Analysis of Vanguard Mutual Funds: 2014 - 2019 Appendix - Figures and Tables

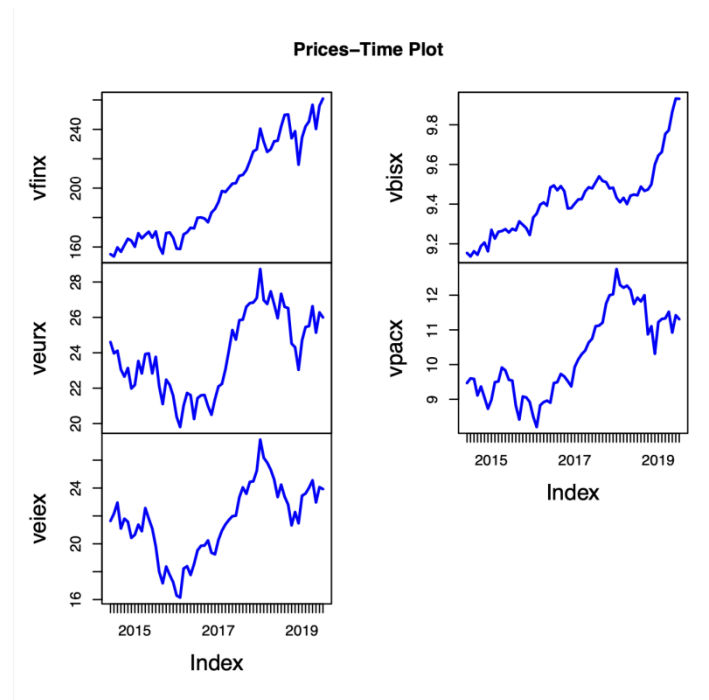## 1. Return calculations and Sample Statistics

