}
$$

$$
\begin{aligned}
v(BB1SL) &= 2S_b + S_s - [(\alpha_3 + \frac{\alpha_2^{eff}}{2} + \alpha_1^{eff})B_{ch} + \frac{\alpha_2^{eff}}{2}B_s] \\
v^*(BB1SL) &= 2S_b + S_s - [\alpha_3 B_b + \frac{\alpha_2^{eff}}{2}B_s]
\end{aligned} \tag{2.13}
$$

$$
\begin{aligned}
v(BB1SR) &= 2S_b + S_s - [(\alpha_3 + \alpha_2^{eff} + \frac{\alpha_1^{eff}}{2})B_{ch} + \frac{\alpha_1^{eff}}{2}B_s] \\
v^*(BB1SR) &= 2S_b + S_s - [\alpha_3 B_b + \frac{\alpha_1^{eff}}{2}B_s]
\end{aligned} \tag{2.14}
$$

$$
\begin{aligned}
v(1B2SL) &= S_b + 2S_s - [\frac{\alpha_3}{2}B_b + \alpha_2^{eff}B_s] \\
v^*(1B2SL) &= v(1B2SL)
\end{aligned} \tag{2.15}
$$

$$
\begin{aligned}
v(1B2SR) &= S_b + 2S_s - [\frac{\alpha_3}{2}B_b + \alpha_1^{eff}B_s] \\
v^*(1B2SR) &= v(1B2SR)
\end{aligned} \tag{2.16}
$$

$$
\begin{aligned}
v(2SBBL) = 2S_b + 2S_s - [(\frac{\alpha_3}{2} + \alpha_1^{eff})B_{ch} + \frac{\alpha_3}{2}B_b + \alpha_2^{eff}B_s \\
v^*(2SBBL) = 2S_b + 2S_s - [\alpha_3 B_b + \alpha_2^{eff}B_s]
\end{aligned}
\tag{2.17}
$$

$$
\begin{aligned}
v(2SBBR) = 2S_b + 2S_s - [(\frac{\alpha_3}{2} + \alpha_2^{eff})B_{ch} + \frac{\alpha_3}{2}B_b + \alpha_1^{eff}B_s \\
v^*(2SBBR) = 2S_b + 2S_s - [\alpha_3 B_b + \alpha_1^{eff}B_s]
\end{aligned}
\tag{2.18}
$$

$$
\begin{aligned}
v(1SBB1S) = 2S_b + 2S_s - [(\alpha_3 + \frac{\alpha_2^{eff}}{2} + \frac{\alpha_1^{eff}}{2})B_{ch} + (\frac{\alpha_2^{eff}}{2} + \frac{\alpha_1^{eff}}{2})B_s] \\
v^*(1SBB1S) = 2S_b + 2S_s - [\alpha_3 B_b + (\frac{\alpha_2^{eff}}{2} + \frac{\alpha_1^{eff}}{2})B_s]
\end{aligned}
\tag{2.19}
$$

$$
\begin{aligned}
v(2SBB1SR) = 2S_b + 3S_s - [(\frac{\alpha_3}{2} + \frac{\alpha_1^{eff}}{2})B_{ch} + \frac{\alpha_3}{2}B_b + (\alpha_2^{eff} + \frac{\alpha_1^{eff}}{2})B_s] \\
v^*(2SBB1SR) = 2S_b + 3S_s - [\alpha_3 B_b + (\alpha_2^{eff} + \frac{\alpha_1^{eff}}{2})B_s]
\end{aligned}
\tag{2.20}
$$

$$
\begin{aligned}
v(2SBB1SL) = 2S_b + 3S_s - [(\frac{\alpha_3}{2} + \frac{\alpha_2^{eff}}{2})B_{ch} + \frac{\alpha_3}{2}B_b + (\frac{\alpha_2^{eff}}{2} + \alpha_1^{eff})B_s] \\
v^*(2SBB1SL) = 2S_b + 3S_s - [\alpha_3 B_b + (\frac{\alpha_2^{eff}}{2} + \alpha_1^{eff})B_s]
\end{aligned}
\tag{2.21}
$$

$$
\begin{aligned}
v(GC) = 2S_b + 4S_s - [\alpha_3 B_b + (\alpha_2^{eff} + \alpha_1^{eff})B_s]; \\
v^*(GC) = v(GC)
\end{aligned}
\tag{2.22}
$$

The formulas for values of all 63 coalitions can be found in the Appendix ?. The code for calculation can be found in the Appendix ?.

Table 2.9: Myerson/Theirson analysis for the corruption minimization settings.

