

An Analysis of Vanguard Mutual Funds: 2014 - 2019

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I. Executive Summary

- During 2014-2019, there were major events that happened worldwide that affected stock prices and returns throughout this time period. These events include: Brexit, concerns over Eurozone debt, specifically with the Greek government debt, volatility in Oil Prices, the US-China trade war, and the annexation of Crimea from Russia.
- Within the stock market, it can be observed that the S&P 500(VFINX) was the highest performing stock as compared to its counterparts. As opposed to this, the Emerging Market Funds(VEIEX) is the lowest performing market in terms of average returns and simultaneously the market that has the highest amount of variance. This can generally be explained through the economic instability and political unrest that comes with the formation of emerging markets. It has a lower average return than the average return of the short-term bond funds market(VBISX).
- The skewness of stock funds is negative, and a few of them have several negative outliers, implying very low returns. On the other hand, bond funds tend to be slightly right-skewed and exhibit lower volatility.
- There is a high degree of correlation within the stock funds and the bond funds respectively. However, there is little to no correlation between the stock and bond markets. Therefore, investing in a combination of a stock fund such as VFINX and a bond fund such as VBISX would provide the greatest reduction in risk through diversification.

- VEIEX has the lowest Sharpe Ratio, indicating it provides a lower reward per unit of risk compared to other funds. In contrast, VBISX has the highest Sharpe Ratio, making it a more attractive investment choice as it offers the highest compensation per unit of risk.
- Value-at-Risk (VaR) is a measure that quantifies the potential loss of our initial investment over a holding period with a probability of $\alpha\%$. Both normal and empirical VaR calculations indicate that VEIEX has the highest VaR, while VBISX has the lowest VaR among the assets considered.
- Diversified portfolios tend to be more efficient compared to individual assets, as they offer a higher Sharpe Ratio and a lower Value-at-Risk (VaR). This highlights the advantage of risk reduction through diversification.
- The global minimum variance portfolio, which has the lowest variance and is considered efficient, takes short positions in VEIEX and VBLTX while having a leveraged position in VBISX.
- The tangency portfolio refers to a portfolio of risky assets that maximizes the Sharpe Ratio. In this case, to view the tangency ratio, we take a short position in all assets except for VFINX and VBISX.
- By allowing for short sales, we can construct portfolios that are more efficient, as they have higher Sharpe Ratios and lower Value-at-Risk. However, in certain cases such as mutual funds in 401K, short sales are not permitted.
- The upper part of the Markowitz bullet, which includes the global minimum variance portfolio, forms the set of efficient portfolios of risky assets. When we combine this set with T-bills, we get an efficient portfolio of risk-free and risky

assets. The efficient portfolio of risk-free and risky assets is represented by a line known as the tangent line.

II. Dataset Description

This report will analyze the monthly closing price data of five Vanguard mutual funds over the course of five years from the end of June 2014 through the end of June 2019. Mutual funds pool money together from different investors and invest that capital into different industries and types of securities such as stocks, bonds, and short-term debt. Mutual funds are popular among investors since they are diversified, professionally managed by fund managers, have a low minimum initial investment, can be tax efficient and are liquidable. Vanguard is one of the top three U.S. mutual fund companies alongside BlackRock and Charles Schwab. As of 2022, Vanguard has more than \$8 trillion in assets under management. Vanguard currently offers 267 mutual funds on their official website and is a suitable choice for new and experienced investors. The six Vanguard mutual funds that we will be analyzing in this report include: VFINX, VEURX, VEIEX, VBISX, and VPACX. It's important to note that all six of these mutual funds are currently closed to new investors. Now, we will present a brief summary on each of the mutual funds.

- **S&P 500 Index - VFINX:** This mutual fund attempts to track Standard & Poor's (S&P) 500 Index which is a widely used benchmark of the U.S. stock market's overall performance. The S&P 500 tracks the stock performance of the 500 largest publicly traded companies.

Therefore, this fund invests its money into the same 500 companies at approximately the same proportion as the index.