1) Time Trend

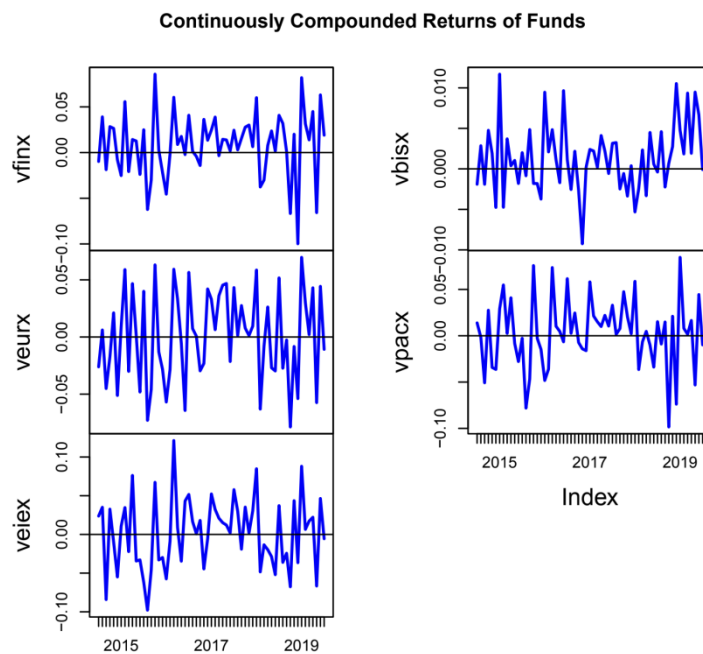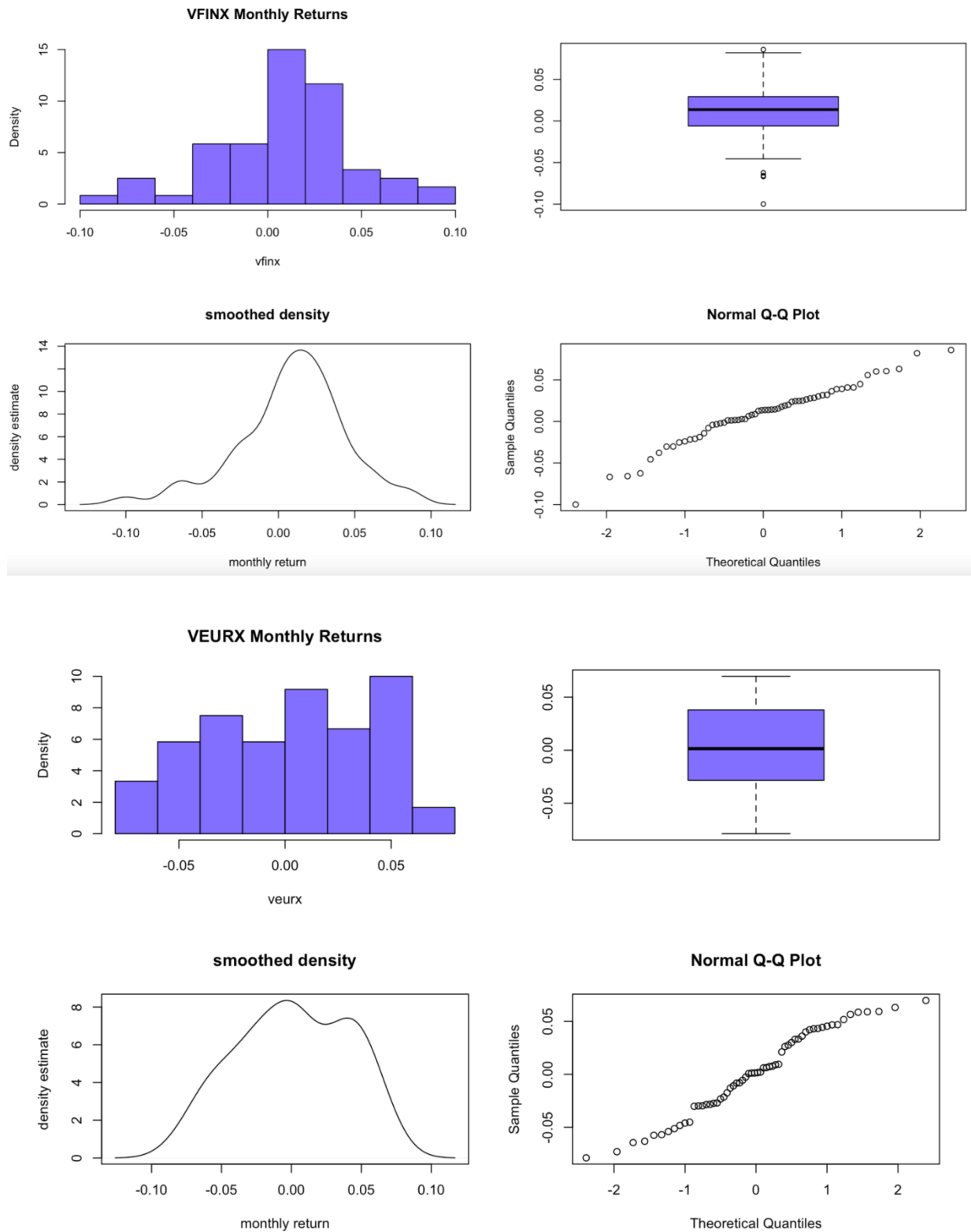Figure 1 - Price Time Plot
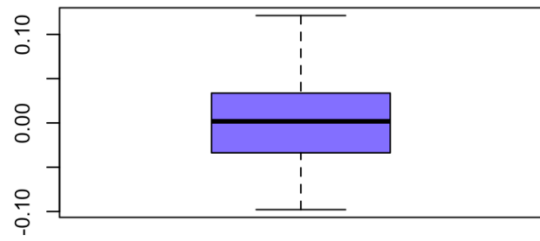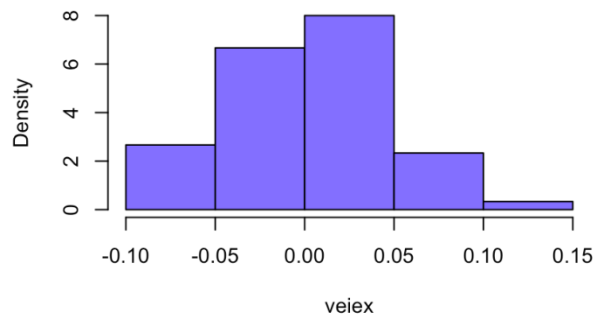

