دانشگاه صنعتی امیرکبیر
(پلی تکنیک تهران)

Electrical and Electronic Circuits

chapter 13. Operational Amplifier

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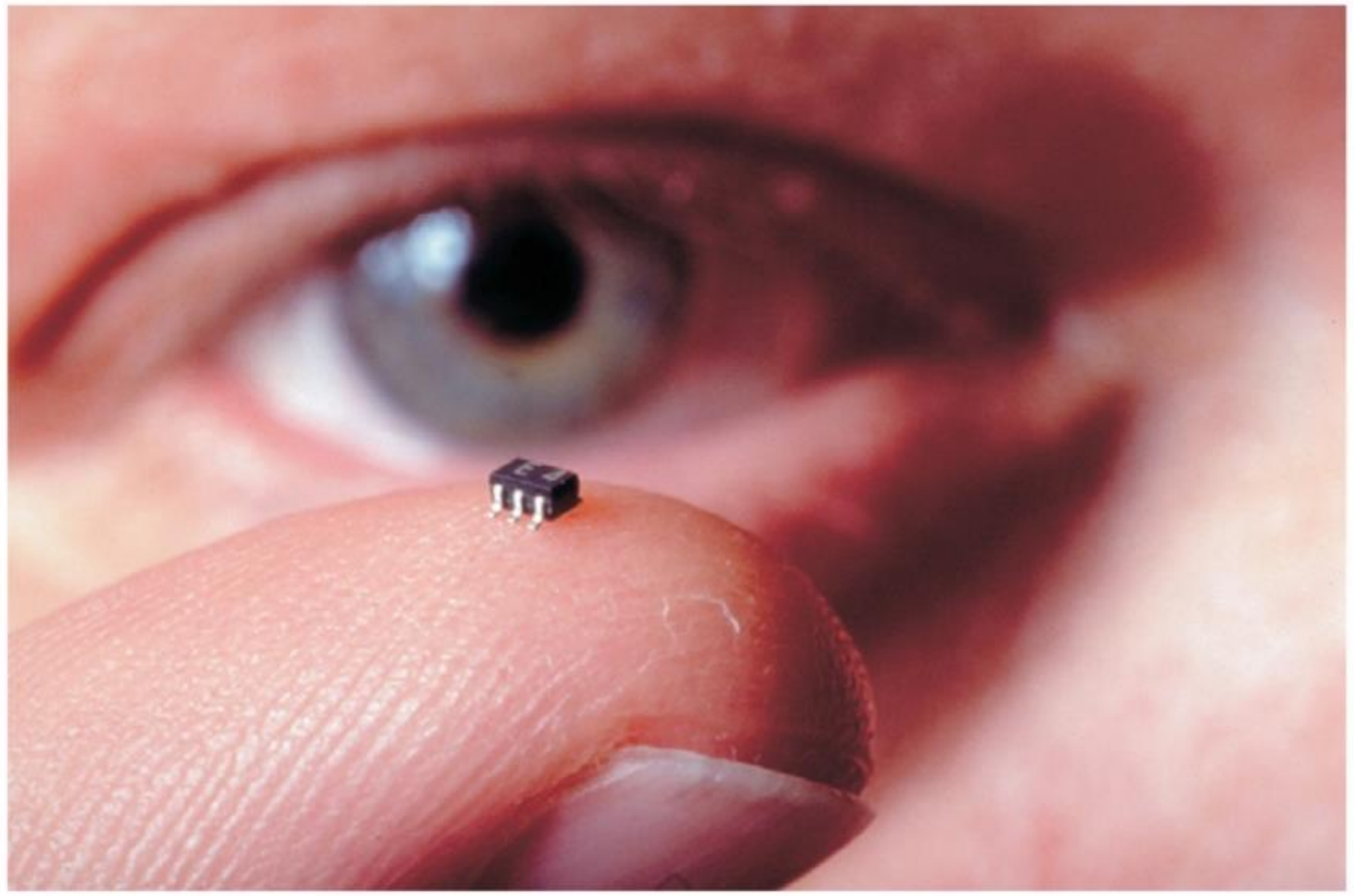
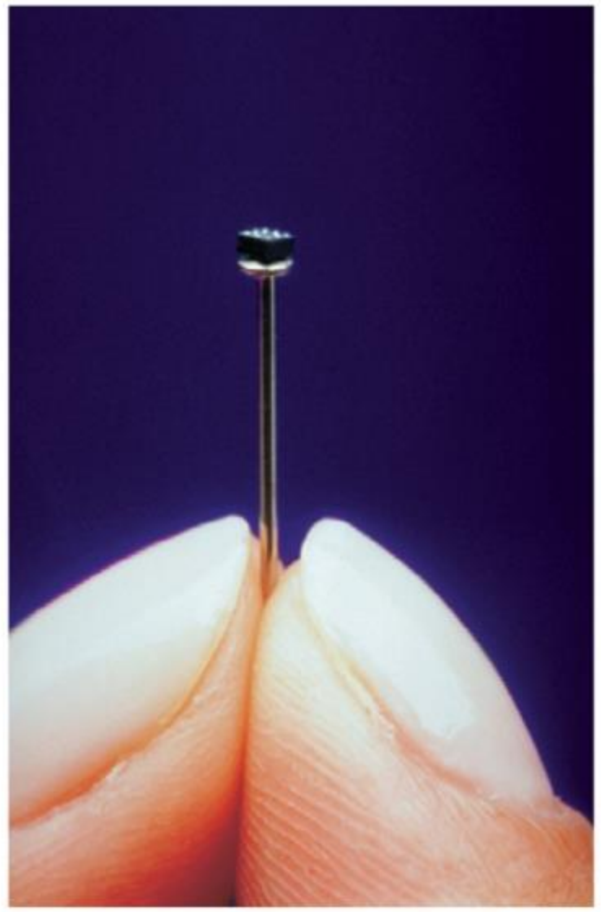
مهر ۱۴۰۳

Objectives of the Lecture

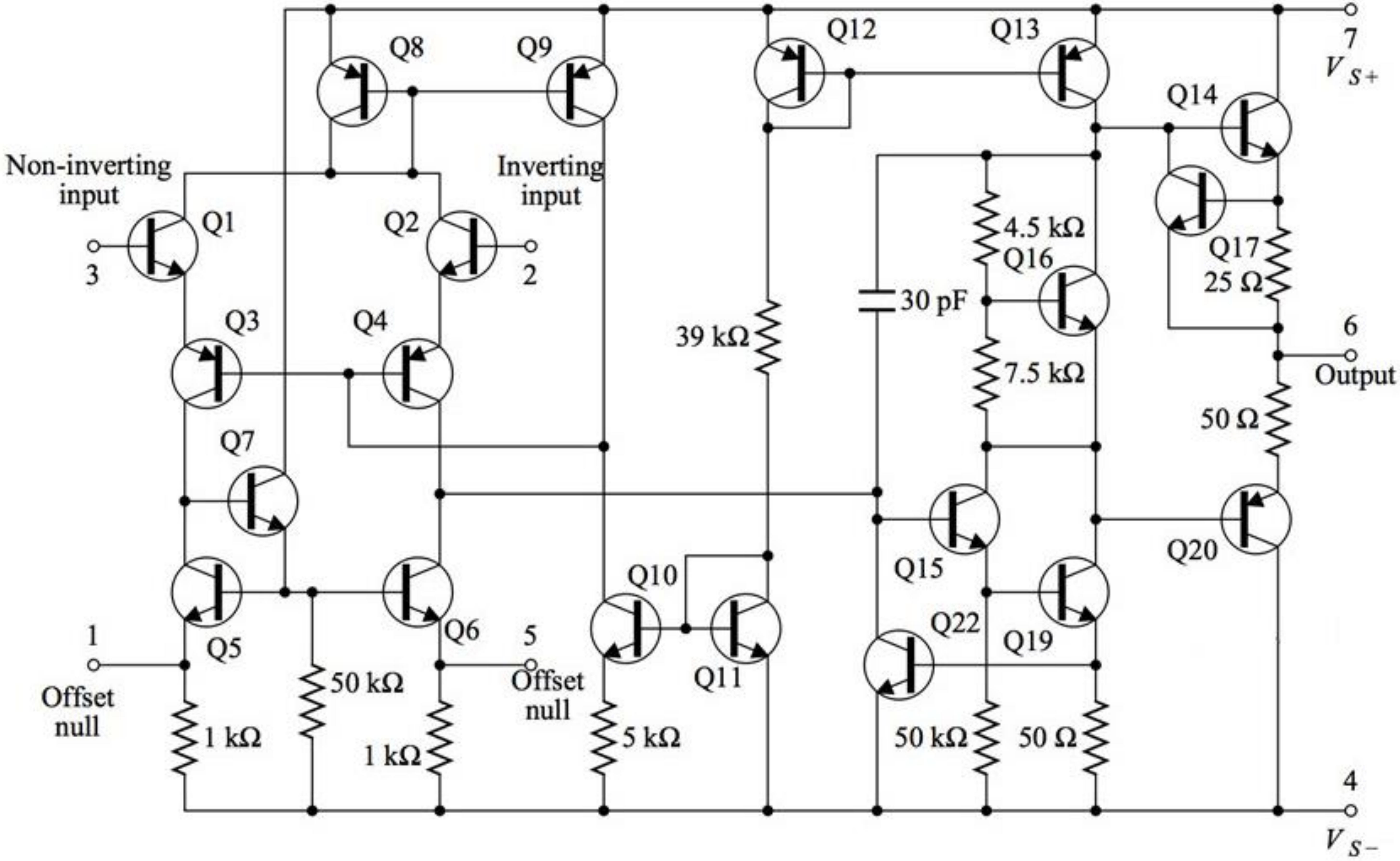
- Introduction to Operational Amplifiers
  - Precise Model of Operational Amplifiers
  - Ideal Model
  - Applications of Operational Amplifiers
    - Inverting Amplifier
    - Non-Inverting Amplifier
    - Voltage Follower (Buffer)
    - Multi-Stage Amplifier
    - Ideal Voltage and Current Sources
- Comparator Circuit
- Several Examples

- The **Operational Amplifier (Op-Amp)** is an integrated circuit (IC) widely utilized as an amplifier in various applications.
- Its origins date back to the 1940s when it was employed in analog computational circuits for constructing adders, sub tractors, and analog multipliers.





