Hierarchical Model of Corruption: Game-Theoretic Approach

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Research Object & Aim

Research object is corruption (embezzlement and bribery) within a hierarchy.

Aim of this study is to analyze corruption in hierarchical context and find conditions under which it is minimal.

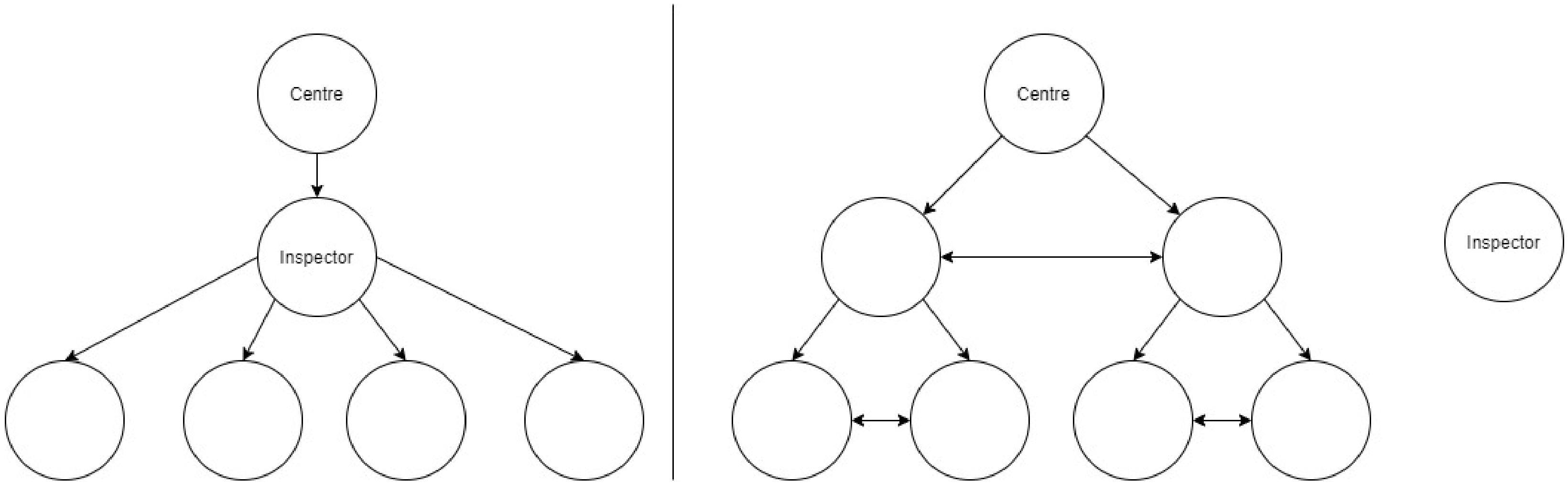
Research Objectives

1. Study the relevant literature.
2. Create and study the hierarchical model of corruption (both non-cooperative and cooperative cases).
3. Write a code simulation for the model.
4. Solve the particular case of the model.
5. Analyze the solution.
6. Find the conditions for corruption minimization.

Literature review

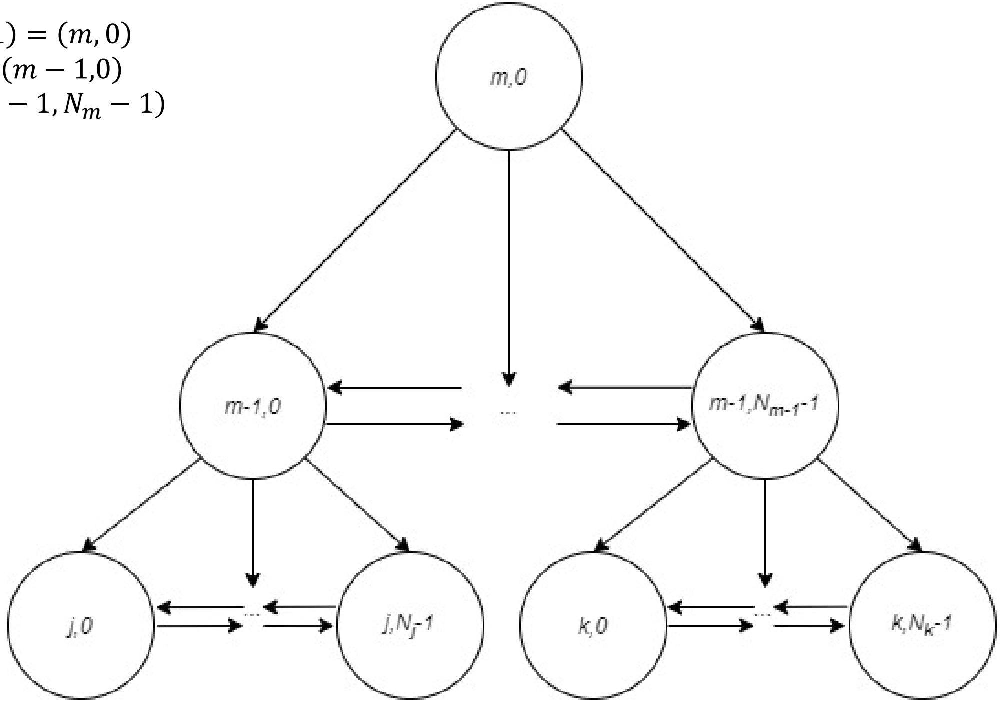
№	Author(s)	Year	Problem	№ of players	Form	Solution
1	Spengler	2014	Client-Off-Insp	3	Extensive	MNE
2	Attanasi et al.	2016	Donor-Inter-Recip	3	Extensive	Experiment
3	Shenje	2016	Briber-Bribee	2	Normal	MNE
4	Song et al.	2016	Comm-Dep	2	Normal	MNE
5	Kumacheva	2013	Tax evasion & bribery: adm, insp, taxpayers	1+1+n	Hierarchical, multi-stage	Probabilistic
6	Vasin & Panova	2000	Tax evasion & bribery: center, auditor, taxpayers	1+1+n	Hierarchical, multi-stage	Probabilistic cut-off rule

Novelty of the Model – Less Homogenous Hierarchy



Stage 1 – Embezzlement

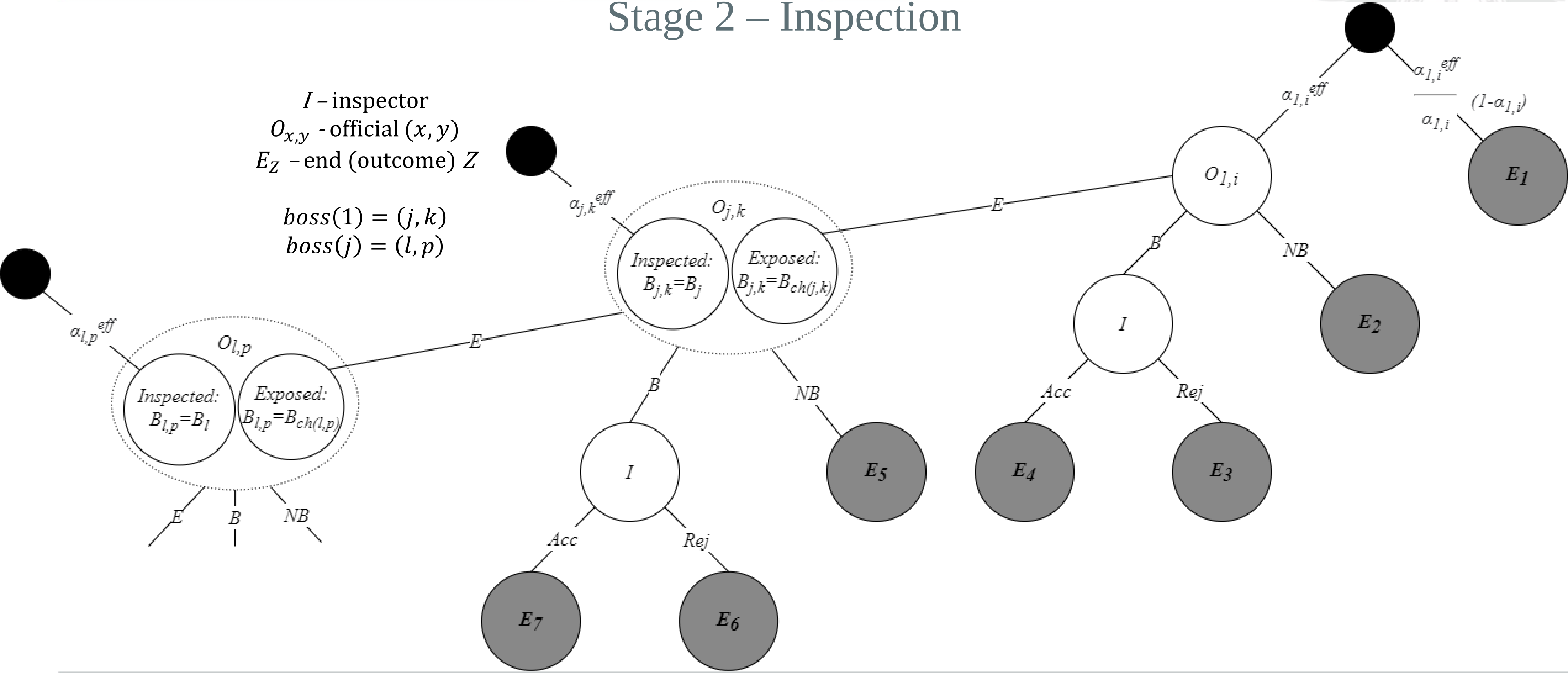
$$boss(m-1) = (m, 0)$$
$$boss(j) = (m-1, 0)$$
$$boss(k) = (m-1, N_m - 1)$$



Stage 2 – Inspection

I – inspector
$O_{x,y}$ - official (x, y)
E_Z – end (outcome) Z

$boss(1) = (j, k)$
$boss(j) = (l, p)$



Possible Actions of Officials on Different Levels

$$A_{n,i} \in \{B, NB, E\} \quad n \neq m-1, m \;\; i = 0 \ldots N_n - 1$$

$$A_{m-1,i} \in \{B, NB\} \quad i = 0 \ldots N_{m-1} - 1$$

$$A_{m,0} \in \emptyset$$

where B – bribe;
NB – not bribe;
E – expose boss.

$$A_I \in \{Acc, Rej\}$$

where Acc – accept the bribe;
Rej– reject the bribe.