Figure 2 - Continuously Compounded Returns
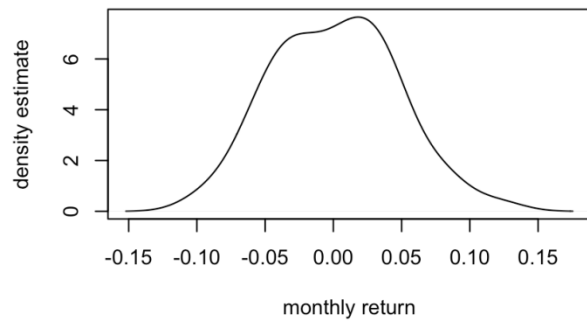
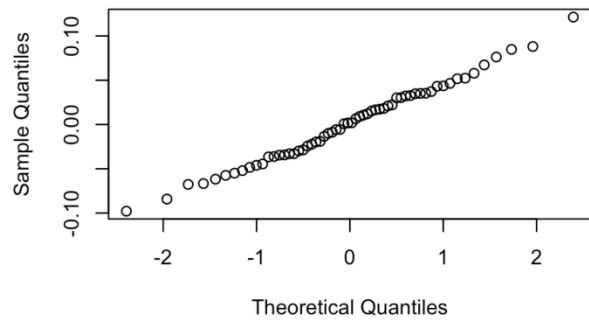## 2) Distribution of Returns

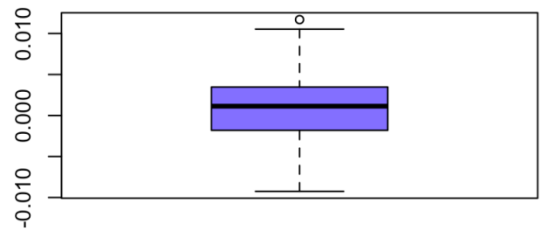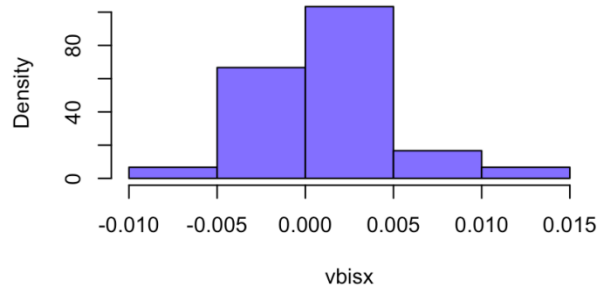Figure 3 - Panel Diagnostic Plots