The Internal Circuit of a Typical Operational Amplifier



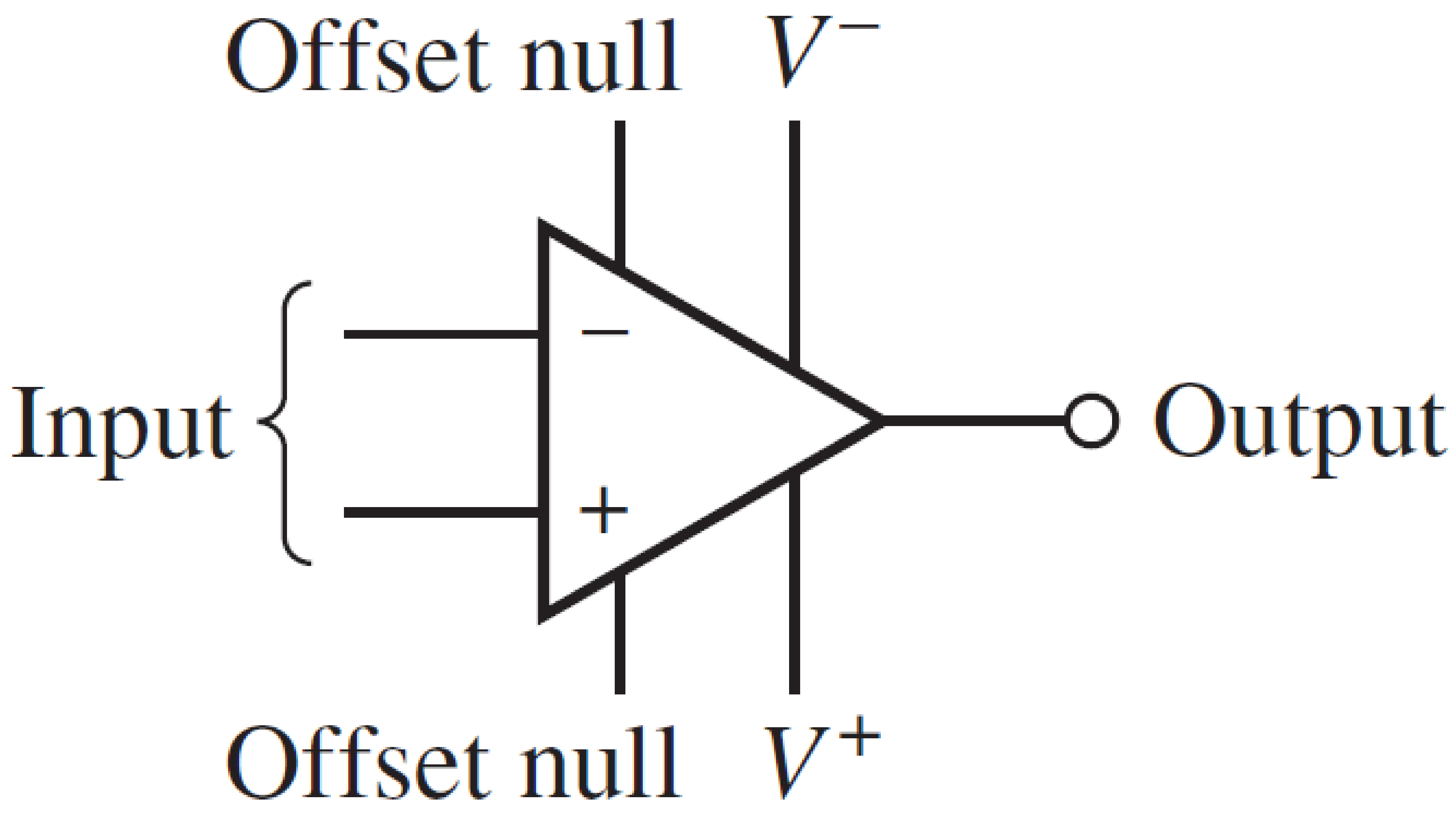
- An operational amplifier (Op-Amp) is generally represented by the following symbol and includes the following pins:

- **Input Pins**: $Input$

- **Output Pin**: $Output$

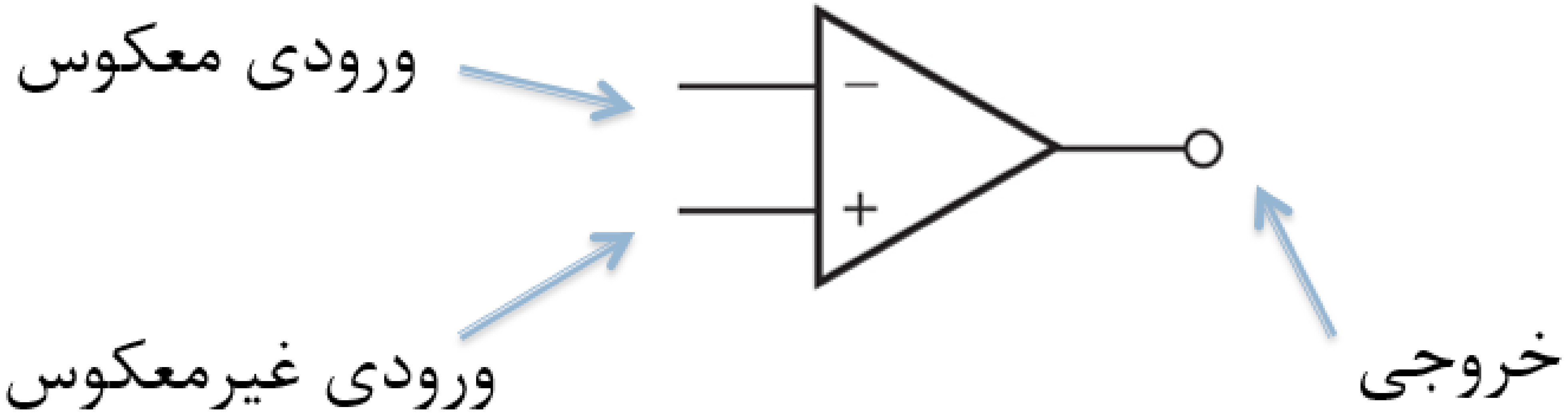
- **Power Supply Pins**: V^+, V^-

- **Offset Adjustment Pins**: $Offset$



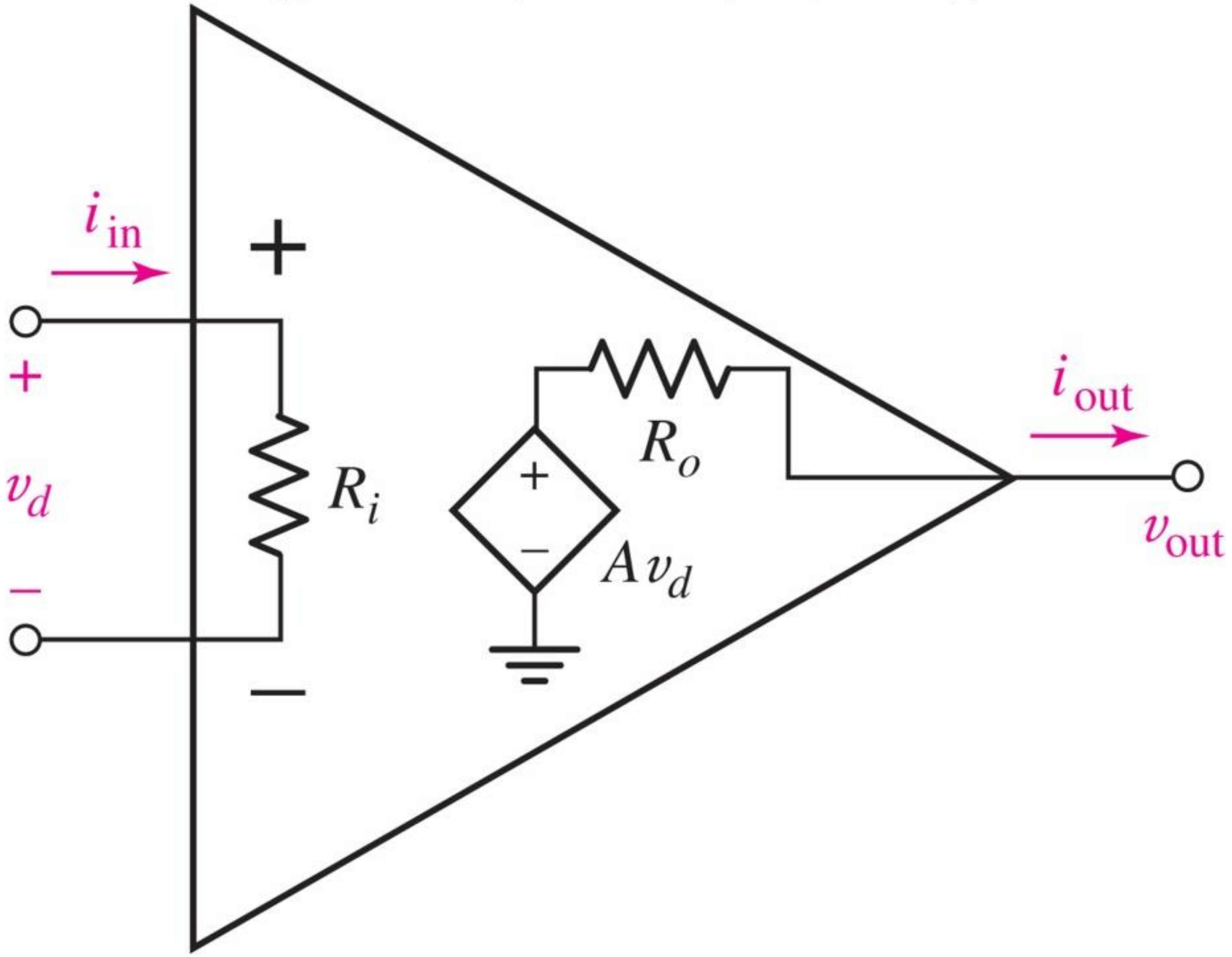
Op-Amp Symbol

- ✓ Let us focus on the **input** and **output pins**, assuming the other pins are connected to appropriate voltage levels.
- ✓ An operational amplifier functions as a **differential amplifier**, meaning it amplifies the difference between its input signals.