Stage 2 – Inspection Outcomes' Payoffs

End	$U_{j,k}$	$U_{1,i}$	$U_I)$
1	$W_{j,k} + S_{j,k}$	$W_{1,i} + S_{1,i}$	W_I
2	$W_{j,k} + S_{j,k}$	$W_{1,i} + \kappa_{1,i} S_{1,i} - F(W_{1,i}, S_{1,i})$	$W_I + R(S_{1,i}) - Ci_1$
3	$W_{j,k} + S_{j,k}$	$W_{1,i} + \kappa_{1,i} S_{1,i} - (F(W_{1,i}, S_{1,i}) + B_{1,i} + Fb(B_{1,i}))$	$W_I + R(S_{1,i}) - Ci_1$
4	$W_{j,k} + S_{j,k}$	$W_{1,i} + S_{1,i} - B_{1,i}$	$W_I + B_{1,i} - Ci_1 - Cu(S_{1,i})$
5	$W_{j,k} + \kappa_{j,k} S_{j,k} - F(W_{j,k}, S_{j,k})$	$W_{1,i} + \kappa_{1,i} S_{1,i} - \theta_{1,i} F(W_{1,i}, S_{1,i})$	$W_I - (Ci_1 + Ci_j) + R(S_{1,i}) + R(S_{j,k})$
6	$W_{j,k} + \kappa_{j,k} S_{j,k} - (F(W_{j,k}, S_{j,k}) + B_{j,k} + Fb(B_{j,k}))$	$W_{1,i} + \kappa_{1,i} S_{1,i} - \theta_{1,i} F(W_{1,i}, S_{1,i})$	$W_I - (Ci_1 + Ci_j) + R(S_{1,i}) + R(S_{j,k})$
7	$W_{j,k} + S_{j,k} - B_{j,k}$	$W_{1,i} + S_{1,i}$	$W_I + B_{j,k} - (Ci_1 + Ci_j + Cu(S_{1,i}) + Cu(S_{j,k}))$

End	Description
1	No inspection.
2	Subordinate is inspected, no bribe.
3	Subordinate is inspected, bribe is rejected.
4	Subordinate is inspected, bribe is accepted.
5	Boss is exposed by the subordinate, no bribe.
6	Boss is exposed by the subordinate, bribe is rejected.
7	Boss is exposed by the subordinate, bribe is accepted.

Formulas for optimal stealing and probability of inspection

$$S^*_{n,i} = \frac{G_n - M_n}{N_n}$$

$$S_n = \sum_{i=0}^{N_n-1} S_{n,i}$$

$$\alpha_n = \frac{\sum_{j=n}^{m-1} S_j}{M_m}$$

$$\alpha_n^{eff} = \alpha_n \prod_{k=n+1}^{m} (1 - \alpha_k)$$

$$S_m = 0 \rightarrow \alpha_m^{eff} = \alpha_m = 0$$

$$\alpha_{n,i} = \frac{\alpha_n}{N_n} \quad \alpha_{n,i}^{eff} = \frac{\alpha_n^{eff}}{N_n}$$

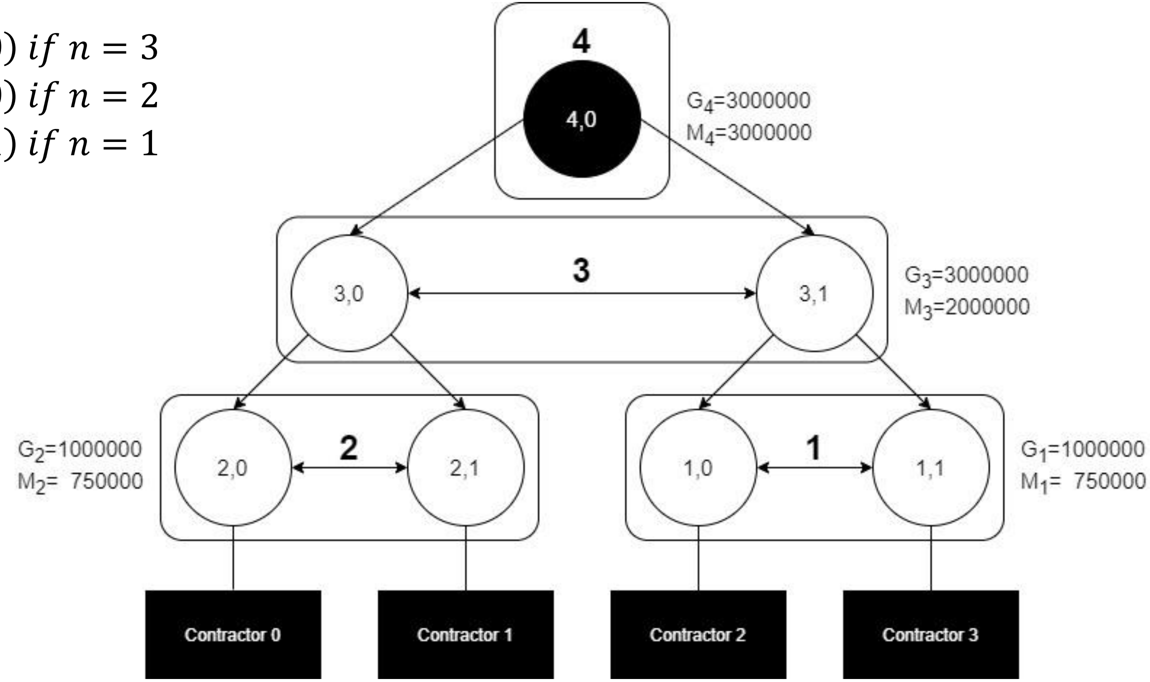
where $S^*_{n,i}$ – optimal stealing of official i from level n;
M_n – cut-off value of level n;
G_n – amount of money entering level n;
N_n – amount of officials on level n;
S_n – total stealing on level n;
M_m – total amount of money given;

α_n – probability of inspecting level n;
α_n^{eff} – effective probability of inspecting level n;

$\alpha_{n,i}$ – probability of inspecting official i from level n.
$\alpha_{n,i}^{eff}$ – effective probability of inspecting official i from level n.

Hierarchy of Officials in Example

$$boss(n) = \begin{cases} (4,0)\ if\ n = 3 \\ (3,0)\ if\ n = 2 \\ (3,1)\ if\ n = 1 \end{cases}$$



Values of officials' characteristics

$O_{n,i}$	$W_{n,i}$	$S_{n,i}$	$\kappa_{n,i}$	$\theta_{n,i}$	$\alpha_{n,i}$	$B_{n,i}$	$F(S_{n,i})$	$Fb(B_{n,i})$
$3, i$	90,000	500,000	0.600	–	0.167	150,000	1,620,000	5,625,000
$2, i$	40,000	125,000	0.300	0.010	0.208	62,500	720,000	2,812,500
$1, i$	40,000	125,000	0.300	0.010	0.250	62,500	720,000	2,812,500

Values of Inspector's characteristics

W_I	$Ci_{\{1,2\}}$	Ci_3	$R(S_{\{1,2\},i})$	$R(S_{3,i})$	$Cu(S_{\{1,2\},i})$	$Cu(S_{3,i})$
70,000	10,000	25,000	40,000	75,000	5,000	12,500

Results of simulation for the initial settings

	OptOpt_EB	OptOpt_BB	NoneOpt_NBB	OptNone_BNB	NoneNone_NBNB
$(3,0)$	523,136	564,934	565,055	90,000	90,000
$(3,1)$	535,972	565,004	564,835	90,000	90,000
$(2,0)$	165,000	156,277	40,000	162,407	40,000
$(2,1)$	165,000	156,294	40,000	162,405	40,000
$(1,0)$	165,000	158,935	40,000	160,187	40,000
$(1,1)$	165,000	158,975	40,000	160,240	40,000
I	156,602	131,233	105,137	81,219	70,000
State	1,090,000	1,090,000	1,590,000	2,090,000	2,590,000
LoC	0.500	0.500	0.333	0.167	0.000