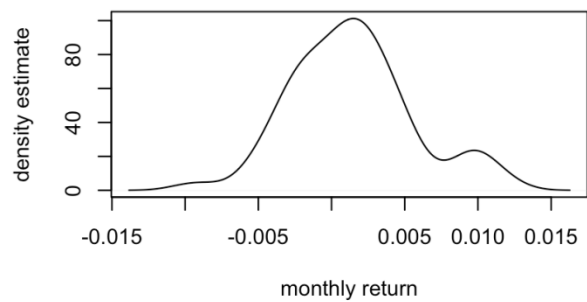## VEIEX Monthly Returns

smoothed density

Normal Q-Q Plot

VBISX Monthly Returns

smoothed density

Normal Q-Q Plot

VPACX Monthly Returns

smoothed density

Normal Q-Q Plot
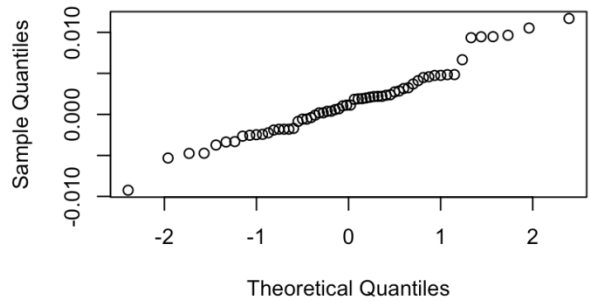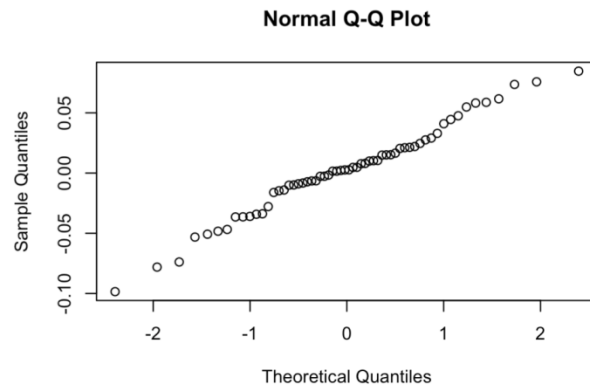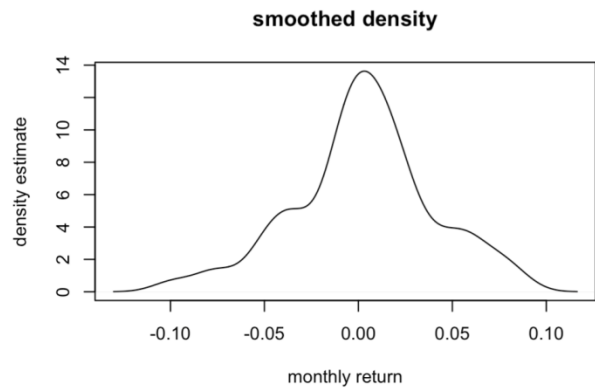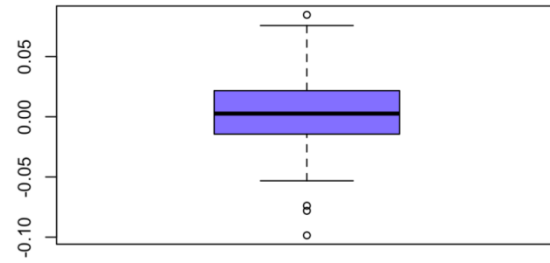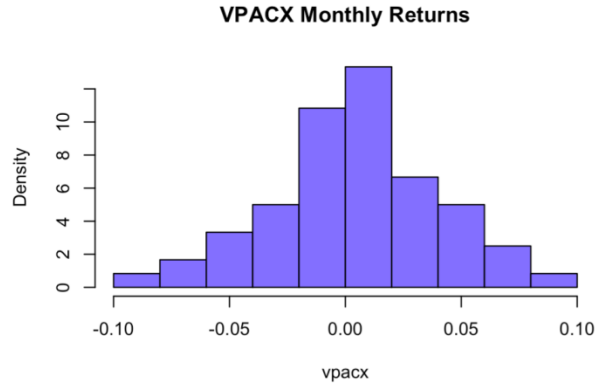
Table 1- Descriptive Statistics

	vfinx	veurx	veiex	vbisx	vpacx
Variance	0.00126153	0.00157403	0.00207061	0.000017447	0.00141323
Std. Deviation	0.035518	0.039674	0.045504	0.004177	0.037593
Skewness	0.5343	-0.1445	0.1676	0.3741	-0.1983
Excess Kurtosis	0.7964	-1.002	-0.2871	0.2916	0.2256
Quantiles					
Min	-0.09984	-0.078744	-0.097871	-0.009249	-0.098485
Q1	-0.00512	-0.02833	-0.033366	-0.001806	-0.014259
Median	0.013646	0.00148	0.001857	0.001135	0.002623
Mean	0.008831	0.001355	0.001278	0.001389	0.002726
Q3	0.028823	0.037063	0.033184	0.003358	0.021496
Max	0.085818	0.06969	0.121272	0.011691	0.084628