Precise Model of an Op-Amp

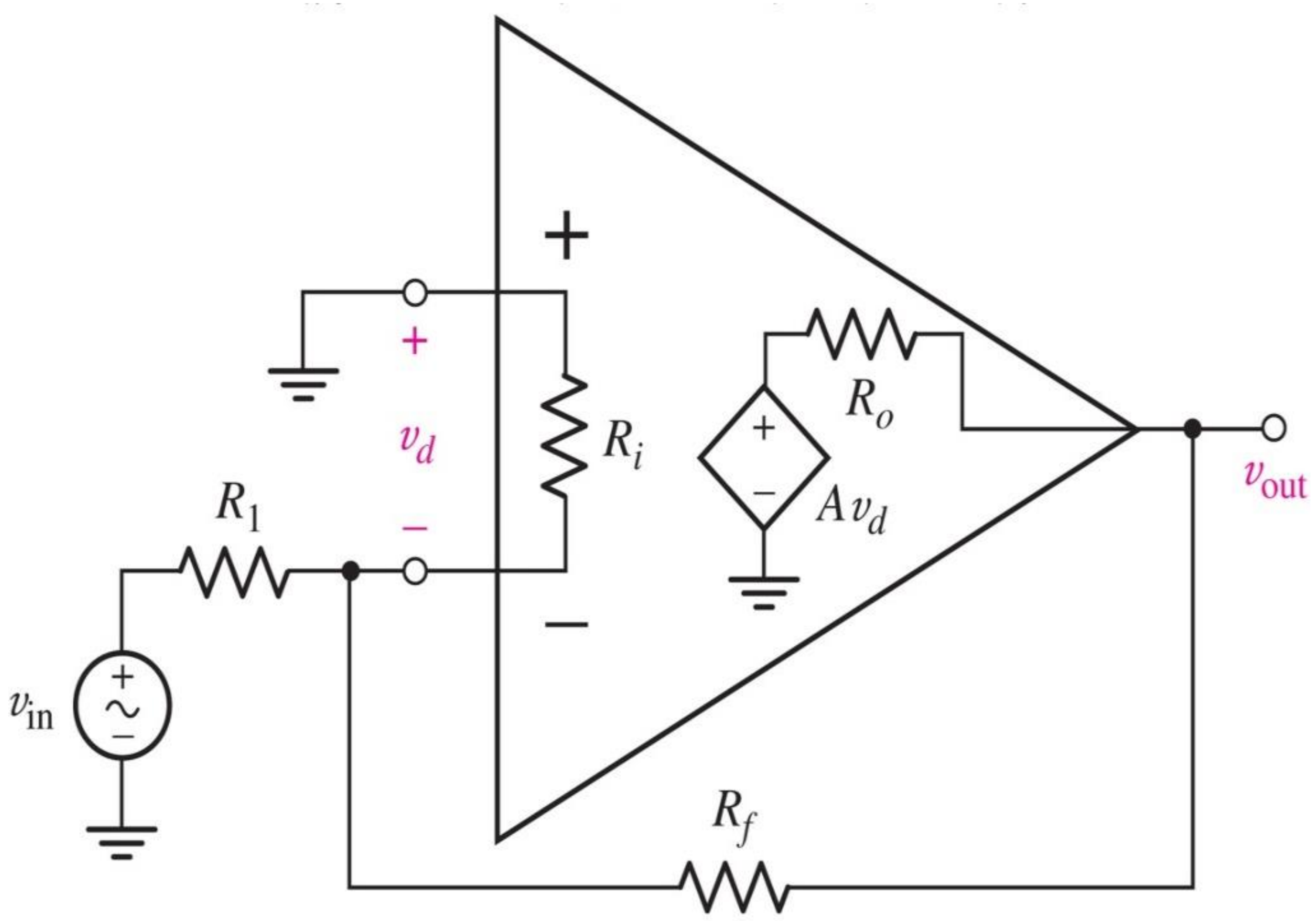
- A circuit that operates as a voltage amplifier can be modelled using the following equivalent circuit.
- This model includes:
  - **Input Resistance (R_i)**
  - **Output Resistance (R_o)**
  - **Open-Loop Gain (A)**



- ✓ In the circuit below, what is the gain of the amplifier?

- ➢ $KCL_1: \frac{-v_d - v_{in}}{R_1} + \frac{-v_d - v_{out}}{R_f} + \frac{-v_d}{R_i} = 0$

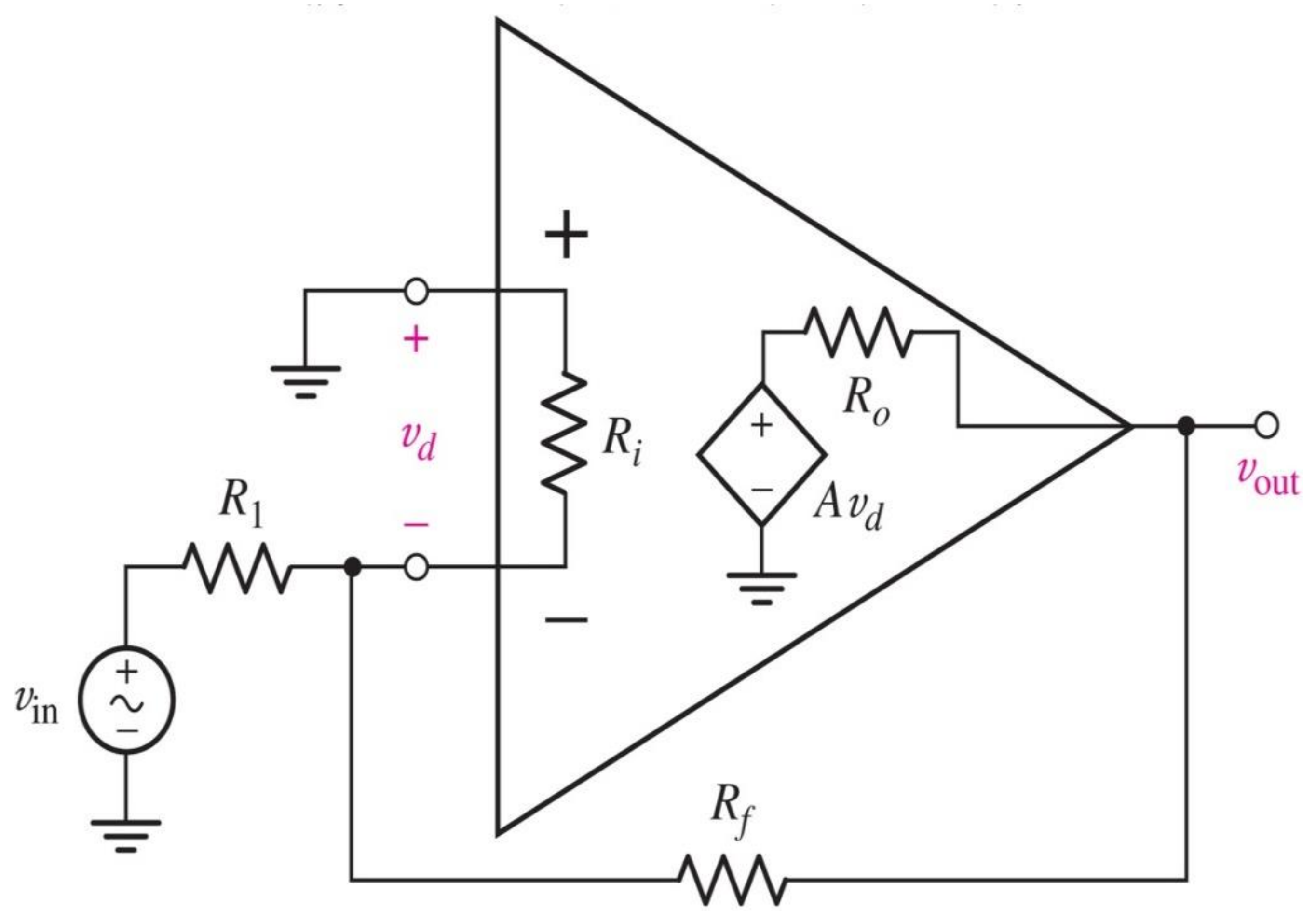
- ➢ $KCL_2: \frac{v_{out} + v_d}{R_f} + \frac{v_{out} - Av_d}{R_o} = 0$



✓ By solving the system of equations and eliminating v_d, we obtain:

- $A_v = \frac{v_{out}}{v_i} = \left[\frac{R_o+R_f}{R_o-AR_f}\left(1+\frac{R_1}{R_f}+\frac{R_1}{R_i}\right)-\frac{R_1}{R_f}\right]^{-1}$

- Example Solution for the Inverting Amplifier Using LM741 Specifications
- $A = 200000$
- $R_i = 2M\Omega$
- $R_o = 75\Omega$
- If $R_f = 47$ and $R_1 = 4.7$
- Then: $A_v = -9.999$