Nash-Like Equilibrium

$$NLE\colon \left(S^*_{n,i}, B^*_{n,i}, A^*_{n,i}\right) = argmax\left\{U_{n,i}\left(S_{n,i}\ , B_{n,i}, A_{n,i}\right) \mid B_{n,i} \geq B^{v}_{n,i}\right\}$$

$$ss_{\{1,2\},i} = \left\{(0,0,NB); \left(S^*_{\{1,2\},i}, 0, E\right); \left(S^*_{\{1,2\},i}, B^*_{\{1,2\},i}, B\right); \dots\right\}$$

$$ss_{3,i} = \left\{(0,0,NB); \left(S^*_{3,i}, B^*_{3,i}, B\right); \dots\right\}$$

$$NLE = \left\{\left(S^*_{1,i}, 0, E\right); \left(S^*_{2,i}, 0, E\right); \left(S^*_{3,i}, B^*_{3,i}, B\right)\right\}\ \ i = 0,1$$

Corruption Minimization (1)

$$U_I(Acc) \gtreqless U_I(Rej) \;\rightarrow\; B_{n,i} - \sum_{(l,j)\in T} Cu(S_{l,j}) \gtreqless \sum_{(l,j)\in T} R_I(S_{l,j})$$

$$B_{n,i} - \sum_{(l,j)\in T} Cu(S_{l,j}) \leq \sum_{(l,j)\in T} R_I(S_{l,j})$$

$$\sum_{(l,j)\in T} [R(S_{l,j}) + Cu(S_{l,j})] \geq B_{n,i}$$

Corruption Minimization (2)

$$U_{n,i}(S^*_{n,i}, B^*_{n,i}, B) - U_{n,i}(0,0,NB) = S^*_{n,i} - \alpha^+_{n,i} B^*_{n,i} \leq 0$$

$$\alpha^+_{n,i} = \alpha^{eff}_{n,i} + \sum_{(l,j) \in SE(n,i)} \alpha^+_{l,j},$$

$$\text{where } SE(n,i) = \{(v,p)\} : \ (v,p) \in subs(n,i) \ \& \ A_{v,p} = E$$

Corruption Minimization (3)

$$\sum_{(l,j)\in T} [R(S_{l,j}) + Cu(S_{l,j})] \geq \frac{S^*_{n,i}}{\alpha^+_{n,i}} \; \forall T,$$

that must be satisfied in the best case for $T = \{O_{n,i}\}$, in the worst case –

$$T = \{O_{n,i}, O_{j,k}, O_{l,p}, \dots\} \; O_{j,k} \in SE(n,i); \; O_{l,p} \in SE(j,k)$$

Formula for the Optimal Bribe

In order to be accepted, the bribe for inspected chain T must be:

$$B_{optT} > \sum_{(l,j)\in T} [R(S_{l,j}) + Cu(S_{l,j})]$$

$$B_{optT}(\zeta) = \sum_{(l,j)\in T} [R(S_{l,j}) + Cu(S_{l,j})] + \zeta$$

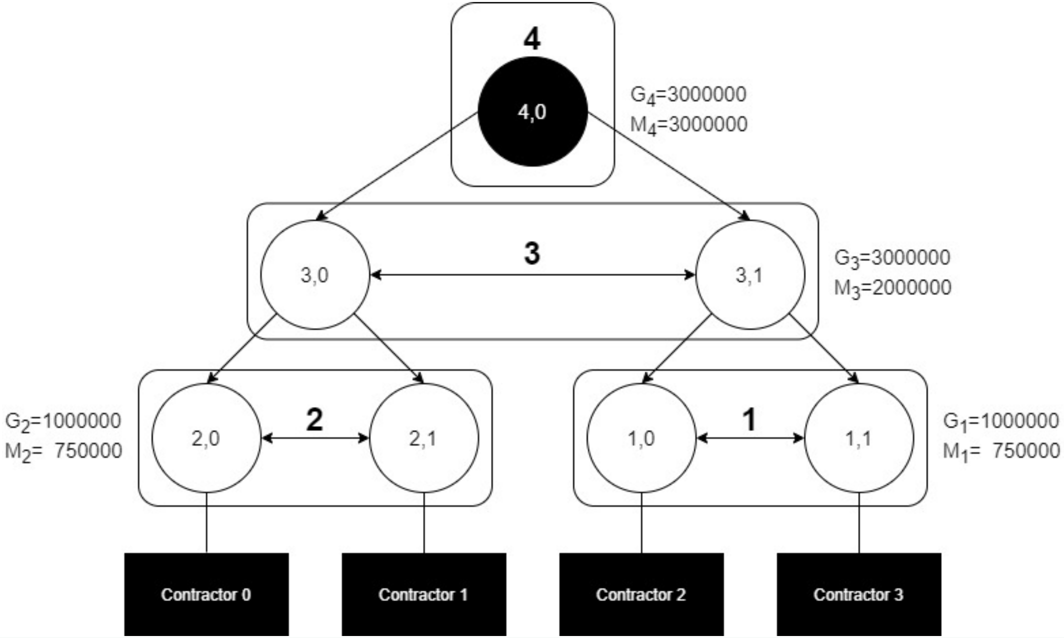
For the corruption minimization, it must hold that

$$B_{optT}(\zeta) \geq \frac{S^*_{n,i}}{\alpha^+_{n,i}}$$

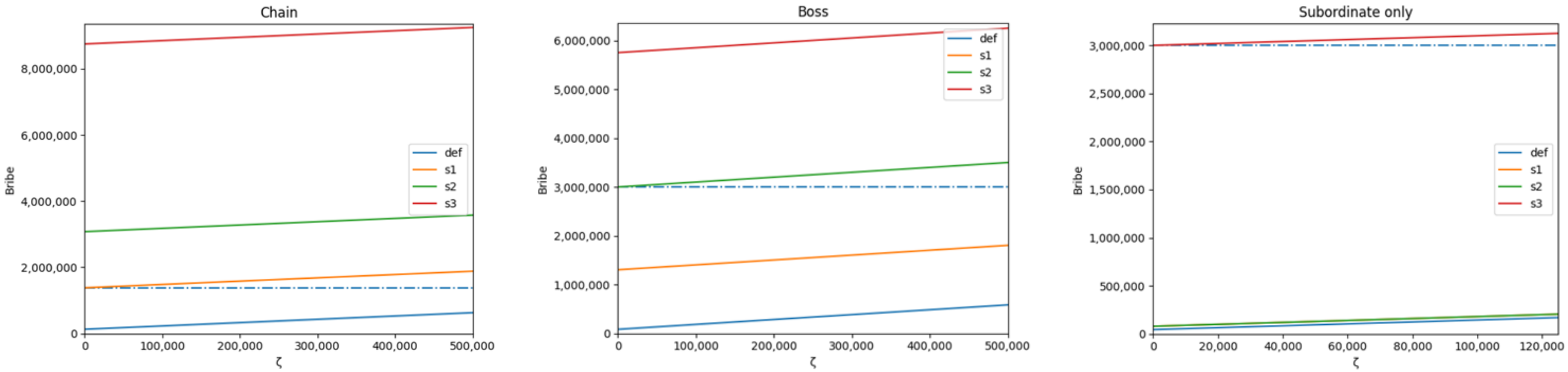
All conclusions valid for $\zeta = x > 0$ are valid for any $\zeta > x$.

Chains of Officials in Example

Chains: $T_s = \{O_{2,i}\}; \{O_{1,i}\}$ $T_b = \{O_{3,i}\}$ $T_{ch} = \{O_{2,i}, O_{3,0}\}; \{O_{1,i}, O_{3,1}\}$