- **European Stock Index - VEURX:** This mutual fund attempts to track the FTSE Developed Europe All Cap Index which is a benchmark for stocks in the major developed European markets. Therefore, this fund holds all stocks in the index with the same weight capitalization.
- **Emerging Market Fund - VEIEX:** This mutual fund attempts to track the FTSE Emerging Markets All Cap China A Inclusion Index which is a market-capitalization-weighted index that is made up of approximately 3,550 common stocks of companies located in emerging markets worldwide, with the two biggest countries being China and India. This fund works by diversifying into different security types that when aggregated, mimic the results of the index.
- **Short-Term Bond Fund - VBISX:** This mutual fund attempts to track the performance of the Barclays Capital U.S. 1-5 year Government/Credit Float Adjusted Index which is a benchmark for the short-term, investment grade U.S. bond market. Therefore, this fund consists of U.S. investment grade bonds having an average maturity of 1-5 years. The structure of the fund is about 30% in corporate bonds and about 70% in U.S. government bonds.
- **Pacific Stock Index - VPACX:** This mutual fund attempts to track the performance of the FTSE Developed Asia Pacific All Cap Index which is a benchmark made up of approximately 2,150 common stock

issued by companies located in Japan, Australia, South Korea, Hong Kong, Singapore, and New Zealand. This fund holds all stocks in the same weight capitalization as the index.

Sources: Vanguard Website

III. Analysis

1.) Return Calculations and Sample Statistics

Stock Funds Time Trends

Looking at **figures 1 and 2**, we can see that most stocks in general move in a positive upward sloping fashion. International stocks such as VEURX, VEIEX, and VPACX, all show a similar stock trend through the years starting from 2014 and ending in 2019. Between the years 2014-2016 however, we can clearly see a copious amount of negative returns in each of the indexes. However, in 2015, there seems to be extremely high returns resulting from a 25% increase in the federal reserve rate since nearly a decade¹.

The low rates of return on the VEURX Stock Index could be the result of the Greek debt crisis and Brexit. Between the years of 2008 and 2018, the Greece government accumulated a substantial amount of debt from the European Union because of its failure to pay IMF and EU loans². Along with this, the official announcement of Brexit in 2016 led to the lowest returns of the stock index. The uncertainty of the United Kingdom breaking from the European Union

combined with the Greece financial debt lead to a level of financial insecurity in the eurozone. These results lead to the low returns shown in the European Stock Index between the periods of 2014 to 2016.

With respect to the performance of VEIEX, the drop in returns can be explained through the slowdown of the Chinese market from 2014-2016, and its substantial share of influence on the stock index. In fact, the Chinese market had reached a 25 year low during 2015. From June 2015 to February 2016, the Shanghai stock market suffered a sharp drop of 30% within a mere three weeks, leading to a highly negative outlier in the returns of the VEIEX fund in mid-2015. This also led to the market crashing to half its price in one month³. The negative returns observed across all stock funds during this period were largely due to China's significant influence on the global economy. Brazil, another country with a substantial influence on the index, experienced political and economic turmoil, with mass protests erupting in 2015-2016 to denounce the corruption of former President Rousseff⁴. Brazil's GDP had contracted by a whopping 3.8% in 2015, reflecting a severe economic crisis that had started during mid-2014⁵. Combined with these two countries, Russia was also grappling with its own financial crisis during 2014-2015, which resulted from a sharp devaluation of the ruble brought on by falling oil prices and the international sanctions imposed following the annexation of Crimea and the Russo-Ukrainian War⁶. Also in 2015, Taiwan recorded its slowest economic growth since the global financial crisis of 2008⁷.

Given these circumstances, the negative performance and falling prices of the VEIEX fund during this time were easily predicted.

While there are not many significant events to explain the decline of VPACX, we observe a period of low and even negative economic growth for major Pacific/Asian countries during 2014-2016. The Japanese economy went into recession in 2015⁸, and the South Korean economy was also underperforming due to slumps in key sectors such as exports and manufacturing⁹. These sectors constitute a large portion of the South Korean economy. Many Pacific/Asian countries heavily rely on China as their trading partner, given their geographic and cultural proximity. Thus, it is highly likely that they were impacted by the deceleration of the Chinese economy.