Advantages of Using an Op-Amp as a Voltage Amplifier

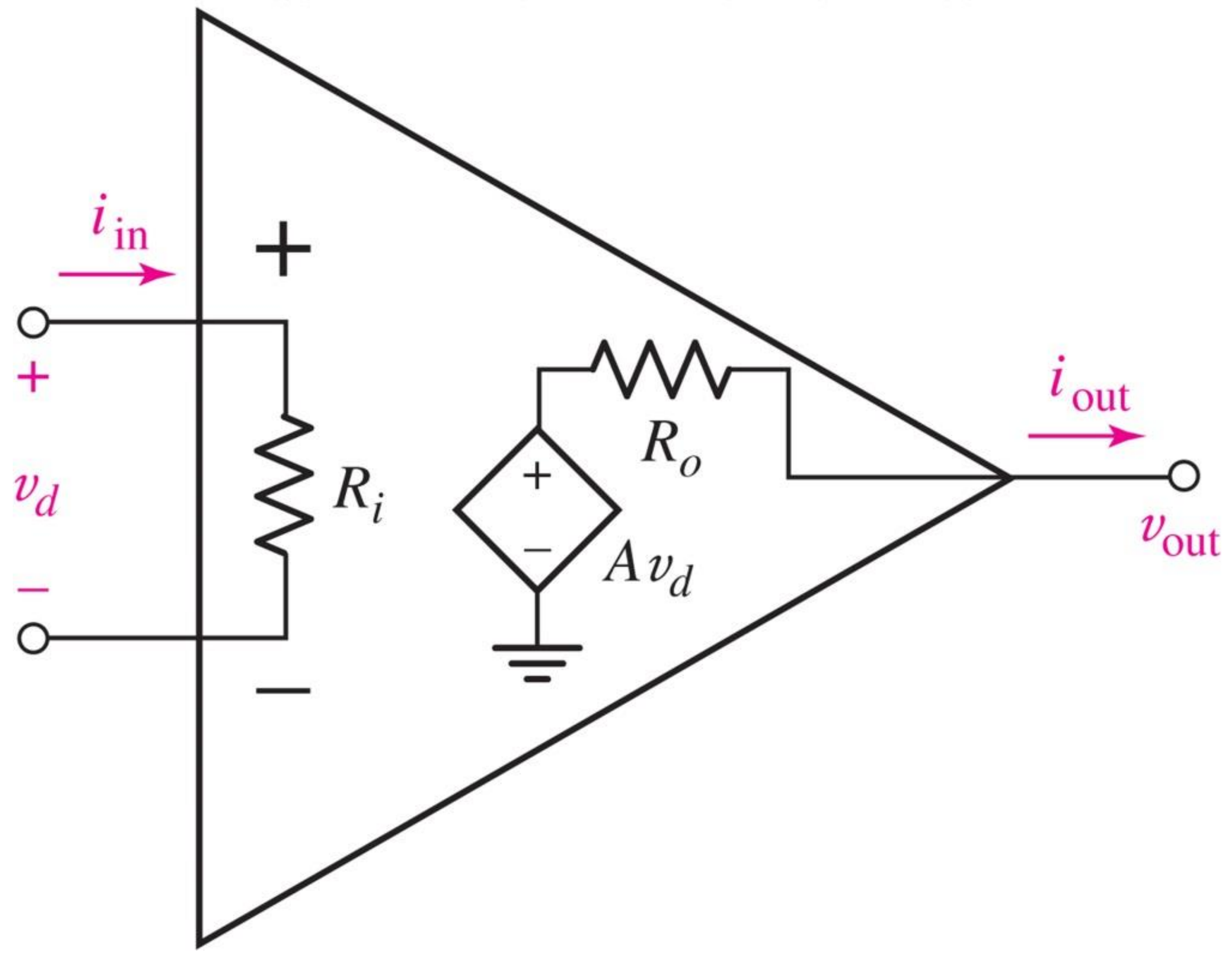
- ✓ **High Input Resistance**:(in the range of mega-ohms to tera-ohms)
  - ➢ To ensure that the maximum voltage of the source is applied across its input.
- ✓ **Low Output Resistance**:(in the range of a few ohms to a few tens of ohms)
  - ➢ to ensure that the entire output voltage of the amplifier is delivered to the load.
- ✓ **Adjustable Gain**:(achieved by applying feedback resistance):
  - ➢ As observed in the circuit shown, provided that A is sufficiently large, the gain of the amplifier can be approximated with high accuracy as $Av \approx -\frac{R_f}{R_1}$.

Ideal Model

- In the ideal case, when $A = \infty$, $R_i = \infty$, and $R_o = 0$, the behaviour of the operational amplifier can be described as follows:

- Since v_{out} has a finite value (it cannot exceed the supply voltage), $v_d = \frac{v_{out}}{A} \approx 0$.

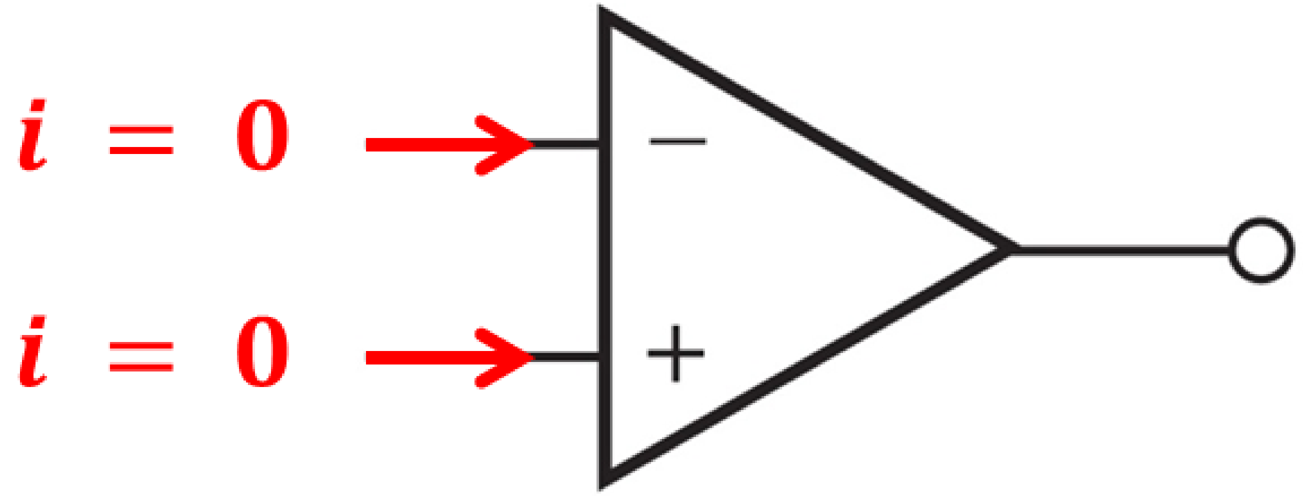
- As a result, $i_{in} \approx 0$.



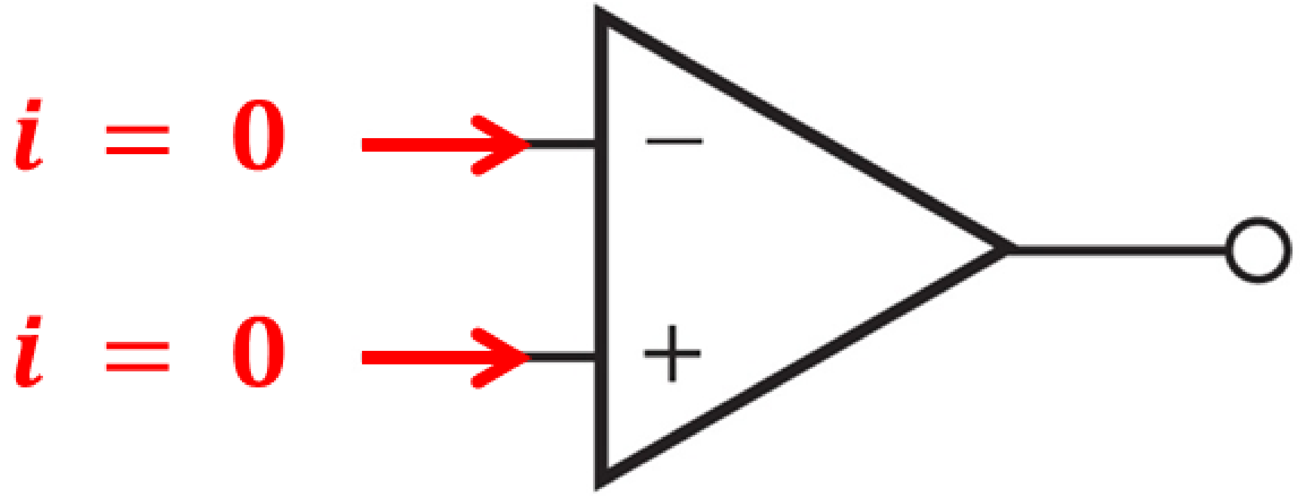
Ideal Model

Rules of the Ideal Model:

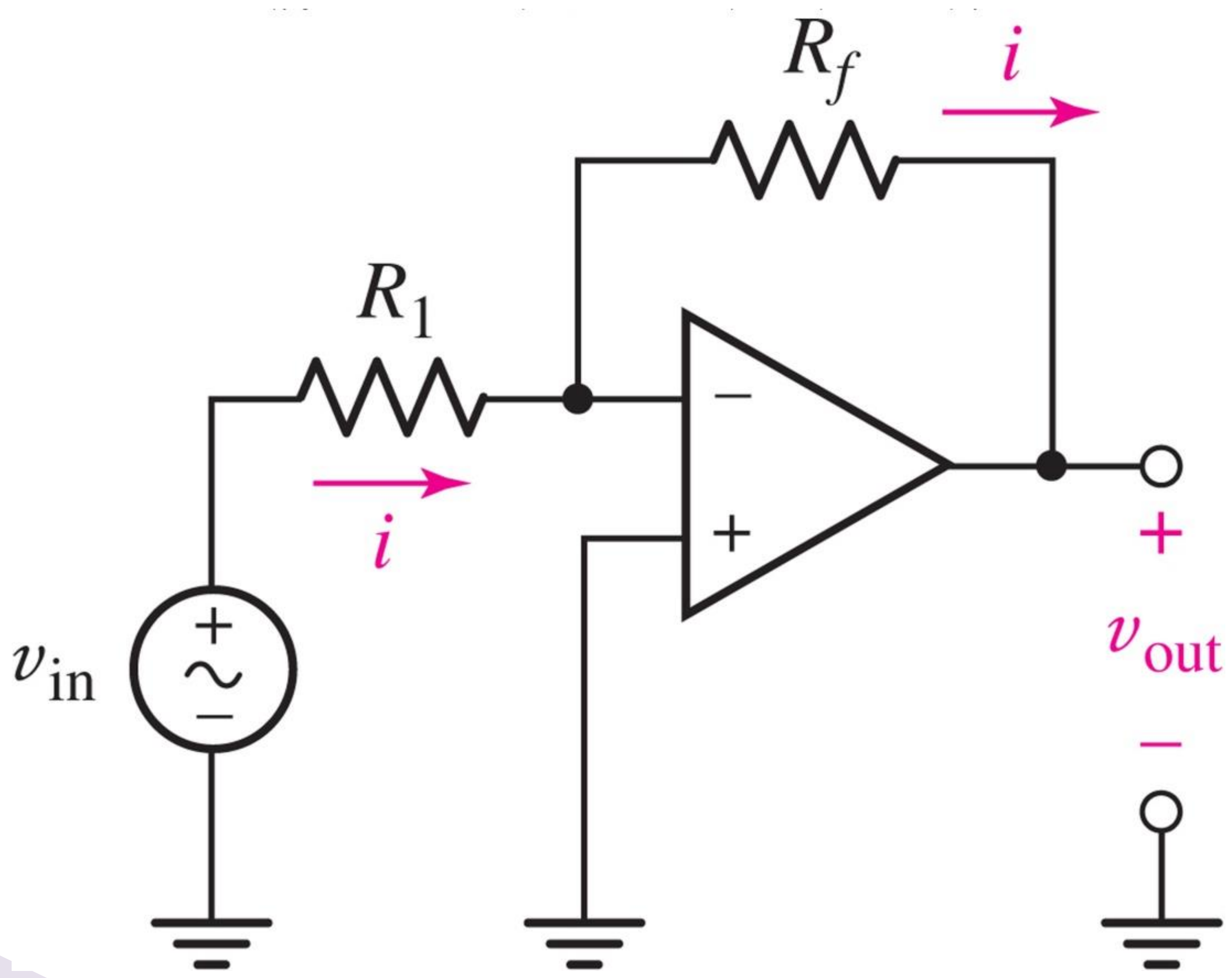
1.No current flows through the input terminals.