Corruption Minimization Settings

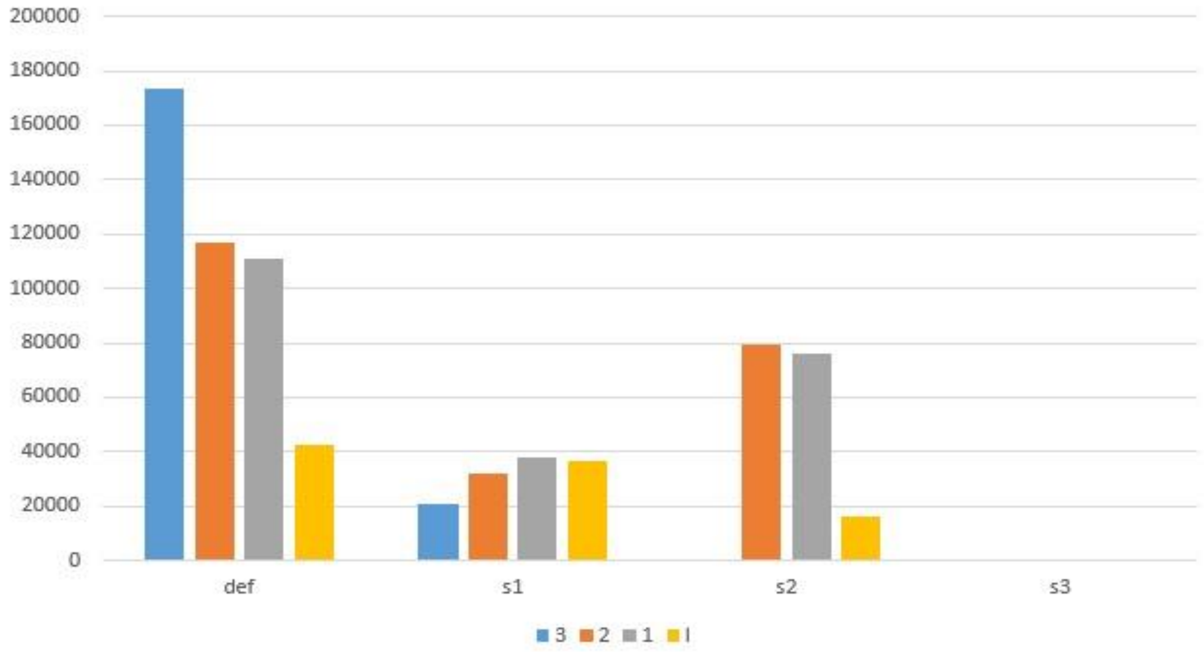


Setting	$R(S_{\{1,2\},i})$	$Cu(S_{\{1,2\},i})$	$R(S_{3,i})$	$Cu(S_{3,i})$	$B_{suff-ch}$	B_{suff-b}	B_{suff-s}	T	B_{optT}
Default	40,000.0	5,000.0	75,000.0	11,250.0	131,251.0	86,251.0	45,001.0	-	-
1	60,000.0	20,000.0	875,000.0	429,615.4	1,384,616.4	1,304,615.4	80,000.0	ch	1,384,615.4
2	60,000.0	20,000.0	2,000,000.0	1,000,000.0	3,080,000.0	3,000,000.0	80,000.0	b	3,000,000.0
3	2,000,000.0	1,000,000.0	3,250,000.0	2,500,000.0	8,750,000.0	5,750,000.0	3,000,000.0	s	3,000,000.0

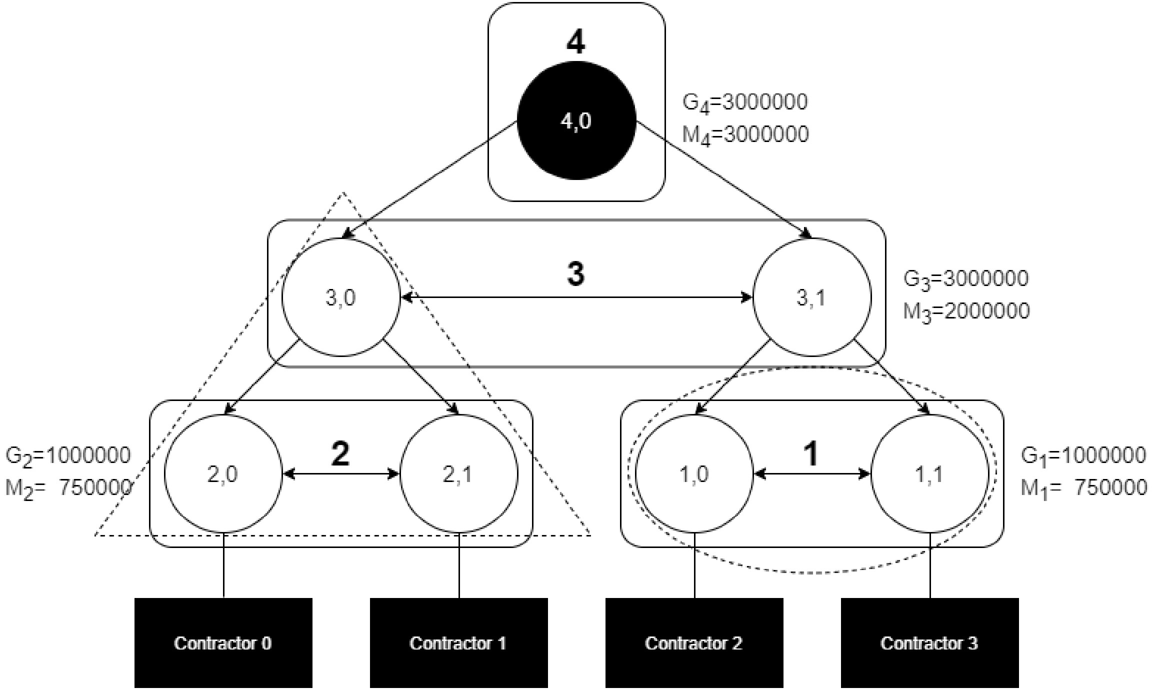
Corruption Minimization Effect

AVG	def	$s1$	$s2$	$s3$		$def \to s1$	$def \to s2$	$def \to s3$
(3, 0)	143,336.69	0.00	0.00	0.00		-100.00 %	-100.00 %	-100.00 %
(3, 1)	147,691.36	0.00	0.00	0.00		-100.00 %	-100.00 %	-100.00 %
(2, 0)	109,345.65	80,560.62	80,554.08	0.00		-26.32 %	-26.33 %	-100.00 %
(2, 1)	109,236.93	80,548.62	80,554.81	0.00		-26.26 %	-26.26 %	-100.00 %
(1, 0)	96,252.46	78,231.37	78,253.14	0.00		-18.72 %	-18.70 %	-100.00 %
(1, 1)	96,099.14	78,242.55	78,230.66	0.00		-18.58 %	-18.59 %	-100.00 %
Inspector	36,663.69	11,026.42	11,018.90	0.00		-69.93 %	-69.95 %	-100.00 %

Corruption Minimization Effect Chart



Cooperative Element (1)



Cooperative Element (2)

$$C = \bigcup_{(n,i)\in C} \{(n,i)\} = \bigcup_{n\in C} C_n \qquad N_C = |C|$$

$$C_j = \bigcup_{(j,i)\in C} \{(j,i)\} \qquad N_{C,j} = \sum_{(j,i)\in C} 1 = |C_j| \le N_j$$

$$RU^C_{n,i} = U_{n,i}(S_{n,i}, 0, BC) - W_{n,i}$$

$$S_{n,i} > 0 \;\&\; A_{n,i} = BC \quad \forall (n,i) \in C$$

$$S_C = \sum_{(n,i)\in C} S_{n,i}$$

$$\alpha_C = \bigcup_{(n,i)\in C} \alpha^{+}_{n,i}$$

$$\alpha_C = (\alpha_{ch}; \alpha_b; \alpha_s)$$

$$B_C = (B_{ch}; B_b; B_s)^T$$

$$\alpha_C B_C = \alpha_{ch} B_{ch} + \alpha_b B_b + \alpha_s B_s$$

Stability

$$I(v) = \{X \in R^{N_C} \mid X(C) = v(C), \quad X_{n,i} \geq v(\{(n,i)\})\ \forall (n,i) \in C\}$$

$$C(v) = \{X \in R^{N_C} \mid X(C) = v(C), \quad X(S) \geq v(S)\ \forall S \subset C\}$$

Preanalysis

Rule	EQ		SS	
Coalition	I	C	I	C
1B1SL	MB	MB	MB	MB
1B1SR	MB	MB	MB	MB
BB1SL	MB	MB	MB	MB
BB1SR	MB	MB	MB	MB
1B2SL	MB	MB	MB	MB
1B2SR	MB	MB	MB	MB
2SBBL	MB	MB	MB	MB
2SBBR	MB	MB	MB	MB
1SBB1S	MB	MB	MB	MB
2SBB1SL	MB	MB	MB	MB
2SBB1SR	MB	MB	MB	MB
GC	MB	MB	MB	MB

$$EQU_{n,i}^{C} = \frac{S_C - \alpha_C B_C}{N_C} \quad \forall (n,i) \in C$$

$$SSU_{n,i}^{C} = S_{n,i} - \begin{cases} \frac{\alpha_C B_C + |C \cap \bigcup_{(n,i)\notin C_{bl}} \{(n,i)\}| \cdot \xi}{N_{C,bl}} & if\ n = bl \\ -\xi\ otherwise \end{cases} \quad \forall (n,i) \in C$$