3) Sharpe Ratio

Table 2 - Monthly and annual Sharpe ratios

	vfinx	veurx	veiex	vbisx	vpacx
Monthly Sharpe Ratio	0.23692	0.02364	0.01894	0.23284	0.06144
Annual Sharpe's Ratio	0.82071	0.08189	0.0656	0.80657	0.21284

4) Future Value of Investment

Table 3 - Growth of $1 in five years

Figure 5 - $1 growth between 2014 - 2019
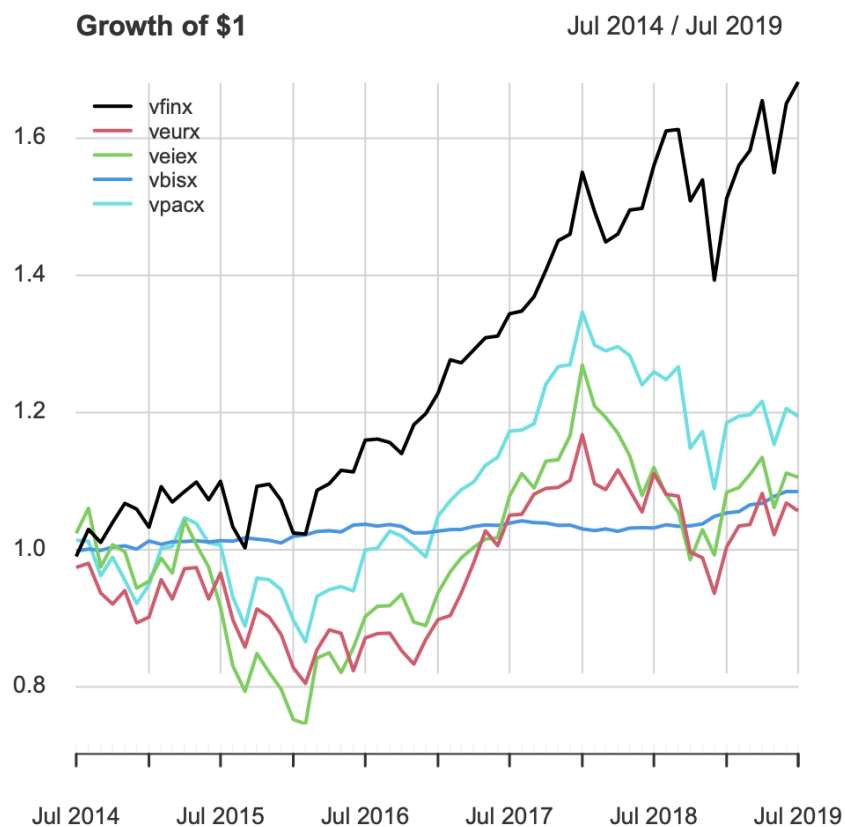
5) Relationship between Assets

Figure 6 - Pairwise Scatterplot

Table 4 - Sample covariance matrix

	vfinx	veurx	veiex	vbisx	vpacx
vfinx	0.0012615	0.0010960	0.0011129	-0.0000341	0.0010834
veurx	0.0010960	0.0015740	0.0013130	-0.0000181	0.0012248
veiex	0.0011129	0.0013130	0.0020706	0.0000200	0.0014584
vbisx	-0.0000341	-0.0000181	0.0000200	0.0000175	-0.0000106
vpacx	0.0010834	0.0012248	0.0014584	-0.0000106	0.0014132