2.The voltage difference between the two input terminals is zero.

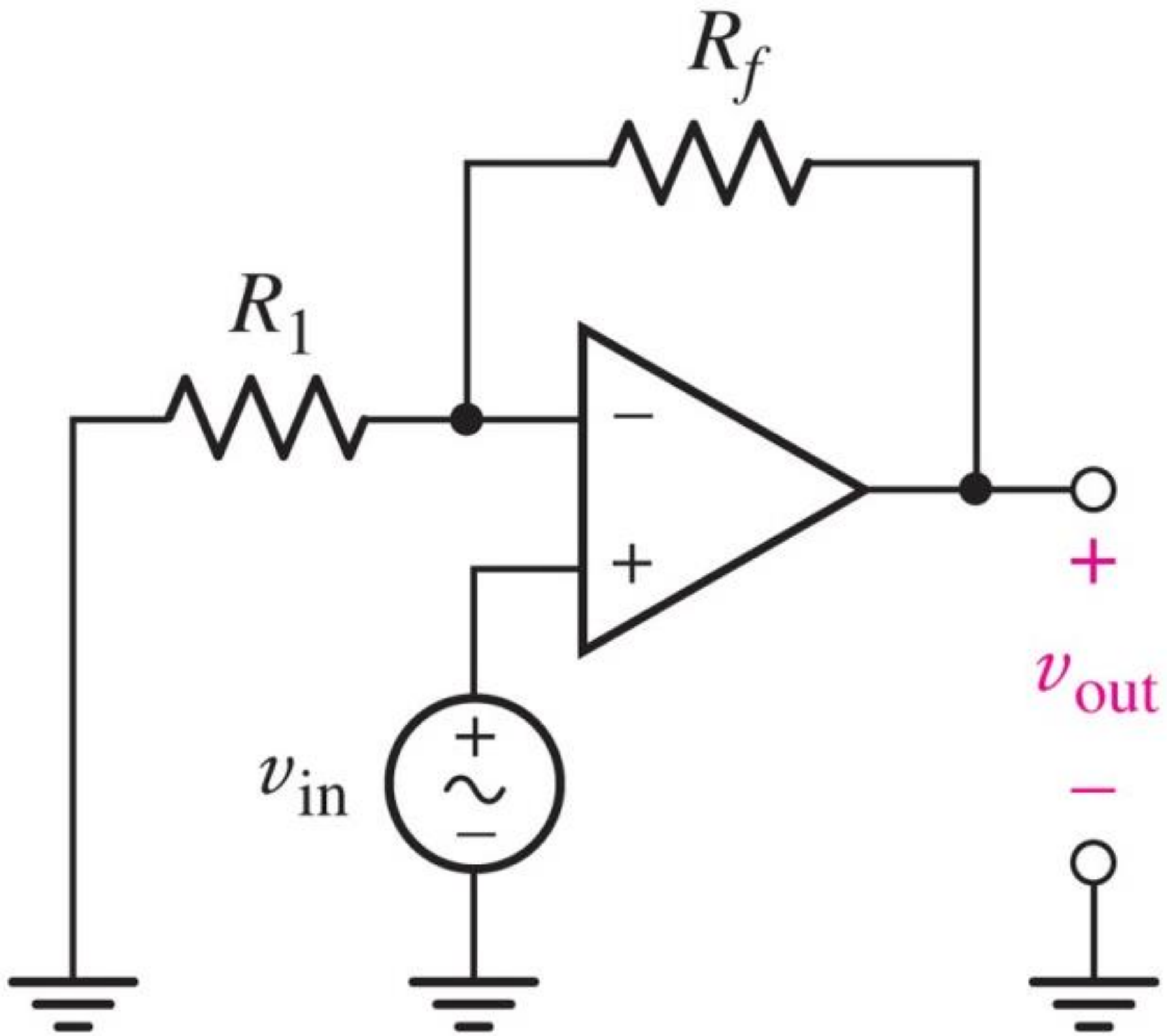


✓ By applying KVL and using the rules of the ideal operational amplifier, we have:



$$\boldsymbol{v_{out} = -\frac{R_f}{R_1} v_{in}}$$

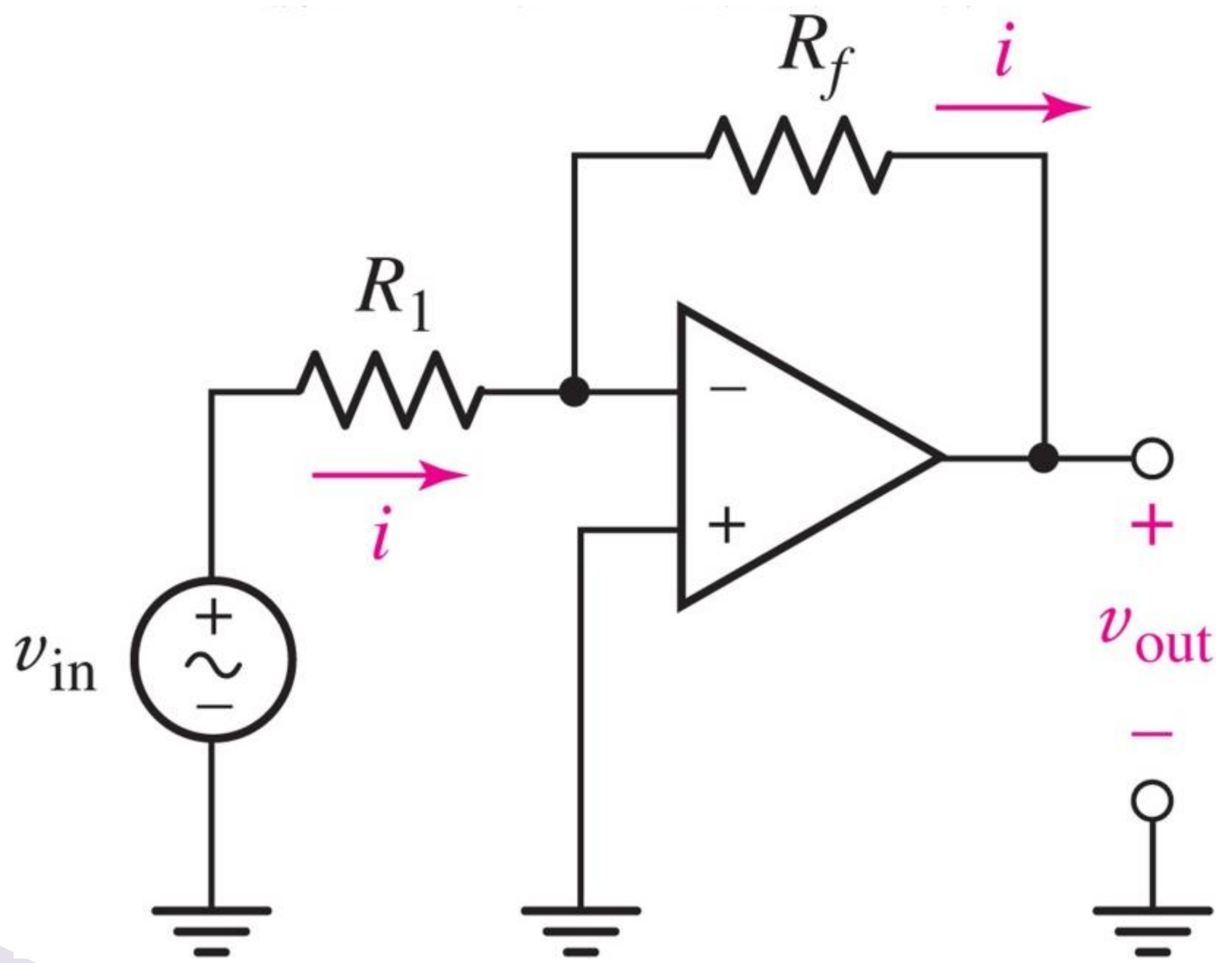
Non-Inverting Amplifier

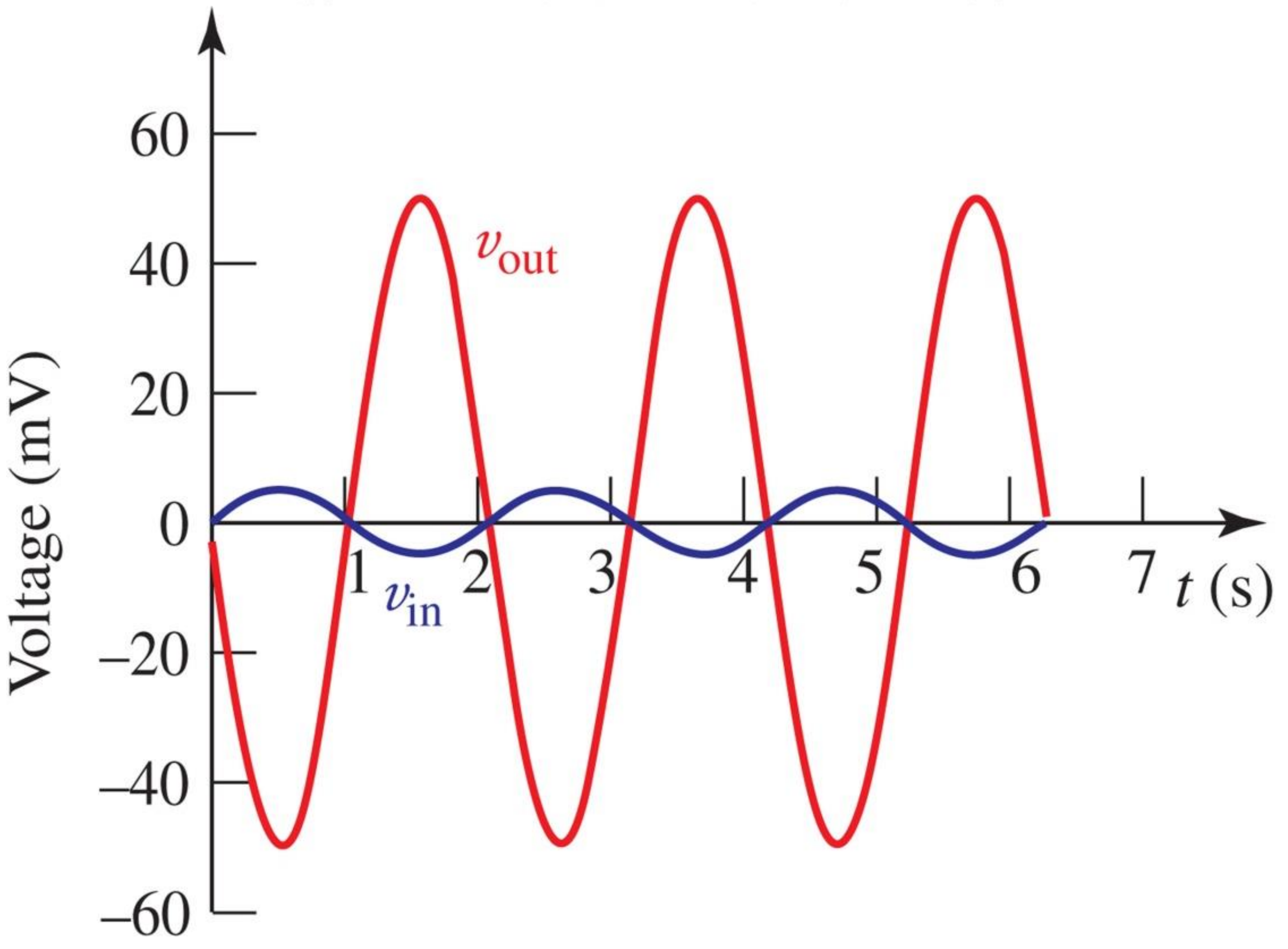


$$v_{out} = \left(1 + \frac{R_f}{R_1}\right) v_{in}$$

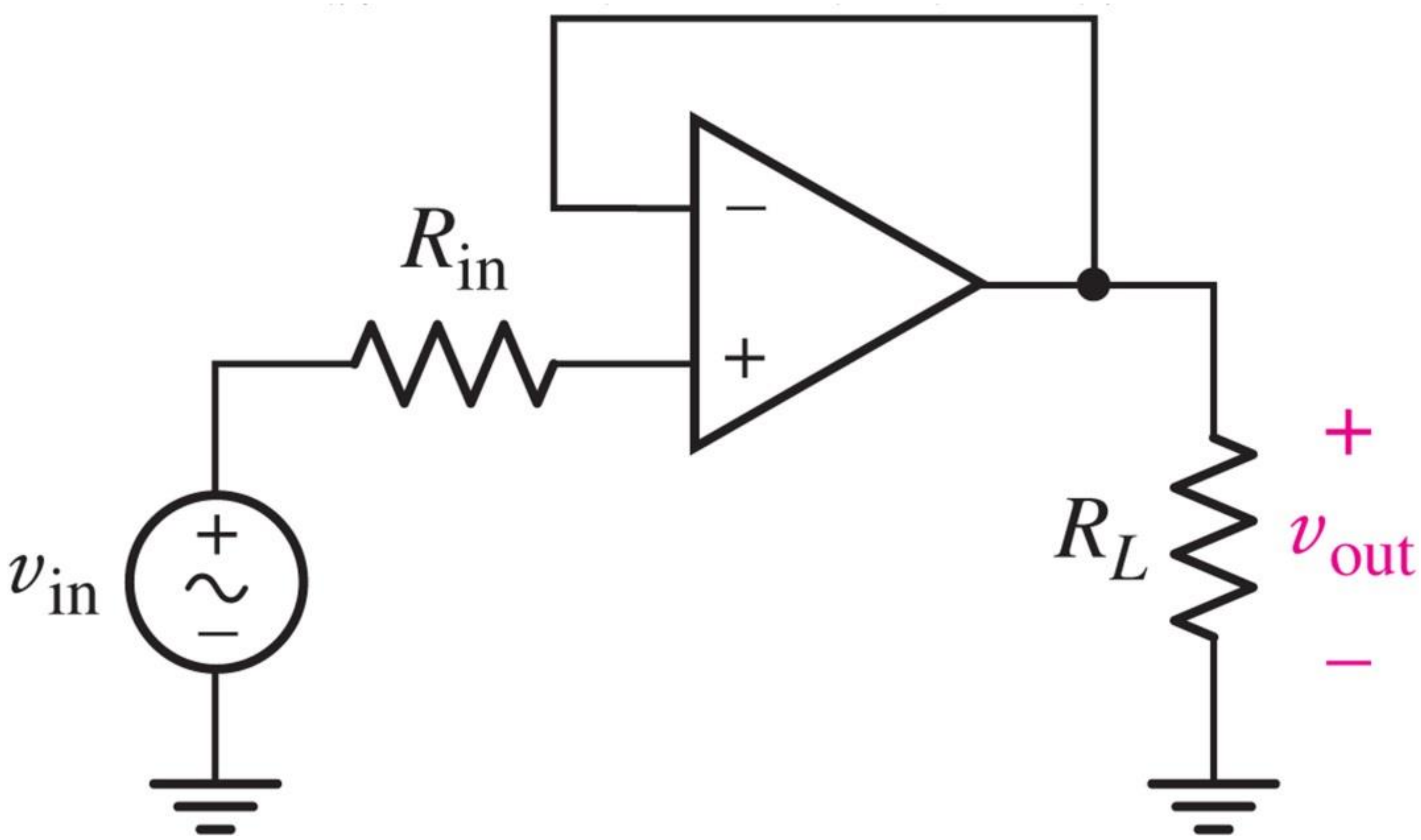
- $v_{in}(t) = 5\sin 3t\ mV, R_f = 47K\Omega, R_1 = 4.7\text{K}\Omega$

$$\boldsymbol{v_{out}(t) = -50\ sin\ 3t\ mV}$$





Voltage Follower (Buffer)

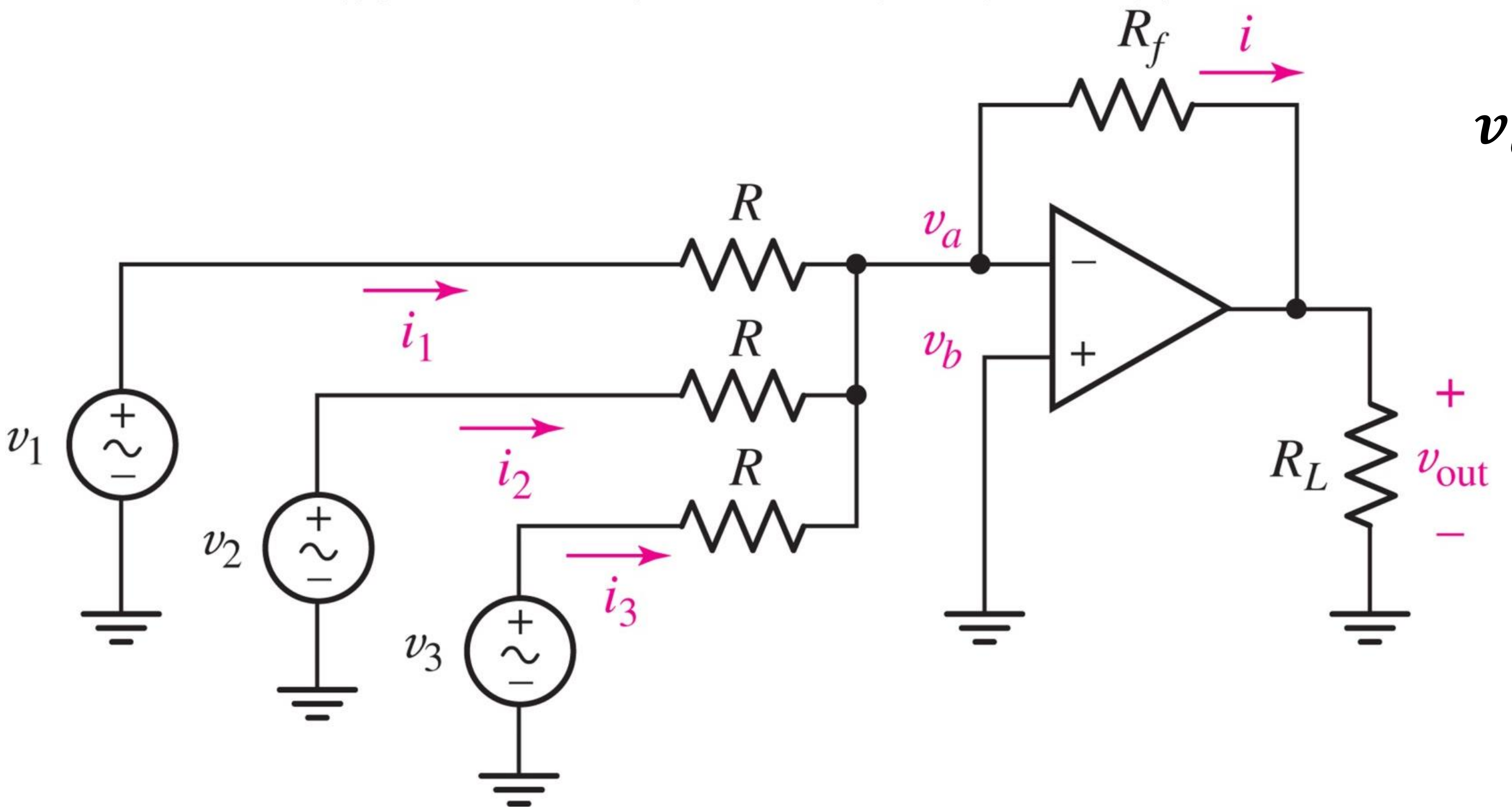


$$v_{out}(t) = vin(t)$$

1.The output voltage of the buffer is independent of the load resistance (R_L). The buffer can supply the required current to maintain a constant output voltage, even if R_L changes.