Analysis of the Results

Setting	def		s1		s2		s3		z1		z3	
Coalition \ Rule	**EQ**	**SS**	**EQ**	**SS**	**EQ**	**SS**	**EQ**	**SS**	**EQ**	**SS**	**EQ**	**SS**
{(3,0),(2,0)}	N	N	N	N	N	N	N	N	N	N	N	N
{(3,0),(2,1)}	N	N	N	N	N	N	N	N	N	N	N	N
{(3,1),(1,0)}	N	N	N	N	N	N	N	N	N	N	N	N
{(3,1),(1,1)}	N	N	N	N	N	N	N	N	N	N	N	N
{(3,0),(2,0),(3,1)}	N	N	N	N	N	N	N	N	N	N	N	N
{(3,0),(2,1),(3,1)}	N	N	N	N	N	N	N	N	N	N	N	N
{(3,1),(1,0),(3,0)}	N	N	N	N	N	N	N	N	N	N	N	N
{(3,1),(1,1),(3,0)}	N	N	N	N	N	N	N	N	N	N	N	N
{(3,0),(2,0),(2,1)}	N	N	N	N	N	N	N	N	N	C	N	C
{(3,1),(1,0),(1,1)}	N	N	N	N	N	N	N	N	N	N	N	N
{(3,0),(2,0),(2,1),(3,1)}	N	C	N	C	N	N	N	N	N	C	N	C
{(3,1),(1,0),(1,1),(3,0)}	N	N	N	N	N	N	N	N	N	N	N	N
{(2,0),(3,0),(3,1),(1,0)}	N	N	N	N	N	N	N	N	N	N	N	N
{(2,0),(3,0),(3,1),(1,1)}	N	N	N	N	N	N	N	N	N	N	N	N
{(2,1),(3,0),(3,1),(1,0)}	N	N	N	N	N	N	N	N	N	N	N	N
{(2,1),(3,0),(3,1),(1,1)}	N	N	N	N	N	N	N	N	N	N	N	N
{(3,0),(2,0),(2,1),(3,1),(1,0)}	N	C	C	C	N	N	N	N	N	C	N	C
{(3,0),(2,0),(2,1),(3,1),(1,1)}	N	C	C	C	N	N	N	N	N	C	N	C
{(3,1),(1,0),(1,1),(3,0),(2,0)}	N	C	N	C	N	N	N	N	N	C	N	C
{(3,1),(1,0),(1,1),(3,0),(2,1)}	N	C	N	C	N	N	N	N	N	C	N	C
{(2,0),(2,1),(3,0),(3,1),(1,0),(1,1)}	N	C	C	C	N	N	N	N	N	C	N	C

Myerson/Theirson

$$v^g(S) = \sum_{C \in S|_g} v(C)$$

$$M_{n,i}(v) = \sum_{S \subseteq H \setminus \{(n,i)\}} \frac{|S|!(|H|-1-|S|)!}{|H|!}[v(S \cup \{(n,i)\}) - v(S)],$$

where H – hierarchy, set of all officials.

$$T_{n,i}(v) = M_{n,i}(v^*) = \sum_{S \subseteq H \setminus \{(n,i)\}} \frac{|S|!(|H|-1-|S|)!}{|H|!}[v^*(S \cup \{(n,i)\}) - v^*(S)]$$

Myerson/Theirson Results

Setting	def		s1		s2		s3	
O	My > BST	Th > BST	My > BST	Th > BST	My > BST	Th > BST	My > BST	Th > BST
(3, 0)	FALSE	FALSE	TRUE	TRUE	FALSE	FALSE	FALSE	FALSE
(3, 1)	FALSE	FALSE	TRUE	TRUE	FALSE	FALSE	FALSE	FALSE
(2, 0)	FALSE	FALSE	TRUE	FALSE	TRUE	FALSE	TRUE	FALSE
(2, 1)	FALSE	FALSE	TRUE	FALSE	TRUE	FALSE	TRUE	FALSE
(1, 0)	FALSE	FALSE	TRUE	FALSE	TRUE	FALSE	TRUE	FALSE
(1, 1)	FALSE	FALSE	TRUE	FALSE	TRUE	FALSE	TRUE	FALSE
Conv_fail	274	1044	306	982	308	888	348	1028

Zetting	z1		z3	
O	My > BST	Th > BST	My > BST	Th > BST
(3, 0)	TRUE	TRUE	TRUE	TRUE
(3, 1)	FALSE	TRUE	FALSE	TRUE
(2, 0)	TRUE	FALSE	TRUE	FALSE
(2, 1)	TRUE	FALSE	TRUE	FALSE
(1, 0)	TRUE	FALSE	TRUE	FALSE
(1, 1)	TRUE	FALSE	TRUE	FALSE
Conv_fail	344	856	290	910

Results

1. Literature: hierarchical context is not often analyzed.

2. Hierarchical non-cooperative and cooperative models of corruption were built.

3. Code simulations for both models were written.

4. The equilibrium situations of particular cases of models were found.

5. Equilibriums were analyzed: non-cooperative is pessimistic, cooperative is too (but somewhat less).

6. The corruption minimization conditions were found: non-cooperative works for cooperative.

Approbation

The different parts of this work were presented at CPS 2020 and МТУиП-2020 and published in its respective proceedings.

УДК 519.83

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Пример решения коррупционной игры с иерархической схемой

Рекомендовано к публикации старшим преподавателем Кумачевой С. Ш.

Материалы 13-й мультиконференции по проблемам управления, 2020 г.

И. М. ОРЛОВ, С. Ш. КУМАЧЕВА (СПбГУ,Санкт-Петербург)

ИЕРАРХИЧЕСКАЯ МОДЕЛЬ КОРРУПЦИИ: ТЕОРЕТИКО-ИГРОВОЙ ПОДХОД

В работе представлена модель хищения и взяточничества, выполненная в форме субиерархической игры, построен и решен частный пример и предложены условия, минимизирующие коррупцию.

It was also presented at the Fourteenth International Conference on Game Theory and Management (GTM2020) and CPS 2021 and will be published in its respective proceedings.

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Thank you for your time and attention,
I am ready to answer your questions

Results of Simulation

- The model was simulated 500,000 times for 5 different pairs of strategies.
- The equilibrium is Expose / Bribe with Optimal / Optimal stealing.
- The resulting situation is pessimistic: corruption is not punished, but multiplied.

Results of Corruption Minimization

- The model was simulated 500,000 times for 5 different pairs of strategies for 4 different settings.
- There are situations (settings and bribes) in which not stealing is the most beneficial action for $O_{3,i}$.
- The changes in the settings reduce corruption.
- It is possible to eliminate the corruption in the model, but the means are extreme.
- High-level officials need some mechanism of protection from subordinates exposing them.

Mild Corruption Minimization Settings

Chains: $T_{12} = \{o_{2,i}\}; \{o_{1,i}\}$ $T_3 = \{o_{3,i}\}$ $T_C = \{o_{2,i}, o_{3,0}\}; \{o_{1,i}, o_{3,1}\}$

Let us limit $B_{suff-X} \leq S^*_{n,i}$, then with default we have 4 *zettings*:

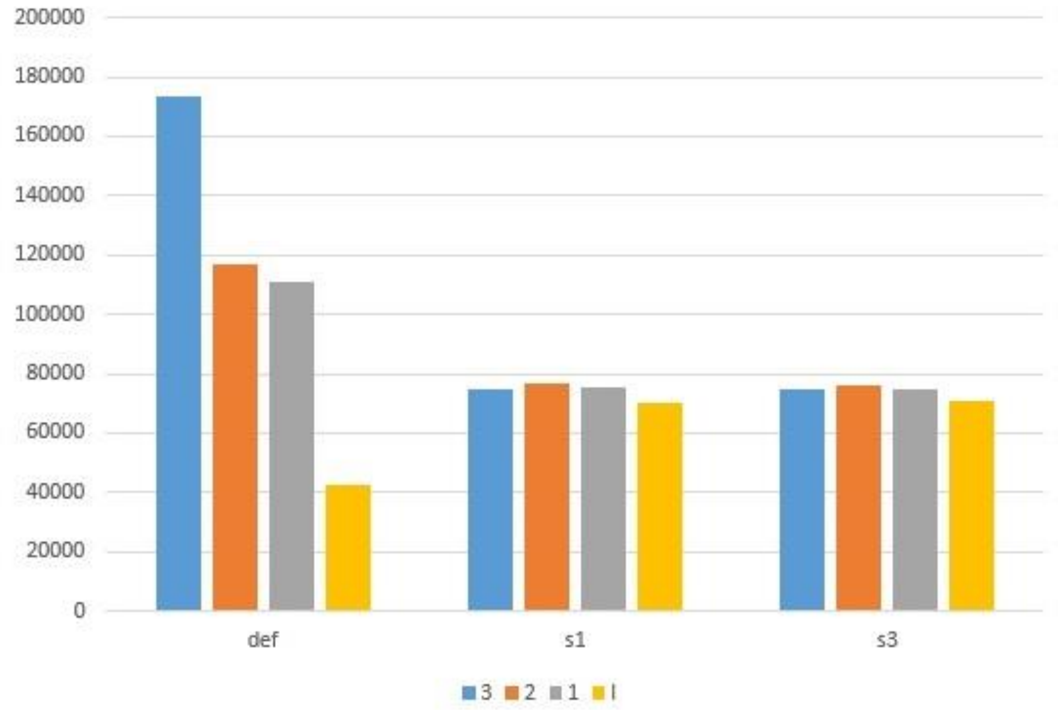
Zetting	$R(S_{\{1,2\},i})$	$Cu(S_{\{1,2\},i})$	$R(S_{3,i})$	$Cu(S_{3,i})$	$B_{suff-ch}$	B_{suff-b}	B_{suff-s}	T	B_{optT}
Default	40000	5000	75000	11250	131251	86251	45001	-	-
1	70000	35000	270000	124999	500000	395000	105001	ch	500000
2	0	0	300000	199999	500000	500000	1	b	500000
3	85000	39999	250000	125000	500000	375001	125000	s	125000

Zetting 2 is unrealistic (no reward and cover-up cost for “small” stealing).