Between 2016 and 2018, there is a consistent and gradual recovery in stock index prices, indicating a period of stable and successful economic growth. The VFINX index in particular displays this bullish market trend, which can be attributed to the pro-business and growth-oriented policies implemented by the Trump administration, such as reductions in corporate taxes and regulatory measures. Additionally, the impressive performance of information-technology firms, which constitute a substantial portion of VFINX's holdings, is reflected in this market trend.

Towards the end of 2018, there were significant drops in stock index prices and returns. This was due to the looming threat of a trade war between the United States and China, concerns of a potential economic slowdown, and

worries that the Federal Reserve may have made an error in its monetary policy¹⁰. These factors contributed to a decline in investor confidence and market volatility. However, the market showed signs of recovery at the start of 2019 when President Trump announced a phase-one trade deal with China, which helped to ease tensions and mitigate some of the uncertainty surrounding the trade conflict¹¹.

The price movements of the two bond funds, VBLTX and VBISX, appear to be closely related. Unlike the stock funds however, their plots of price over time show much smoother fluctuations. VBISX, in particular, displays a smooth and steady upward price trend, which is not unexpected given its low volatility. The decline in bond returns at the end of 2016 can be attributed to President Trump's election victory. Many investors shifted from the bond market to the equity market because they did not want to miss out on the economic opportunities that would come with Trump's corporate stimulus plans. As the U.S. economy recovered towards the end of 2018, the bond market also began to rebound.

Covariance Stationary

Upon examining the time series displayed in **Figure 2**, there is a possibility of covariance non-stationarity in continuously compounded returns. Covariance stationarity requires constant mean and volatility, but there appears to be less volatility in 2017-2018 due to economic stability. This leads to some skepticism about whether the returns stem from the same underlying probability distribution function. However, for the purposes of this report, we will disregard this concern.

2.) Distribution of Returns

Funds Distribution

Figure 3 shows the QQ-plot, boxplot, histogram, and smoothed histogram of each asset's monthly return. **Table 1** shows the descriptive statistics of each of the monthly assets. One thing to note is that the X and Y values for each graph are different because of their intervals. Both of these help us observe the distribution of the assets along with its normality.

When we look at the distribution of VFINX based on the four graphs, we can observe a relatively normal distribution from both the bar graph and the smoothed density curve. The QQ plot also shows this, however, the ends of the plot heavily deviate from the theoretical values. This implies that there is a heavy amount of excess kurtosis, which can be observed in **table one**, as the value of excess kurtosis of VFINX is 0.7965. In normal distributions, the excess kurtosis value would normally have been 0. This is also corroborated with the fact that the boxplot has a singular positive outlier, however it also has several negative outliers, implying an extremity of incidences.

The distributions of the three funds - VEURX, VEIEX, and VPACX - show varying degrees of deviation from normality. VEURX exhibits a narrow distribution, with returns concentrated around the center and thin left and right tails, suggesting a period of relatively stable economic conditions in Europe. This is reflected in the negative excess kurtosis of -1.002 and negative skewness of -0.1455. On the other hand, VEIEX displays right skewness, with a few negative

outliers, as shown in the upward deviation of the QQ plot in the lower end, but overall a normal distribution. The positive skewness of 0.1676 and negative excess kurtosis of -0.2871 characterize this distribution. Finally, VPACX's distribution exhibits fat left and right tails, several negative outliers, and a few positive outliers, as seen in the positive excess kurtosis of 0.2256 and negative skewness of -0.1983.

The short-term bond market VBISX, has a fairly normal distribution within the histogram and smoothed density. Its box plot however indicates a high outlier of positive numbers. This is even more strongly supported by the fact that there is a slightly positive skew value of 0.3741. While looking at the QQ-plot, we can also see a large amount of variance in the right portion of the graph. This is also supported by the positive value of its kurtosis, being 0.2919. Overall however, there is very little variance, and this can also be seen in the bonds funds time trend.