2.This circuit eliminates the effect of the source's input resistance by presenting a very high input resistance to the source and a very low output resistance to the load.

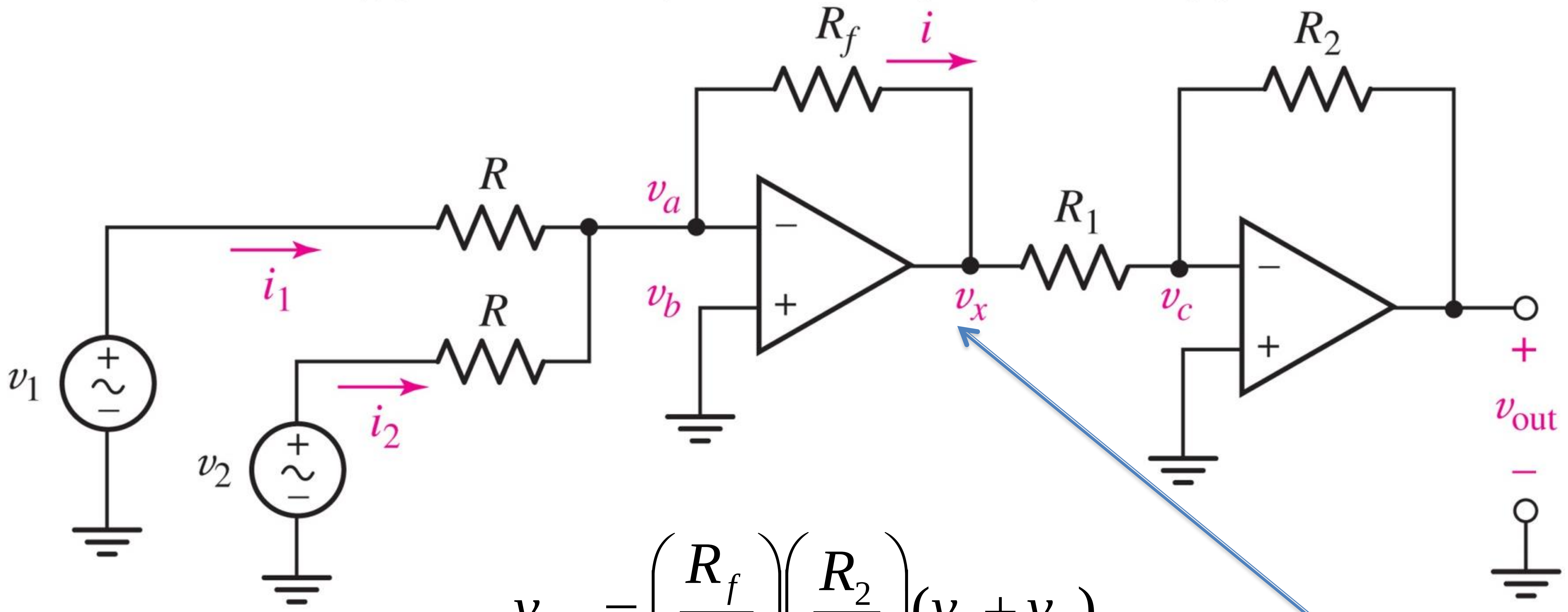
Analog Summing Circuit



$$v_{out} = -\frac{R_f}{R_1}(v_1 + v_2 + v_3)$$

✓ This circuit performs the addition of input signals and amplifies the result by a factor of $-\frac{R_f}{R_1}$.

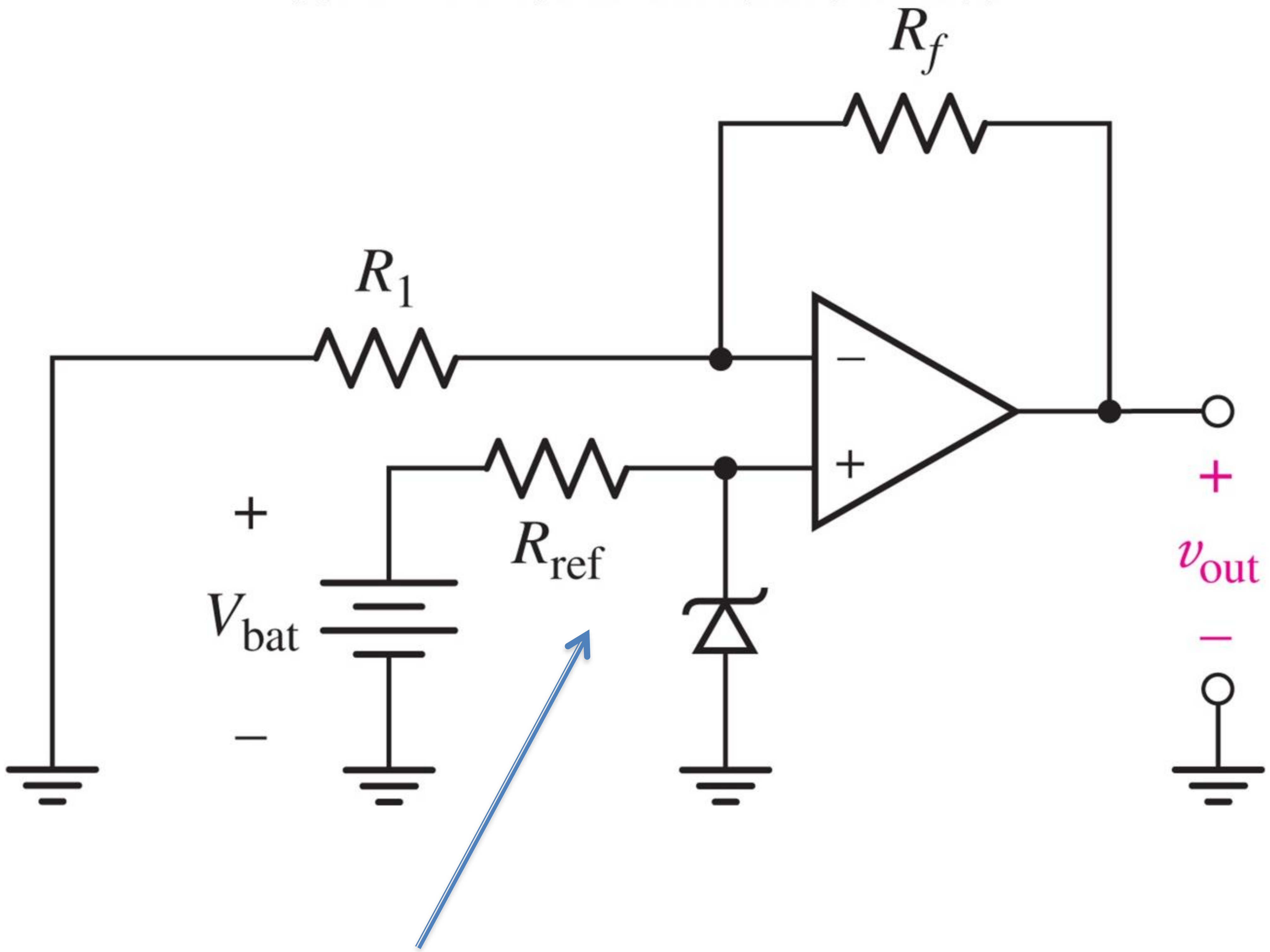
Cascading Multiple Operational Amplifiers



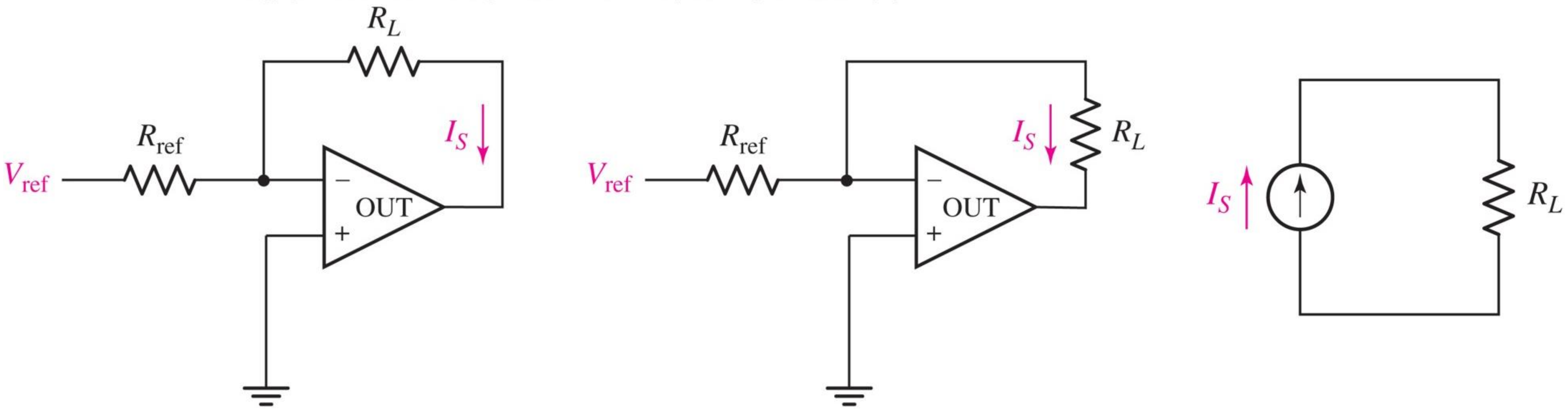
$$v_{out} = \left(\frac{R_f}{R}\right)\left(\frac{R_2}{R_1}\right)(v_1 + v_2)$$

- ✓ The output voltage of each stage is independent of the subsequent stage. **(Why?)**
- ✓ As a result, **Op-Amp** can be cascaded without any alteration in their individual gains.

$$v_{out} = \left(1 + \frac{R_f}{R_1}\right) V_Z$$



This circuit resembles the voltage regulator using a Zener diode, which we have encountered before.