Mild Corruption Minimization Effect

AVG	def	$z1$	$z3$		$def \to z1$	$def \to z3$	$z1 \to z3$
(3, 0)	143,336.69	69,432.38	69,307.13		-51.56 %	-51.65 %	-0.18 %
(3, 1)	147,691.36	79,857.00	79,864.25		-45.93 %	-45.92 %	0.01 %
(2, 0)	109,345.65	76,485.57	76,168.38		-30.05 %	-30.34 %	-0.41 %
(2, 1)	109,236.93	76,497.06	76,163.28		-29.97 %	-30.28 %	-0.44 %
(1, 0)	96,252.46	75,106.62	74,542.06		-21.97 %	-22.56 %	-0.75 %
(1, 1)	96,099.14	75,127.30	74,531.44		-21.82 %	-22.44 %	-0.79 %
Inspector	36,663.69	70,989.22	71,822.14		93.62 %	95.89 %	1.17 %

Mild Corruption Minimization Effect Chart



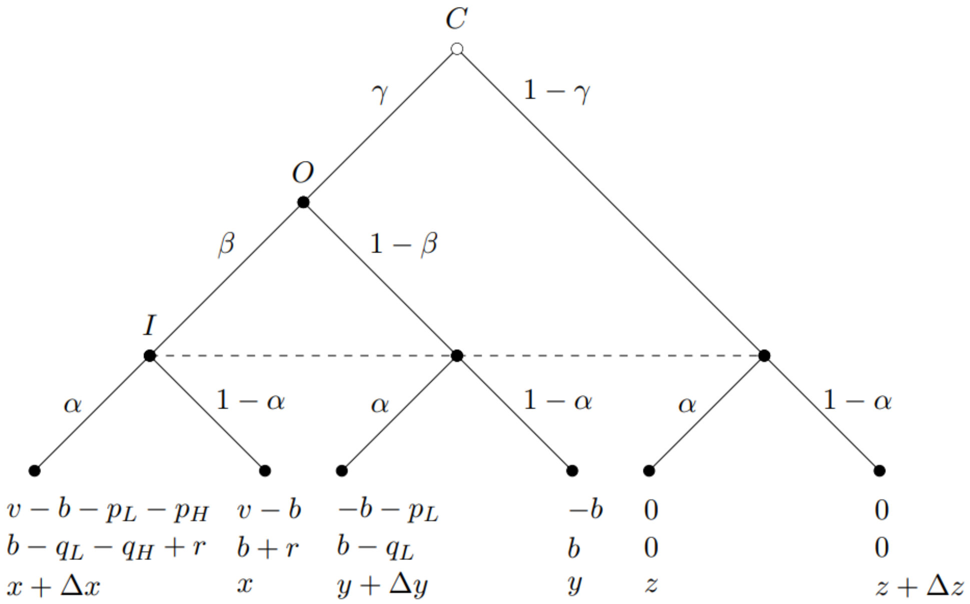
Results of Mild Corruption Minimization

- The model was simulated 500,000 times for 5 different pairs of strategies for 3 different zettings.
- There are situations (zettings and bribes) in which not stealing is the most beneficial action for $O_{3,i}$.
- The zettings reduce corruption, decrease revenue for $O_{n,i}$ and weakly increase for I.
- Mild Corruption Minimization is less extreme, effective, but less so than Corruption Minimization.
- High-level officials need some mechanism of protection from subordinates exposing them.

Further Research

- Analysis of κ and θ, measuring them in real world.
- Real-world experiments.
- Analysis of fine functions' effect.
- Larger hierarchies.
- Repeater game mechanism: orphans and punishment.
- Imperfect inspection.
- Changing the inspection direction.

Spengler D. Detection and Deterrence in the Economics of Corruption: a Game Theoretic Analysis and some Experimental Evidence



Attanasi et al. Embezzlement and Guilt Aversion

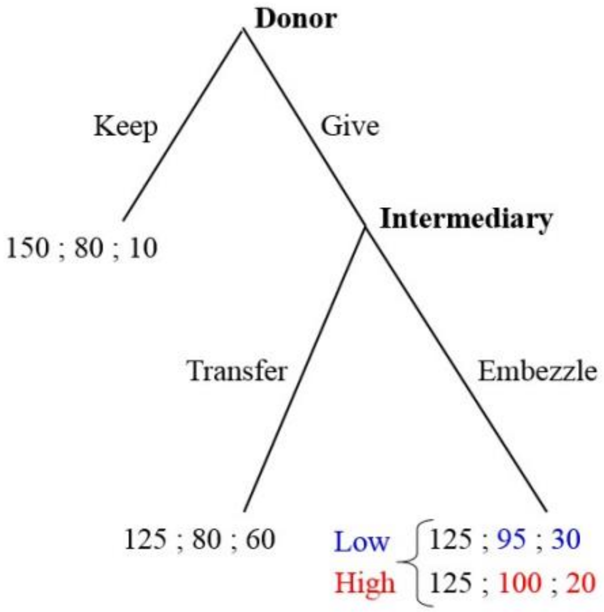


Figure 1: The Embezzlement Mini-Game(s)

Song et al. Game-theoretic Approach for Anti-corruption Policy Between Investigating Committee and Inspected Departments in China

TABLE I. GAME ANALYSIS MODEL OF ANTI-CORRUPTION

		Department being inspected	
		Corruption	*Non-corruption*
Committee investigating corruption	*Investigation*	R – C, – R	– C, 0
	Non-investigation	– R, R	0, – R

Shenje T. Investigating the Mechanism of Corruption and Bribery Behavior: A Game-Theoretical Methodology

Table 1: Payoff Matrix of the Game between Briber and Bribee

B \ A	Bribery	No Bribery
Bribery	$(w-b)$ $(x-y)$	b $-z$
No Bribery	0 0	0 0

Formula for Utility of *i-th* Official from Level *n*

$$U_{n,i}(S_{n,i}, B_{n,i}, A_{n,i}) = W_{n,i} + S_{n,i} - \alpha^{+}_{n,i} L(A_{n,i}, A_{-n,i})$$

$$A_{-n,i} = (A_{k,j}, \ldots, A_I) \ \ \forall (k,j) \neq (n,i)$$

where $S_{n,i}$ – official's steal;
$W_{n,i}$ – official's wage;
$B_{n,i}$ – official's bribe;
$L(A_{n,i}, A_{-n,i})$ – part of utility, dependent on players' actions;
$A_{n,i}$ – official's action;
$A_{-n,i}$ – other players' (officials' and inspector's) actions.

Stage 2 – Inspection Outcomes' Payoffs

End	$U_{j,k}$	$U_{1,i}$	$U_I)$
1	$W_{j,k} + S_{j,k}$	$W_{1,i} + S_{1,i}$	W_I
2	$W_{j,k} + S_{j,k}$	$W_{1,i} + \kappa_{1,i} S_{1,i} - F(W_{1,i}, S_{1,i})$	$W_I + R(S_{1,i}) - Ci_1$
3	$W_{j,k} + S_{j,k}$	$W_{1,i} + \kappa_{1,i} S_{1,i} - (F(W_{1,i}, S_{1,i}) + B_{1,i} + Fb(B_{1,i}))$	$W_I + R(S_{1,i}) - Ci_1$
4	$W_{j,k} + S_{j,k}$	$W_{1,i} + S_{1,i} - B_{1,i}$	$W_I + B_{1,i} - Ci_1 - Cu(S_{1,i})$
5	$W_{j,k} + \kappa_{j,k} S_{j,k} - F(W_{j,k}, S_{j,k})$	$W_{1,i} + \kappa_{1,i} S_{1,i} - \theta_{1,i} F(W_{1,i}, S_{1,i})$	$W_I - (Ci_1 + Ci_j) + R(S_{1,i}) + R(S_{j,k})$
6	$W_{j,k} + \kappa_{j,k} S_{j,k} - (F(W_{j,k}, S_{j,k}) + B_{j,k} + Fb(B_{j,k}))$	$W_{1,i} + \kappa_{1,i} S_{1,i} - \theta_{1,i} F(W_{1,i}, S_{1,i})$	$W_I - (Ci_1 + Ci_j) + R(S_{1,i}) + R(S_{j,k})$
7	$W_{j,k} + S_{j,k} - B_{j,k}$	$W_{1,i} + S_{1,i}$	$W_I + B_{j,k} - (Ci_1 + Ci_j + Cu(S_{1,i}) + Cu(S_{j,k}))$

End	Description
1	No inspection.
2	Subordinate is inspected, no bribe.
3	Subordinate is inspected, bribe is rejected.
4	Subordinate is inspected, bribe is accepted.
5	Boss is exposed by the subordinate, no bribe.
6	Boss is exposed by the subordinate, bribe is rejected.
7	Boss is exposed by the subordinate, bribe is accepted.