Setting	def		s1		s2		s3	
O	My > BST	Th > BST	My > BST	Th > BST	My > BST	Th > BST	My > BST	Th > BST
(3, 0)	FALSE	FALSE	TRUE	TRUE	FALSE	FALSE	FALSE	FALSE
(3, 1)	FALSE	FALSE	TRUE	TRUE	FALSE	FALSE	FALSE	FALSE
(2, 0)	FALSE	FALSE	TRUE	FALSE	TRUE	FALSE	TRUE	FALSE
(2, 1)	FALSE	FALSE	TRUE	FALSE	TRUE	FALSE	TRUE	FALSE
(1, 0)	FALSE	FALSE	TRUE	FALSE	TRUE	FALSE	TRUE	FALSE
(1, 1)	FALSE	FALSE	TRUE	FALSE	TRUE	FALSE	TRUE	FALSE
Conv_fail	274	1044	306	982	308	888	348	1028

Table 2.10: Myerson/Theirson analysis for the mild corruption minimization settings.

Setting	z1		z3	
O	My > BST	Th > BST	My > BST	Th > BST
(3, 0)	TRUE	TRUE	TRUE	TRUE
(3, 1)	FALSE	TRUE	FALSE	TRUE
(2, 0)	TRUE	FALSE	TRUE	FALSE
(2, 1)	TRUE	FALSE	TRUE	FALSE
(1, 0)	TRUE	FALSE	TRUE	FALSE
(1, 1)	TRUE	FALSE	TRUE	FALSE
Conv_fail	344	856	290	910

The results of analysis for all the settings are presented in Tables 2.9 and 2.10. "BST" is the average utility of the strategies providing the biggest utilities for respective settings. The column "My > Th" was deleted from the results due to always being TRUE for bosses and FALSE for subordinates. Conclusions from the analysis are as follows:

1. Neither Myerson nor Theirson game is convex: out of all possible $63 \cdot 62 = 3906$ coalition pairs S, T the number of pairs for which the condition $v(S) + v(T) \leq v(S \cup T) + v(S \cap T)$ does not hold is in the Conv_fail row.

2. Theirson always undervalues subordinates compared to Myerson, which

is to be expected since in the former they "give up" their ability to expose.

3. Neither rule provides a stable division for the default setting.

4. Myerson rule provides a stable division in the settings s1, z1 and z3 (in the latter two the differences in the only "FALSE" are 0.69% and 0.55% respectively).

5. Theirson rule does not provide a stable divison in any setting: it satisfies either only bosses (s1, s2, z1, z3) or no one (def, s3).

2.4 Limitations and further work

In the current work, no analysis of effect of the parameters κ and θ was carried out. Doing so or measuring them in an organization or a country and make some prediction based on their effect might be a prospect. Real-life experiments (post-hoc or real-time) also might be useful for tuning of the model.

Fine functions' effect analysis also might be fruitful. It was not done since in the current model the official always prefers not being fined due to losing the part of steal, so it might have not been very informative.

Studying a bigger hierarchy might introduce undiscovered effects and open the possibility of the coalitional wars: multiple corrupt coalitions exposing each other (or bribing inspector to fix the evidence such that the other coalition is fined). It was not done due to the limited computing resources.

Introducing the mechanism of repeated game into the model is another interesting prospect. The one who does will have to solve the problem of orphans in a hierarchy (if the uncovered corrupt official is getting fined, their subordinates become orphans) and different values of the future for the players. It also creates opportunity for punishment strategies (bosses finding out who exposed them and taking revenge), which will surely change the equilibrium situation from the current. It was not done because of the not willingness to add yet another lair of complexity to the model.

Change of inspection direction can be done quite easily in code simulation and quite hard in the formulas. The current top-down approach is based on the mainstream inspection works and the idea of "following the money": when the inspection is checking the organization that received the money, it may try to recreate its path to find the exact place everything went awry. On the other hand, bottom-up approach can be seen as a "reaction to malfunction": something happened and the inspection is reacting to it. The inspector using the first (proactive) approach deals with the corruption until something happened and thus is easier to be bribed, while in the second (reactive) case something has already happened and it is much harder to "cover up".

2.5 Approbation

The work was presented at Control Processes and Stability (CPS'20) [17], MCTaIA-2020 [18] and was published in proceedings. The study was also presented at the Fourteenth International Conference on Game Theory and Management (GTM2020) and Control Processes and Stability (CPS'21) and is in publishing at the moment.