Table 5 - Sample correlation matrix

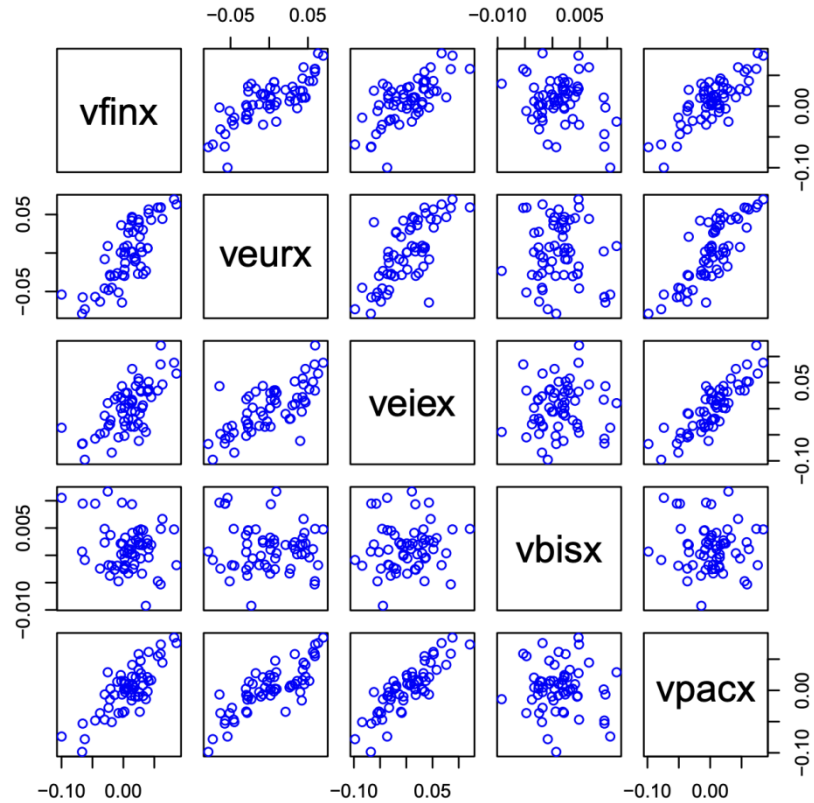|	vfinx	veurx	veiex	vbisx	vpacx
vfinx	1	0.777779	0.6885962	-0.23002476	0.81141563
veurx	0.777779	1	0.7273099	-0.10905481	0.82121794
veiex	0.6885962	0.7273099	1	0.10508198	0.85257025
vbisx	-0.2300248	-0.1090548	0.105082	1	-0.0678158
vpacx	0.8114156	0.8212179	0.8525703	-0.06781576	1

2. Value-at-risk Calculations

Table 6. Value-at-Risk in $

	vfinx	veurx	veiex	vbisx	vpacx
Monthly (Normal Distribution)					
α = 5%	-4838.1	-6190.4	-7092.8	-546.7	-5739.6
α =1%	-7113.8	-8692.8	-9929.7	-829.4	-8123.8
Annual (Normal Distribution					
α = 5%	-9189.9	-18925.7	-21646.5	-710.5	-16597
α =1%	-16494	-26174	-29627	-1685	-23680

3. Portfolio Theory
1) Global Minimum Variance (GMV) Portfolio

Figure 7 - Global minimum variance portfolio weights
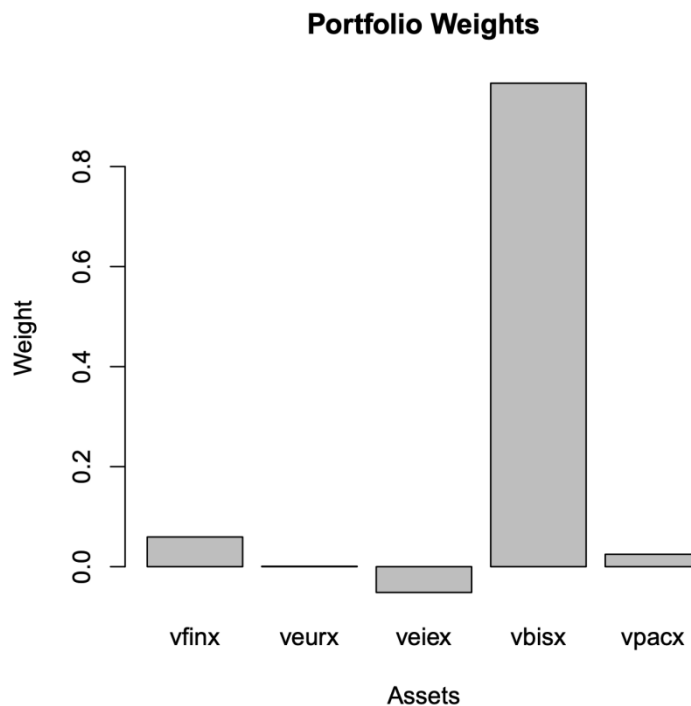