Formula for Inspector's Utility for inspecting official $O_{n,i}$

$$U_I(A_I, A_{n,i}, T) = W_I + \alpha^+_{n,i} K(A_I, A_{n,i}, T),$$

where

$$K(A_I, A_{n,i}, T) = \begin{cases} K(A_I, A_{boss(n)}, T \cup \{(n,i)\}) \; if \, A_{n,i} = E \\ B_{n,i} - \sum_{(l,j)\in T} [Cu(S_{l,j}) + Ci_l] \; if \, A_{n,i} = B \;\&\; A_I = Acc \\ \sum_{(l,j)\in T} [R(S_{l,j}) - Ci_l] \; if \, A_{n,i} \in \{B, NB\} \;\&\; A_I = Rej \end{cases},$$

where W_I – inspector's wage, $T = \{(v,k)\}$ – set of ids of inspected and exposed officials.

Formula for State Utility for inspecting Official *n,i*

$$U_s(A_{n,i}, A_I, T) = M_m - \sum_{j=1}^{m-1} S_j - \sum_{X \in \{I\} \cup H} W_X + \alpha_{n,i}^{+} D(A_{n,i}, A_I, T),$$

where

$$D(A_{n,i}, A_I, T) = \begin{cases} F(S_{n,i}, W_{n,i}) + \sum_{(l,j) \in T} [(1 - \kappa_{l,j}) S_{l,j} - R(S_{l,j})] + \\ + \sum_{(v,p) \in T \setminus \{(n,i)\}} \theta_{v,p} F(W_{v,p}, S_{v,p}) \; if \; A_{n,i} = NB \\ D(A_{boss(n)}, A_I, T \cup \{(n,i)\}) \; if \; A_{n,i} = E \\ D(NB, A_I, T) + B_{n,i} + Fb(B_{n,i}) \; if \; A_{n,i} = B \,\&\, A_I = Rej \\ 0 \; if \; A_{n,i} = B \,\&\, A_I = Acc \end{cases},$$

Formula for the Level of Corruption

$$LoC = \frac{\sum_{j=1}^{m-1} S_j}{M_m}$$

Results of Simulation

(22501, 43126)	**OptOpt_EB**	**OptOpt_BB**	**NoneOpt_NBB**	**OptNone_BNB**	**NoneNone_NBNB**
(3, 0)	-1198754.0	-80099.8	-78691.7	90000.0	90000.0
(3, 1)	-860900.0	-80550.4	-79753.8	90000.0	90000.0
(2, 0)	151774.6	-19181.1	40000.0	109342.1	40000.0
(2, 1)	151907.5	-19598.7	40000.0	109249.0	40000.0
(1, 0)	155765.8	35309.0	40000.0	63326.8	40000.0
(1, 1)	155846.5	36242.7	40000.0	63528.9	40000.0
Inspector	126438.3	101642.9	83753.5	77095.2	70000.0
State	4295082.2	3014021.8	2903494.0	2395092.9	2590000.0
LoC	0.5	0.5	0.3	0.2	0.0

B12/B3	**43126**	**86251**	**108751**	**131251**	**196877**
22501	OptNone_BNB	OptNone_BNB	OptNone_BNB	OptOpt_EB	OptNone_BNB
45001	OptNone_BNB	OptNone_BNB	OptNone_BNB	OptOpt_EB	OptOpt_EB
67502	OptNone_BNB	OptNone_BNB	OptNone_BNB	OptOpt_EB	OptOpt_EB

Cooperative Analysis Assumptions (1)

Assumptions:

1. Default setting.

2. $S^*_{1,i} = S^*_{2,i} = S_s \quad S^*_{3,i} = S_b$

3. If official is indifferent between being in coalition and not being in one, they choose not being.

Cooperative Analysis Assumptions (2)

From **Assumption 1** we get

$$\sum_{(l,j)\in T} [R(S_{l,j}) + Cu(S_{l,j})] < B^*_{n,i} < \frac{S^*_{n,i}}{\alpha^+_{n,i}} \quad \forall T, \tag{2.15}$$

and that gives us

$$S^*_{n,i} > 0 \quad \forall (n,i) \in H \;\rightarrow\; S_s > 0, \tag{2.16}$$

$$S_s - \alpha^+_{n,i} B_s = S_s - \frac{\alpha^{eff}_n}{2} B_s > 0 \quad n = 1,2 \;\; i = 0,1 \tag{2.17}$$

$$S_b - \alpha^+_{3,j} B_{ch} = S_b - (\frac{\alpha_3}{2} + \alpha^{eff}_k) B_{ch} > 0 \quad (j,k) = (1,1),(2,0) \tag{2.18}$$

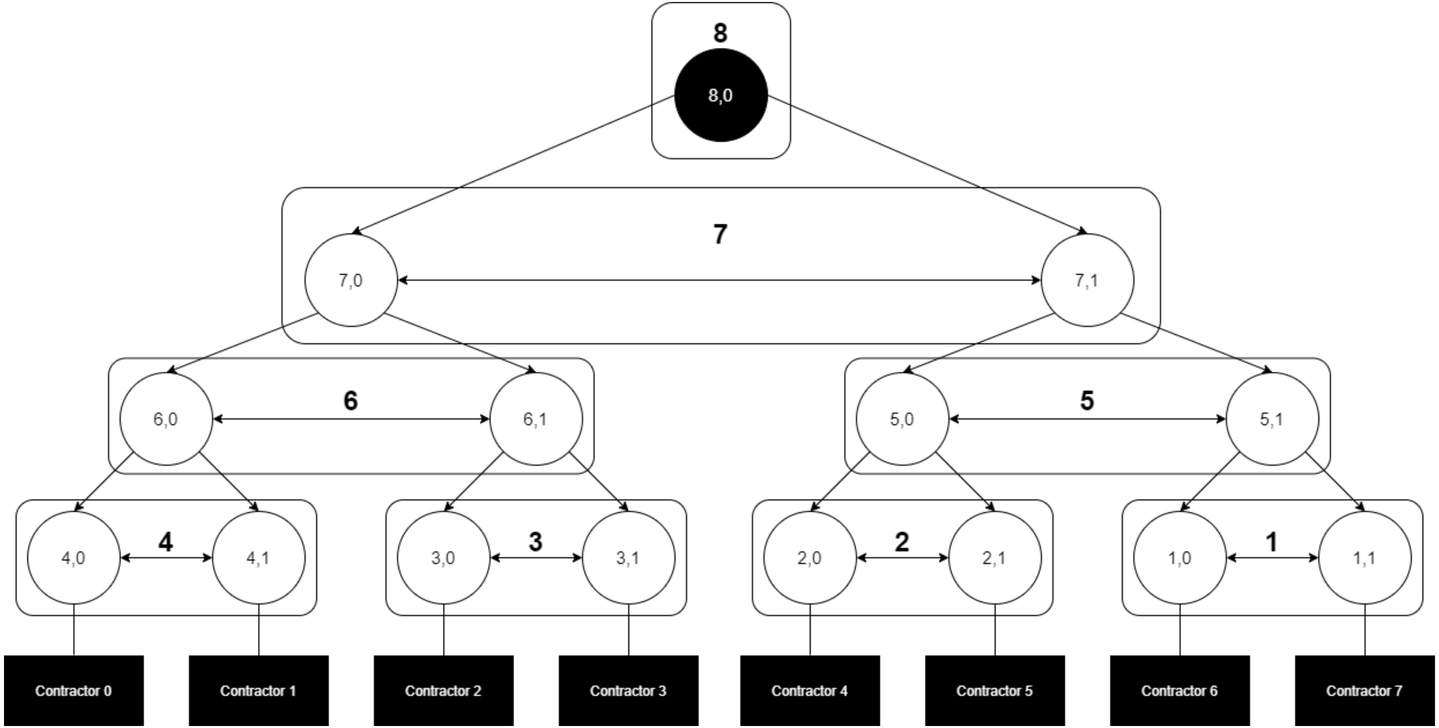
$$B_{ch} > B_b > B_s \tag{2.19}$$

For Imputation the test is against (2.16) and (2.18), for Coalition – against any other proper subcoalition.