3 Conclusion

The study of the literature shows researches that do not take hierarchical relations of players into account and analyze "simple" games between two-three agents are more common. The similar claim is made in Gorbaneva et al. [10].

The difference between the current study and hierarchical studies [10, 11] lies in the construction of hierarchy: in the works mentioned above hierarchies are of "administration-inspector-client" type with no differentiation in the last class, while this work focuses on the "superior-subordinate" type (which provides a feature of subordinate having the ability to expose the bigger stealer, for example, their superior) with inspector being outside the hierarchy. Another difference is in the future development of cooperative element. The semblance can be found in absence of corruption on the highest level of the hierarchy and the use of hierarchy itself.

The model of hierarchical corruption is built. It consists of two stages: at the first stage each official decides how much money he or she embezzles, at the second stage inspector investigates the stealing and inspected official chooses the size of bribe and action (bribe, not bribe or expose).

The particular case with two levels and six officials is built and is solved via computer simulation. The result is an equilibrium in which each inspected subordinate (official from level 1 or 2) exposes their boss who then gives sufficient bribe to the inspector and each inspected official from level 3 gives sufficient bribe. This equilibrium situation is pessimistic, because corruption is not punished, but causes even greater corruption.

The inequalities connecting the decision-making of inspector and official in general form are suggested and used to find the corruption minimization settings in the example. Their simulations are carried out: two settings decrease corruption and one eradicates it. Mild cooperation minimization *zettings* with the sufficient bribe being limited to the steal are also suggested and simulated.

The cooperative element is introduced; rules for forming coalition and allocating the steal and bribe are suggested; criteria for stability are described. Code simulation is run under all settings that were not analytically proven to be unstable under any circumstances, the results are analyzed. The Myerson and its suggested modified version Theirson values are calculated. The results are analyzed.

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Appendix

Appendix A.

The transition probabilities $\pi(\omega''/\omega', \eta)$ from state $\omega' \in \Omega$ to state $\omega'' \in \Omega$ using the mixed stationary strategy profile η is shown in Table

Appendix B.

Appendix C.

Appendix D.

Appendix E.

Table 3.1: Values of all coalitions for Myerson/Theirson.