It's worth noting that starting from 2015, VEURX, VEIEX, and VPACX began using different benchmark indexes. In November 2015, VEIEX transitioned from using the FTSE Emerging Index to the FTSE Emerging Markets All Cap China A Index, and then switched again to the FTSE Emerging Markets All Cap China A Inclusion Index in September 2016. In September 2015, VPACX replaced its FTSE Developed Asia Pacific Index with the FTSE Developed Asia Pacific All Cap Index. And finally, VEURX shifted from utilizing the FTSE Developed Europe Index to the FTSE Developed Europe All Cap Index in

September 2015. These modifications could significantly impact the behavior of Vanguard mutual funds, given that they track the performance of indexes.

Summary

Among the funds analyzed, VFINX has the highest monthly average return at approximately 0.8659%, while VEIEX has the lowest at approximately 0.1395%. The strong performance of VFINX can be attributed to the tech industry boom and the robustness of the US economy during the period under consideration. Meanwhile, VEIEX's poor performance aligns with that of major emerging markets such as China, Brazil, Russia, and Taiwan, which were affected by political instability, currency fluctuations, and economic downturns. In terms of the mean returns, the ranking from highest to lowest is VFINX, VBLTX, VPACX, VEURX, VBISX, and VEIEX.

VEIEX has the highest standard deviation of 4.55%, which is not surprising given that the stocks in emerging markets are typically more volatile than those of developed countries. While it may offer greater long-term returns, it can also result in lower and more volatile realized returns. Therefore, despite its potential for higher long-term gains, VEIEX is a poor investment in the five-year period used due to its low returns and high volatility. On the other hand, it is expected that VBISX, being a short-term and low-risk bond, has the lowest volatility. The assets are ranked in order of standard deviation from highest to lowest as follows: VEIEX, VEURX, VPACX, VFINX, VBLTX, VBISX. The efficient portfolio

selection process considers this volatility ranking to identify the most efficient portfolio based on an investor's risk preferences.

One thing that must be kept in mind is that the mean and standard deviation are subject to change from outliers, which are either extremely small or large values, as displayed by some of the funds. Therefore if we change the metric by which we analyze stocks, we can have a more clear and accurate picture of each index. Instead of looking at mean and standard deviation, we switch to looking at the median and interquartile range. This slightly changes the results of the dataset. The highest median is still VFINX, however the lowest median is given by VEIEX instead of VBISX. The highest and lowest interquartile range are given by both VEIEX and VBISX, which were the same results as the standard deviation of both bond indexes.

3.) Sharpe Ratio

Monthly Sharpe Ratio

The Sharpe Ratio is a performance metric that reflects the compensation per unit of risk an investor receives when investing in a risky asset. Typically, rational investors seek to minimize risk while maximizing returns and prefer investments with higher Sharpe Ratios. Using a monthly continuously compounded risk-free rate of 0.04167% (shown in **Table 2**), VFINX has the highest monthly Sharpe Ratio at 0.23692, while VEIEX has the lowest at 0.01894. The Sharpe Ratio ranked from highest to lowest are VFINX, VBISX,

VPACX, VEURX, and VEIEX. Thus, VFINX appears to be the most attractive investment, providing the highest reward per unit of risk taken.

Annualized Sharpe Ratio

By converting the monthly means and standard deviations to annual estimates and using the annual risk-free rate of 0.5%, we can calculate the annualized Sharpe ratios. Similar to the monthly Sharpe ratios, VFINX has the highest annualized Sharpe ratio at 0.82071, and VEIEX has the lowest annualized Sharpe ratio at 0.06560. The ranking of the Sharpe ratios from highest to lowest remains the same: VFINX, VBISX, VPACX, VEURX, and VEIEX. Therefore, VFINX is still the most desirable investment for a rational investor.

4.) Future Value of Investment

In order to analyze the future value of investment of each of the different stock indexes, we must look at **Figure 5** and **Table 3** respectively in order to see the growth of each fund over time.

Figure 5 presents an equity curve that allows for easy comparison of the growth of \$1 invested in each asset over the five-year period. The graph indicates that VFINX provides the highest future value, with \$1 invested in 2014 growing to just over \$1.60 in 2019. In contrast, VEURX, VEIEX, and VBISX yield the lowest future values, at around \$1.10. However, as typical investors invest much larger amounts than \$1, it is more practical to consider an initial investment of \$100,000. With this investment, the initial amount would have grown to

approximately \$160,000 for VFINX and \$110,000 for VEURX, VEIEX, and VBISX in 2019, representing a difference of nearly \$50,000.