Table 7-a.Global minimum variance portfolio weights

vfinx	veurx	veiex	vbisx	vpacx
0.0626	0.0061	-0.0536	0.9661	0.0188

Table 7-b.Global minimum variance portfolio Characteristics

```
Portfolio expected return:      0.001886
Portfolio standard deviation:   0.003653
Portfolio Sharpe Ratio:         0.4022
```

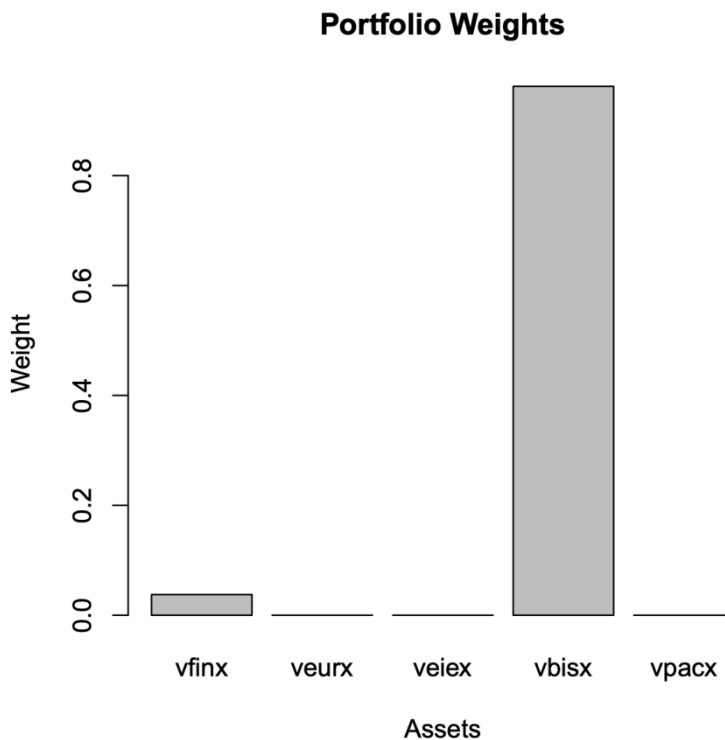Figure 8 - GMV portfolio (no short-sales) weights

Portfolio Weights

Table 8.a- GMV portfolio (no short-sales) weights

vfinx	veurx	veiex	vbisx	vpacx
0.0383	0	0	0.9617	0

Table 8-b.GMV portfolio (no short-sales) Characteristics

```
Portfolio expected return:      0.001674
Portfolio standard deviation:   0.003934
Portfolio Sharpe Ratio:         0.3197
```
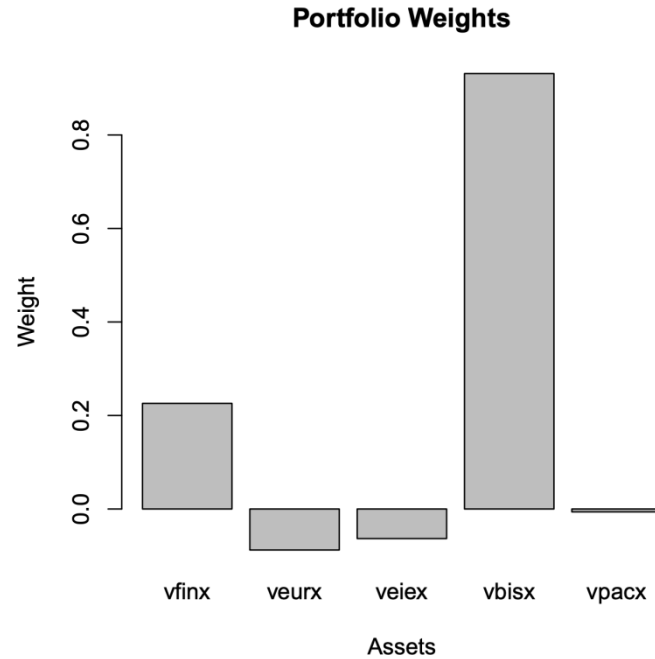
2) Tangency Portfolios