#	(3,0)	(3,1)	(2,0)	(2,1)	(1,0)	(1,1)	v(?)	Fully formable?
1	0	0	0	0	0	1	$\{(1,1)\}$	TRUE
2	0	0	0	0	1	0	$\{(1,0)\}$	TRUE
3	0	0	0	0	1	1	$\{(1,0),(1,1)\}$	TRUE
4	0	0	0	1	0	0	$\{(2,1)\}$	TRUE
5	0	0	0	1	0	1	$\{(2,1)\}+\{(1,1)\}$	FALSE
6	0	0	0	1	1	0	$\{(2,1)\}+\{(1,0)\}$	FALSE
7	0	0	0	1	1	1	$\{(2,1)\}+\{(1,0),(1,1)\}$	FALSE
8	0	0	1	0	0	0	$\{(2,0)\}$	TRUE
9	0	0	1	0	0	1	$\{(2,0)\}+\{(1,1)\}$	FALSE
10	0	0	1	0	1	0	$\{(2,0)\}+\{(1,0)\}$	FALSE
11	0	0	1	0	1	1	$\{(2,0)\}+\{(1,0),(1,1)\}$	FALSE
12	0	0	1	1	0	0	$\{(2,0),(2,1)\}$	TRUE
13	0	0	1	1	0	1	$\{(2,0),(2,1)\}+\{(1,1)\}$	FALSE
14	0	0	1	1	1	0	$\{(2,0),(2,1)\}+\{(1,0)\}$	FALSE
15	0	0	1	1	1	1	$\{(2,0),(2,1)\}+\{(1,0),(1,1)\}$	FALSE
16	0	1	0	0	0	0	$\{(3,1)\}$	TRUE
17	0	1	0	0	0	1	$\{(3,1),(1,1)\}$	TRUE
18	0	1	0	0	1	0	$\{(3,1),(1,0)\}$	TRUE
19	0	1	0	0	1	1	$\{(3,1),(1,0),(1,1)\}$	TRUE
20	0	1	0	1	0	0	$\{(3,1)\}+\{(2,1)\}$	FALSE
21	0	1	0	1	0	1	$\{(3,1),(1,1)\}+\{(2,1)\}$	FALSE
22	0	1	0	1	1	0	$\{(3,1),(1,0)\}+\{(2,1)\}$	FALSE
23	0	1	0	1	1	1	$\{(3,1),(1,0),(1,1)\}+\{(2,1)\}$	FALSE
24	0	1	1	0	0	0	$\{(3,1)\}+\{(2,0)\}$	FALSE
25	0	1	1	0	0	1	$\{(3,1),(1,1)\}+\{(2,0)\}$	FALSE
26	0	1	1	0	1	0	$\{(3,1),(1,0)\}+\{(2,0)\}$	FALSE
27	0	1	1	0	1	1	$\{(3,1),(1,0),(1,1)\}+\{(2,0)\}$	FALSE
28	0	1	1	1	0	0	$\{(3,1)\}+\{(2,0),(2,1)\}$	FALSE
29	0	1	1	1	0	1	$\{(3,1),(1,1)\}+\{(2,0),(2,1)\}$	FALSE
30	0	1	1	1	1	0	$\{(3,1),(1,0)\}+\{(2,0),(2,1)\}$	FALSE
31	0	1	1	1	1	1	$\{(3,1),(1,0),(1,1)\}+\{(2,0),(2,1)\}$	FALSE
32	1	0	0	0	0	0	$\{(3,0)\}$	TRUE
33	1	0	0	0	0	1	$\{(3,0)\}+\{(1,1)\}$	FALSE
34	1	0	0	0	1	0	$\{(3,0)\}+\{(1,0)\}$	FALSE
35	1	0	0	0	1	1	$\{(3,0)\}+\{(1,0),(1,1)\}$	FALSE
36	1	0	0	1	0	0	$\{(3,0),(2,1)\}$	TRUE
37	1	0	0	1	0	1	$\{(3,0),(2,1)\}+\{(1,1)\}$	FALSE
38	1	0	0	1	1	0	$\{(3,0),(2,1)\}+\{(1,0)\}$	FALSE
39	1	0	0	1	1	1	$\{(3,0),(2,1)\}+\{(1,0),(1,1)\}$	FALSE
40	1	0	1	0	0	0	$\{(3,0),(2,0)\}$	TRUE
41	1	0	1	0	0	1	$\{(3,0),(2,0)\}+\{(1,1)\}$	FALSE
42	1	0	1	0	1	0	$\{(3,0),(2,0)\}+\{(1,0)\}$	FALSE
43	1	0	1	0	1	1	$\{(3,0),(2,0)\}+\{(1,0),(1,1)\}$	FALSE
44	1	0	1	1	0	0	$\{(3,0),(2,0),(2,1)\}$	TRUE
45	1	0	1	1	0	1	$\{(3,0),(2,0),(2,1)\}+\{(1,1)\}$	FALSE
46	1	0	1	1	1	0	$\{(3,0),(2,0),(2,1)\}+\{(1,0)\}$	FALSE
47	1	0	1	1	1	1	$\{(3,0),(2,0),(2,1)\}+\{(1,0),(1,1)\}$	FALSE
48	1	1	0	0	0	0	$\{(3,0),(3,1)\}$	TRUE
49	1	1	0	0	0	1	$\{(3,0),(3,1),(1,1)\}$	TRUE
50	1	1	0	0	1	0	$\{(3,0),(3,1),(1,0)\}$	TRUE
51	1	1	0	0	1	1	$\{(3,0),(3,1),(1,0),(1,1)\}$	TRUE
52	1	1	0	1	0	0	$\{(3,0),(3,1),(2,1)\}$	TRUE
53	1	1	0	1	0	1	$\{(3,0),(3,1),(2,1),(1,1)\}$	TRUE
54	1	1	0	1	1	0	$\{(3,0),(3,1),(2,1),(1,0)\}$	TRUE
55	1	1	0	1	1	1	$\{(3,0),(3,1),(2,1),(1,0),(1,1)\}$	TRUE
56	1	1	1	0	0	0	$\{(3,0),(3,1),(2,0)\}$	TRUE
57	1	1	1	0	0	1	$\{(3,0),(3,1),(2,0),(1,1)\}$	TRUE
58	1	1	1	0	1	0	$\{(3,0),(3,1),(2,0),(1,0)\}$	TRUE
59	1	1	1	0	1	1	$\{(3,0),(3,1),(2,0),(1,0),(1,1)\}$	TRUE
60	1	1	1	1	0	0	$\{(3,0),(3,1),(2,0),(2,1)\}$	TRUE
61	1	1	1	1	0	1	$\{(3,0),(3,1),(2,0),(2,1),(1,1)\}$	TRUE
62	1	1	1	1	1	0	$\{(3,0),(3,1),(2,0),(2,1),(1,0)\}$	TRUE
63	1	1	1	1	1	1	GC	TRUE