The high future value of VFINX is not surprising given that it provides exposure to the 500 largest companies in the U.S., which represent a large-cap market. These companies are high-impact and account for a large majority of the U.S. stock market's value, with a diversified and wide-ranging industry representation. Notably, VFINX's largest holdings include prominent technology companies such as Apple, Microsoft, Amazon, Alphabet, and Facebook. The strong performance of the technology industry, which has benefited other industries through significant advancements such as machine learning, 5G, big data, and quantum computing, may have contributed to this growth. Additionally, the increasingly pro-market stance of the U.S. government in recent years, combined with policies such as quantitative easing, low interest rates, corporate tax cuts, and deregulation, along with a stable overall economy since the 2008 financial crisis, may have played a role in VFINX's substantial performance compared to other stocks.

The line representing VBISX is almost flat throughout the period, illustrating the high stability but low growth of the VBISX fund. The low risk of VBISX is accompanied by minimal returns in the bond index, reflecting the risk-return tradeoff. VEIEX and VEURX perform poorly, with investors even losing a portion of their initial investment during 2015-2017. While this loss is recovered by the end of the investment period, these two stock funds have returns similar to

the short-term bond index fund, and investors do not receive a higher compensation for taking on additional risk.

5.) Relationships between Assets

The sample covariance matrix in **Table 4** shows that the diagonal elements represent the variance, while the off-diagonal elements represent the covariance. We observe that all asset pairs have a positive linear association, except for four pairs that exhibit negative associations: VBLTX and VFINX, VBISX and VFINX, VBISX and VEURX, and VBISX and VPACX.

The correlation matrix in Table 5 enables us to measure the degree of linear association between the assets. The diagonal elements, represented by 1, signify perfect positive correlation since an asset is always perfectly correlated with itself. The highest positive correlations are observed between VPACX and VEIEX with a correlation of 0.85257, followed by VPACX and VEURX with a correlation of 0.82122, and VPACX and VFINX with a correlation of 0.81142. These correlations are not surprising as the analysis of time trends revealed that the stock indexes usually moved together. The bond indexes VEURX and VFINX also exhibit a strong correlation of 0.7778. On the other hand, the least correlated assets are VPACX and VFINX with a correlation of -0.06782. The most negatively correlated assets are VBISX and VFINX with a correlation of -0.23002. These correlations align with the expectation that bond and stock indexes do not

move together, and sometimes even exhibit an opposite movement as investors shift away from the bond market to the equity market, and vice versa.

The relationships observed in the sample correlation matrix are further illustrated by the pairwise scatterplots in **Figure 6**. The scatterplots of the bond and stock indexes appear to be scattered in various directions, resembling "shotgun blasts." On the other hand, the scatterplots of two stock indexes or two bond indexes exhibit fairly upward sloping trends.

Based on the observed relationships, diversification can help to reduce risk with the six assets. The extent of risk-return benefits that diversification can provide depends on the correlation between assets. In the two extreme cases, when the correlation is -1 , we get a minimum variance portfolio that has a "perfect hedge" with zero volatility. On the other hand, when the correlation is 1 , there is no benefit from diversification since the risk-return trade-off is linear. Since the six mutual funds are positively correlated but not perfectly correlated, some degree of risk reduction benefits can be obtained from diversification. Among these assets, VBISX is likely to bring about the most risk reduction benefits since it is the least correlated or negatively correlated with other assets, causing the portfolio frontier to stretch more towards the y-axis. Combining a stock and a bond fund can be a highly effective diversification strategy, and can yield a greater amount of returns.

6.) Value-at-Risk(VaR) Calculation

Monthly Value-at-Risk

The Value-at-Risk (VaR) is a measure of the potential loss that investors may incur with a certain probability, $\alpha\%$, over a specific holding period, by investing in a financial asset. As shown in **Table 6**, VEIEX has the highest monthly VaR, while VBISX has the lowest VaR for both 5% and 1% alpha levels. The VaR is calculated based on the normal distribution of the assets. Assuming an initial investment of \$100,000, investing in VEIEX for a month comes with a 5% probability of losing \$7092.80 or more, and a 1% probability of losing \$9929.70 or more. In contrast, investing in VBISX for a month comes with a much lower risk; there is a 5% probability of losing \$546.70 or more and a 1% probability of losing \$829.40 or more of the initial investment. This is expected, as we have shown earlier that VBISX is a low-risk short-term bond. The VaR ranking of the assets, from highest to lowest possible loss, is VEIEX, VEURX, VPACX, VFINX, VBLTX, VBISX.