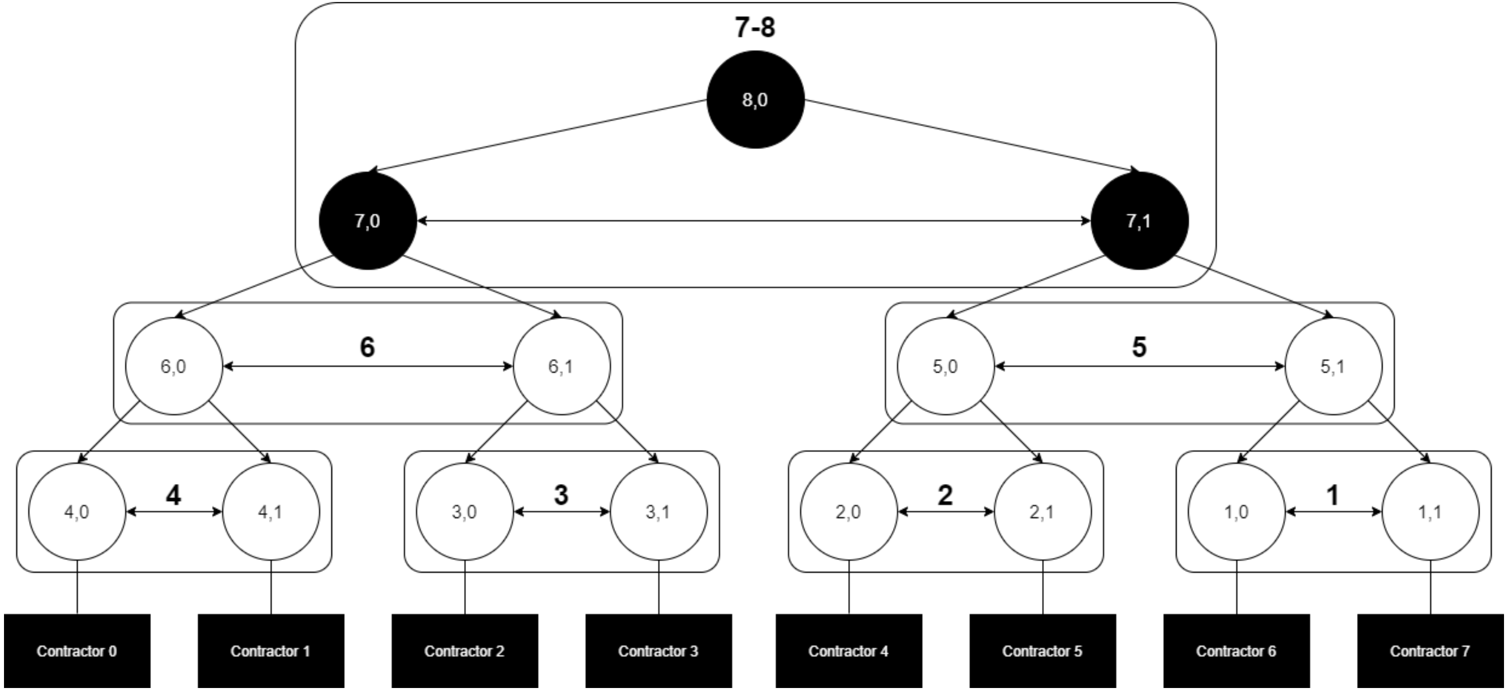
Formulas for all coalitions

#	(3,0)	(3,1)	(2,0)	(2,1)	(1,0)	(1,1)	v(?)	Fully formable?
1	0	0	0	0	0	1	$\{(1,1)\}$	TRUE
2	0	0	0	0	1	0	$\{(1,0)\}$	TRUE
3	0	0	0	0	1	1	$\{(1,0),(1,1)\}$	TRUE
4	0	0	0	1	0	0	$\{(2,1)\}$	TRUE
5	0	0	0	1	0	1	$\{(2,1)\}+\{(1,1)\}$	FALSE
6	0	0	0	1	1	0	$\{(2,1)\}+\{(1,0)\}$	FALSE
7	0	0	0	1	1	1	$\{(2,1)\}+\{(1,0),(1,1)\}$	FALSE
8	0	0	1	0	0	0	$\{(2,0)\}$	TRUE
9	0	0	1	0	0	1	$\{(2,0)\}+\{(1,1)\}$	FALSE
10	0	0	1	0	1	0	$\{(2,0)\}+\{(1,0)\}$	FALSE
11	0	0	1	0	1	1	$\{(2,0)\}+\{(1,0),(1,1)\}$	FALSE
12	0	0	1	1	0	0	$\{(2,0),(2,1)\}$	TRUE
13	0	0	1	1	0	1	$\{(2,0),(2,1)\}+\{(1,1)\}$	FALSE
14	0	0	1	1	1	0	$\{(2,0),(2,1)\}+\{(1,0)\}$	FALSE
15	0	0	1	1	1	1	$\{(2,0),(2,1)\}+\{(1,0),(1,1)\}$	FALSE
16	0	1	0	0	0	0	$\{(3,1)\}$	TRUE
17	0	1	0	0	0	1	$\{(3,1),(1,1)\}$	TRUE
18	0	1	0	0	1	0	$\{(3,1),(1,0)\}$	TRUE
19	0	1	0	0	1	1	$\{(3,1),(1,0),(1,1)\}$	TRUE
20	0	1	0	1	0	0	$\{(3,1)\}+\{(2,1)\}$	FALSE
21	0	1	0	1	0	1	$\{(3,1),(1,1)\}+\{(2,1)\}$	FALSE
22	0	1	0	1	1	0	$\{(3,1),(1,0)\}+\{(2,1)\}$	FALSE
23	0	1	0	1	1	1	$\{(3,1),(1,0),(1,1)\}+\{(2,1)\}$	FALSE
24	0	1	1	0	0	0	$\{(3,1)\}+\{(2,0)\}$	FALSE
25	0	1	1	0	0	1	$\{(3,1),(1,1)\}+\{(2,0)\}$	FALSE
26	0	1	1	0	1	0	$\{(3,1),(1,0)\}+\{(2,0)\}$	FALSE
27	0	1	1	0	1	1	$\{(3,1),(1,0),(1,1)\}+\{(2,0)\}$	FALSE
28	0	1	1	1	0	0	$\{(3,1)\}+\{(2,0),(2,1)\}$	FALSE
29	0	1	1	1	0	1	$\{(3,1),(1,1)\}+\{(2,0),(2,1)\}$	FALSE
30	0	1	1	1	1	0	$\{(3,1),(1,0)\}+\{(2,0),(2,1)\}$	FALSE
31	0	1	1	1	1	1	$\{(3,1),(1,0),(1,1)\}+\{(2,0),(2,1)\}$	FALSE
32	1	0	0	0	0	0	$\{(3,0)\}$	TRUE
33	1	0	0	0	0	1	$\{(3,0)\}+\{(1,1)\}$	FALSE
34	1	0	0	0	1	0	$\{(3,0)\}+\{(1,0)\}$	FALSE
35	1	0	0	0	1	1	$\{(3,0)\}+\{(1,0),(1,1)\}$	FALSE
36	1	0	0	1	0	0	$\{(3,0),(2,1)\}$	TRUE
37	1	0	0	1	0	1	$\{(3,0),(2,1)\}+\{(1,1)\}$	FALSE
38	1	0	0	1	1	0	$\{(3,0),(2,1)\}+\{(1,0)\}$	FALSE
39	1	0	0	1	1	1	$\{(3,0),(2,1)\}+\{(1,0),(1,1)\}$	FALSE
40	1	0	1	0	0	0	$\{(3,0),(2,0)\}$	TRUE
41	1	0	1	0	0	1	$\{(3,0),(2,0)\}+\{(1,1)\}$	FALSE
42	1	0	1	0	1	0	$\{(3,0),(2,0)\}+\{(1,0)\}$	FALSE
43	1	0	1	0	1	1	$\{(3,0),(2,0)\}+\{(1,0),(1,1)\}$	FALSE
44	1	0	1	1	0	0	$\{(3,0),(2,0),(2,1)\}$	TRUE
45	1	0	1	1	0	1	$\{(3,0),(2,0),(2,1)\}+\{(1,1)\}$	FALSE
46	1	0	1	1	1	0	$\{(3,0),(2,0),(2,1)\}+\{(1,0)\}$	FALSE
47	1	0	1	1	1	1	$\{(3,0),(2,0),(2,1)\}+\{(1,0),(1,1)\}$	FALSE
48	1	1	0	0	0	0	$\{(3,0),(3,1)\}$	TRUE
49	1	1	0	0	0	1	$\{(3,0),(3,1),(1,1)\}$	TRUE
50	1	1	0	0	1	0	$\{(3,0),(3,1),(1,0)\}$	TRUE
51	1	1	0	0	1	1	$\{(3,0),(3,1),(1,0),(1,1)\}$	TRUE
52	1	1	0	1	0	0	$\{(3,0),(3,1),(2,1)\}$	TRUE
53	1	1	0	1	0	1	$\{(3,0),(3,1),(2,1),(1,1)\}$	TRUE
54	1	1	0	1	1	0	$\{(3,0),(3,1),(2,1),(1,0)\}$	TRUE
55	1	1	0	1	1	1	$\{(3,0),(3,1),(2,1),(1,0),(1,1)\}$	TRUE
56	1	1	1	0	0	0	$\{(3,0),(3,1),(2,0)\}$	TRUE
57	1	1	1	0	0	1	$\{(3,0),(3,1),(2,0),(1,1)\}$	TRUE
58	1	1	1	0	1	0	$\{(3,0),(3,1),(2,0),(1,0)\}$	TRUE
59	1	1	1	0	1	1	$\{(3,0),(3,1),(2,0),(1,0),(1,1)\}$	TRUE
60	1	1	1	1	0	0	$\{(3,0),(3,1),(2,0),(2,1)\}$	TRUE
61	1	1	1	1	0	1	$\{(3,0),(3,1),(2,0),(2,1),(1,1)\}$	TRUE
62	1	1	1	1	1	0	$\{(3,0),(3,1),(2,0),(2,1),(1,0)\}$	TRUE
63	1	1	1	1	1	1	GC	TRUE

Divide and Conquer



Divide and Conquer



Divide and Conquer