Figure 9 - Tangency Portfolio weights

Portfolio Weights

Table 9a. Tangency portfolio weights

vfinx	veurx	veiex	vbisx	vpacx
0.2185	-0.0632	-0.0681	0.9334	-0.0206

Table 9b. Tangency portfolio characteristics

```
Portfolio expected return:      0.002997
Portfolio standard deviation:   0.004841
Portfolio Sharpe Ratio:         0.5331
```

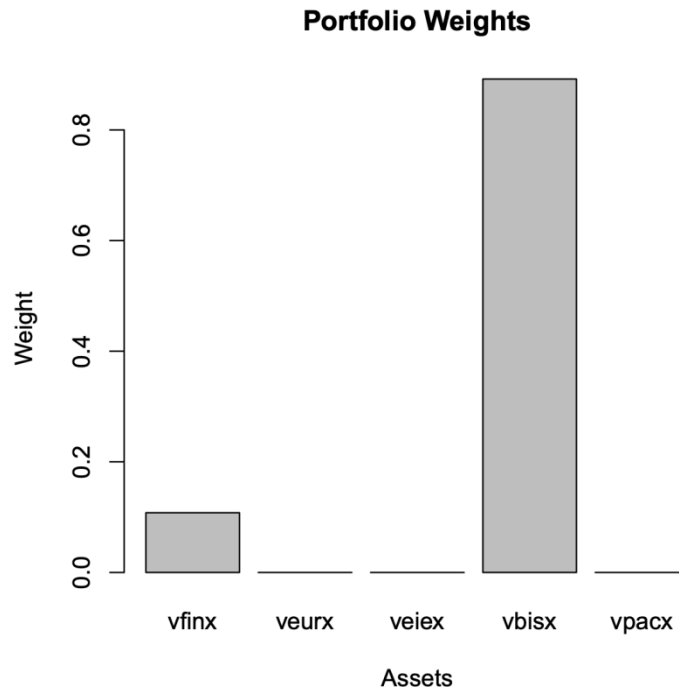Figure 10. Tangency portfolio (no short-sales) weights

Portfolio Weights

Table 10-a. Tangency portfolio (no short-sales) weights

vfinx	veurx	veiex	vbisx	vpacx
0.1063	0	0	0.8937	0

Table 10-b. Tangency portfolio (no short-sales) characteristics

```
Portfolio expected return:      0.00218
Portfolio standard deviation:   0.004659
Portfolio Sharpe Ratio:         0.3786
```

4. Asset Location
1) Efficient Portfolio

Figure 11. Efficient portfolio weights (with target return equal to the max expected return of the assets)
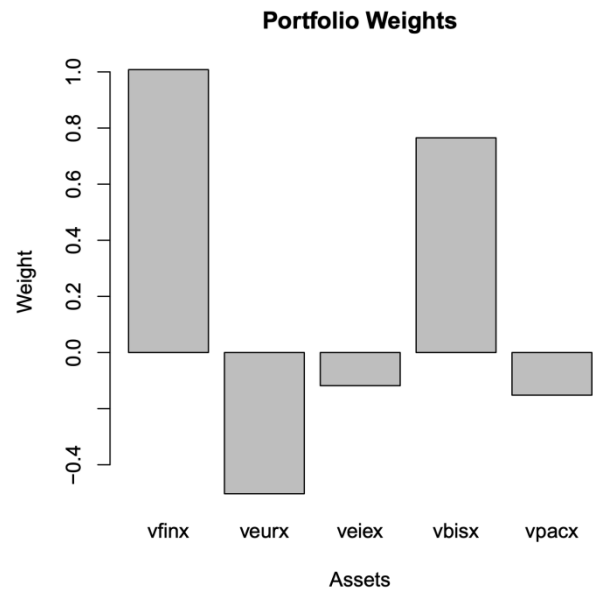
Portfolio Weights

Figure 12. Efficient Portfolio Frontier

Efficient Frontier