Annual Value-at-Risk

Expanding the analysis to cover a one-year holding period, the disparities between high-risk and low-risk assets become even more pronounced. For instance, VEIEX poses a significant risk to investors, as we can observe from **Table 7**. With a 5% probability level, investors may lose \$21,646.50 or more of their initial investment, while with a 1% probability level, they may lose \$29,627 or more of their initial investment by investing in this asset for a year. In contrast, the low-risk VBISX provides much more security to investors. With a 5% probability level, investors may lose only \$710.50 or more, and with a 1%

probability level, they may lose \$1,685 or more of their initial investment by investing in VBISX for a year. Notably, the ordering of the VaR remains the same as in the monthly analysis.

Empirical Value-at-Risk

The initial two VaR computations were based on the assumption of normal distributions of the assets. However, VaR can also be calculated using an empirical quantile and historical simulation. Upon using this approach, we see that the ranking of assets by their VaR is preserved, with VEIEX having the highest VaR and VBISX having the lowest VaR. However, the empirical VaRs for VEIEX, VEURX, VBLTX, and VBISX are lower than those predicted by a normal distribution. This phenomenon is attributable to the thin left tails of monthly returns observed in the 4-panel diagnostic plots we previously created, particularly for VEIEX, VEURX, and VBISX. As a result, the probability of negative extreme returns occurring is lower than what a normal distribution would estimate. VPACX's empirical VaR is lower than the normal VaR at the 5% probability level, but higher at the 1% probability level. In contrast, VFINX has a higher VaR in both the 5% and 1% probability levels. The diagnostic plots show that VPACX and VFINX have fatter left tails than a normal distribution, indicating a greater likelihood of negative extreme returns than what a normal distribution would predict. Consequently, the VaRs are higher when using historical data.

IV.)Portfolio Theory

7.) Global Minimum Variance(GMV) Portfolio

The global minimum variance portfolio is the portfolio with the lowest variance among all possible portfolios. Its weight distribution can be seen visually in **Figure 7** and numerically in **Table 7-a**. The portfolio includes negative weights in VEIEX, with a weight of -0.0536. Additionally, it includes a leveraged position in VBISX with a weight of 0.9661. This leveraged position in VBISX is logical as it is the least volatile asset, and the goal of the GMV portfolio is to minimize variance. It is important to note that the portfolio weights sum up to 1, indicating that the entire wealth is invested in the six assets.

According to Table 7-b, the GMV portfolio's expected monthly return is 0.1886 %, with a standard deviation of 0.3653% and a Sharpe Ratio of 0.4022. Its expected annual return is also the same for all values. The portfolio's standard deviation is lower than that of any individual asset, which aligns with its aim to minimize volatility. The portfolio's expected return is also lower than that of individual assets, reflecting the risk-return tradeoff. When we accept low risk, we must also accept low returns. Nonetheless, the GMV portfolio's Sharpe Ratio is higher than that of any individual asset. As a result, we can achieve higher compensation per unit of risk with the GMV portfolio than with any individual asset. This underscores the benefits of risk reduction through diversification.

No-Short Sales

The previous portfolio was constructed allowing for short-sales, which involves borrowing and selling a stock with the intention of buying it back at a lower price to make a profit. However, mutual funds generally do not allow short-

sales, so we need to compute the GMV portfolio without short-sales. With this constraint, all weights are now either 0 or positive, as shown in **Figure 8** and **Table 8-a**.

According to **Table 8-b**, the expected monthly return of the long-only GMV portfolio is 0.1674%, the standard deviation is 0.3934%, and the Sharpe Ratio is 0.3197. The expected annual return is the same as its monthly return. Compared to the long-short GMV portfolio, the long-only GMV portfolio has higher expected returns and lower standard deviations, which is explained by the smaller weight on VBISX. However, higher risk is still associated with a higher expected return. The Sharpe ratio for the long-only GMV portfolio is lower than the Sharpe ratio when short sales were allowed. Therefore, the long-short GMV portfolio is better as it provides more compensation per unit of risk.

8.) Tangency Portfolio

The tangency portfolio is a collection of risky assets with the highest Sharpe ratio, as it provides the optimal risk-return tradeoff. **Figure 9** and **Figure 9-a** show the weights of the different assets in the portfolio, which were calculated using a monthly risk-free rate of 0.04167%. The tangency portfolio features positive weights on VFINX and VBISX, and negative weights on the other funds. **Table 9-b** shows that the expected monthly return is 0.2997%, the standard deviation is 0.4841%, and the Sharpe Ratio is 0.5295. On an annualized basis, the expected return is 2.997%, the standard deviation is 1.361%, and the Sharpe ratio is 1.835. The tangency portfolio's expected return

falls within the range of expected returns of the individual assets, but its standard deviation is much lower due to diversification. As a result, the tangency portfolio's Sharpe ratios, both monthly and annualized, are significantly higher than those of any of the individual assets. Therefore, the tangency portfolio is more efficient and preferable.

No-Short Sales

When short-sales are not allowed, the tangency portfolio's weights change and are shown in **Figure 10** and **Table 10-a**. **Table 10-b** shows that the expected monthly return is 0.2164%, the standard deviation is 0.4644%, and the Sharpe Ratio is 0.3763. Similarly, the expected annual return is 2.597%, the standard deviation is 1.609%, and the Sharpe ratio is 1.303. Compared to the tangency portfolio without the restriction, the expected returns are lower, and the standard deviations are higher. As a result, the Sharpe ratios are also lower, indicating lower reward per unit of risk. Therefore, prohibiting short-sales leads to a less efficient portfolio.

V.) Asset Allocation

9.) Efficient Portfolio

In **Table 11-a** and **Figure 11**, I derive the weights for an efficient portfolio with a target return equal to the maximum expected return among the assets, which in this case is VFINX with a return of 0.8659%. An efficient portfolio is one that offers the highest expected return for a given level of risk. The characteristics of this efficient portfolio are presented in **Table 11-b**.

10.) Efficient Portfolio Frontier

Efficient Portfolio Frontier - Risky Assets Only

By using the efficient portfolio and global minimum variance portfolio, both of which are efficient portfolios, we can create a graph of the efficient portfolio frontier for the six risky assets. This is because any efficient portfolio can be constructed as a combination of these two portfolios.

The Markowitz bullet, which shows the efficient portfolios, is displayed in **Figure 12**, with the global minimum variance portfolio represented by the orange dot as the portfolio with the lowest variance. The efficient portfolio frontier of risky assets is made up of all portfolios on the upper portion of the Markowitz bullet, including the global minimum variance portfolio. An investor's choice of efficient portfolio depends on their risk and return preferences. Risk-averse investors prefer less volatility and would hold portfolios closer to the minimum variance portfolio. Risk-tolerant investors are willing to accept more volatility in exchange for higher expected returns. It is worth noting that the six individual assets are located in the southeast corner of the plot, with higher volatility and lower expected returns than those on the efficient frontier. Portfolios are more efficient than individual assets as they benefit from risk reduction and diversification.

Efficient Portfolio Frontier - Risky Assets + Risk-Free Assets

The highest Sharpe's Ratio is represented by the tangency portfolio, indicated by a red dot on the graph. The green tangent line shows the efficient frontier for portfolios consisting of combinations of T-bills and the tangency

portfolio. This aligns with the mutual fund separation theorem, which suggests that the efficient portfolios for risk-free and risky assets are a blend of T-bills and the tangency portfolio. Investors of all risk preferences would hold risky assets in proportions determined by the tangency portfolio. Risk-averse investors would prefer portfolios with more T-bills and lower risk levels, while risk-tolerant investors would invest more in the tangency portfolio to achieve higher expected returns, accepting more risk.

-End of Report-

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