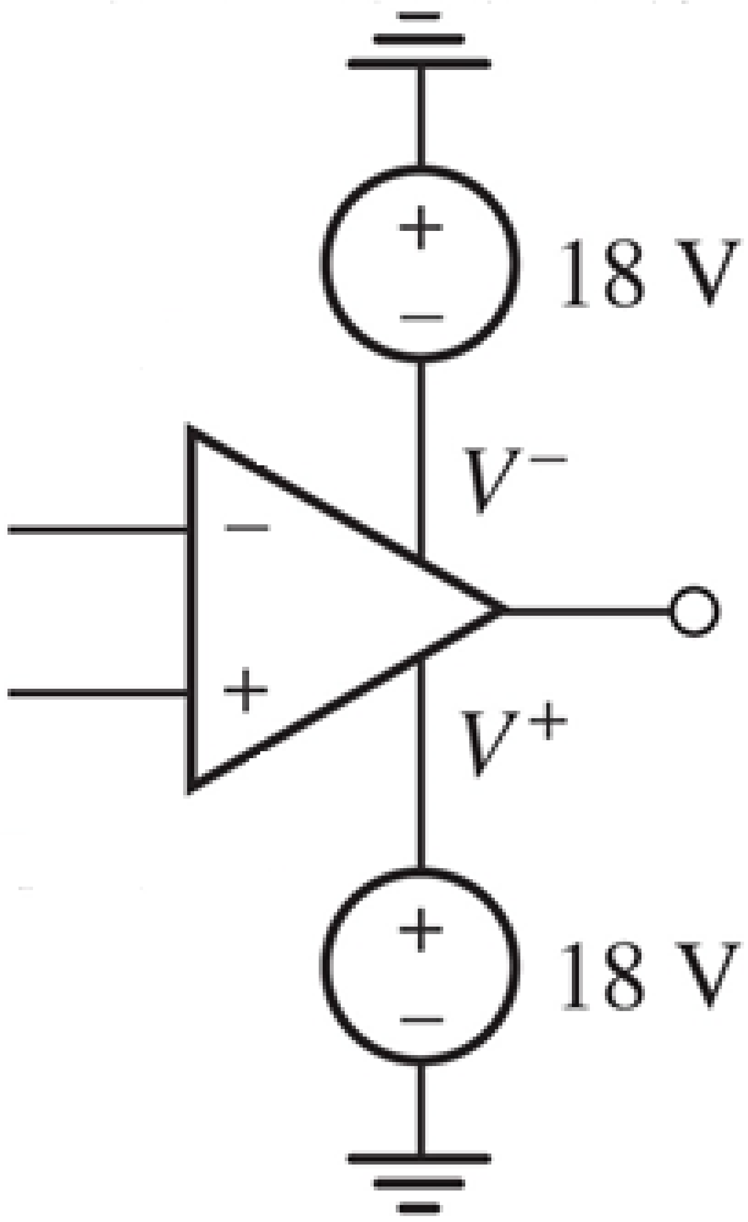
- Using a reference voltage V_{ref} and a resistor V_{ref}, an ideal current source can be constructed with a current:

$$I_S = \frac{V_{ref}}{R_{ref}}$$

- This current is independent of the load resistance R_L,

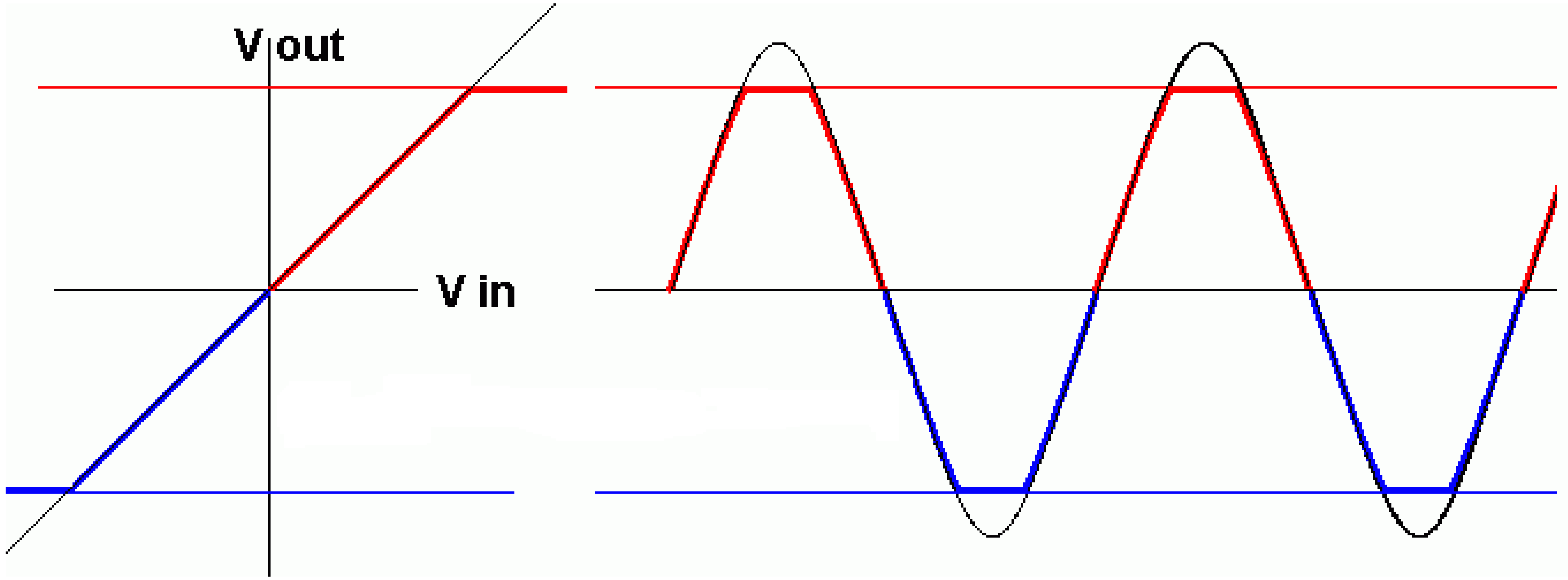
Power Supplies for Op-Amp

- An operational amplifier requires power to function and perform amplification. Therefore, it must be connected to a power supply.
- Typically, equal and opposite voltage values are applied to the V^+ and V^- terminals. These voltage levels usually range between ±5V and ±24V.
- The ground of the power supplies must be connected to the input and output ground reference of the circuit to ensure proper operation and signal integrity.



Output Saturation

- ✓ If the input signal is such that the amplified output exceeds V^+ or falls below V^-, the output will saturate.
- ✓ In this state, the output voltage remains clamped at V^+ or V^-, depending on the direction of the input signal.



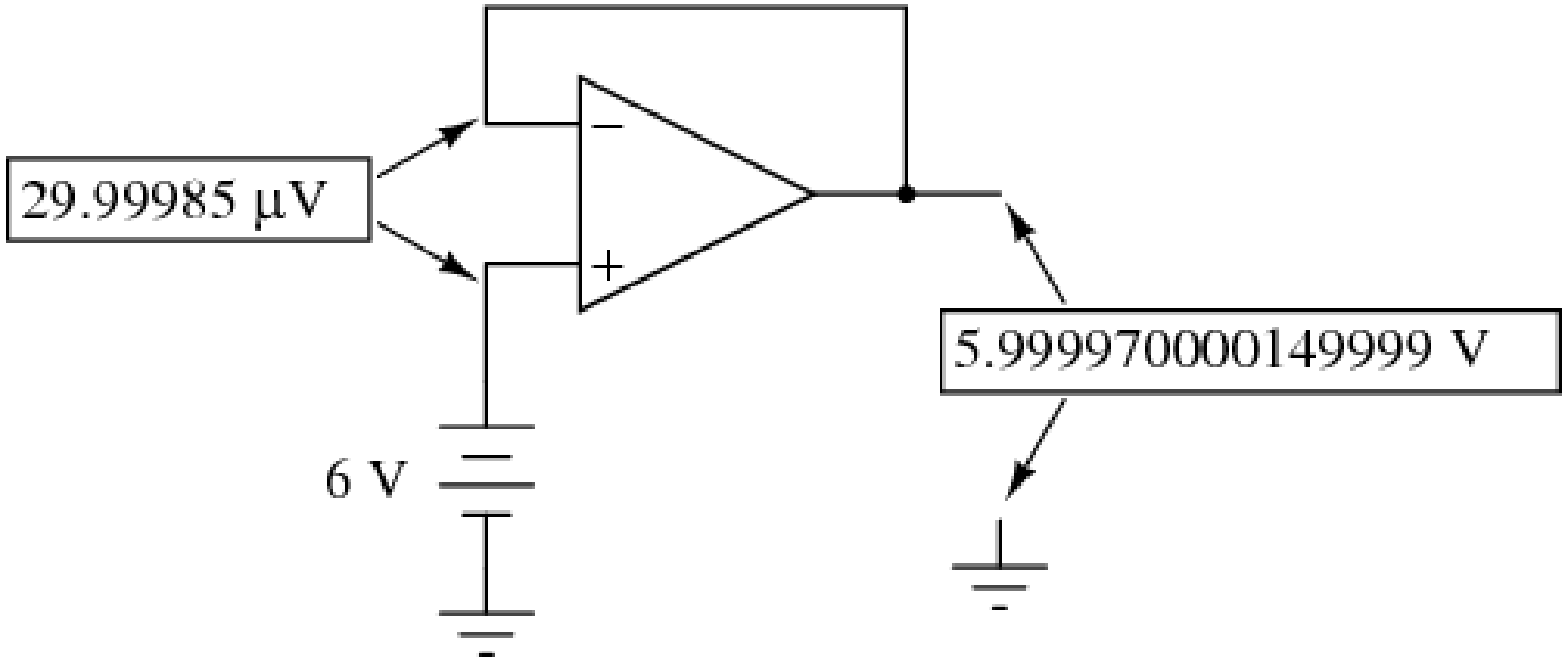
The output signal enters saturation and becomes clipped

Role of Negative Feedback

- The role of negative feedback is to transform a large and undefined gain into a fixed, controlled gain.
- Why Don't We Apply Feedback to the Positive Terminal?

$$v_+ \uparrow \;\rightarrow\; v_d \uparrow \;\xrightarrow{v_{out} = Av_d}\; v_{out} \uparrow \;\rightarrow\; v_- \uparrow \;\rightarrow\; v_d \downarrow \;\ldots$$

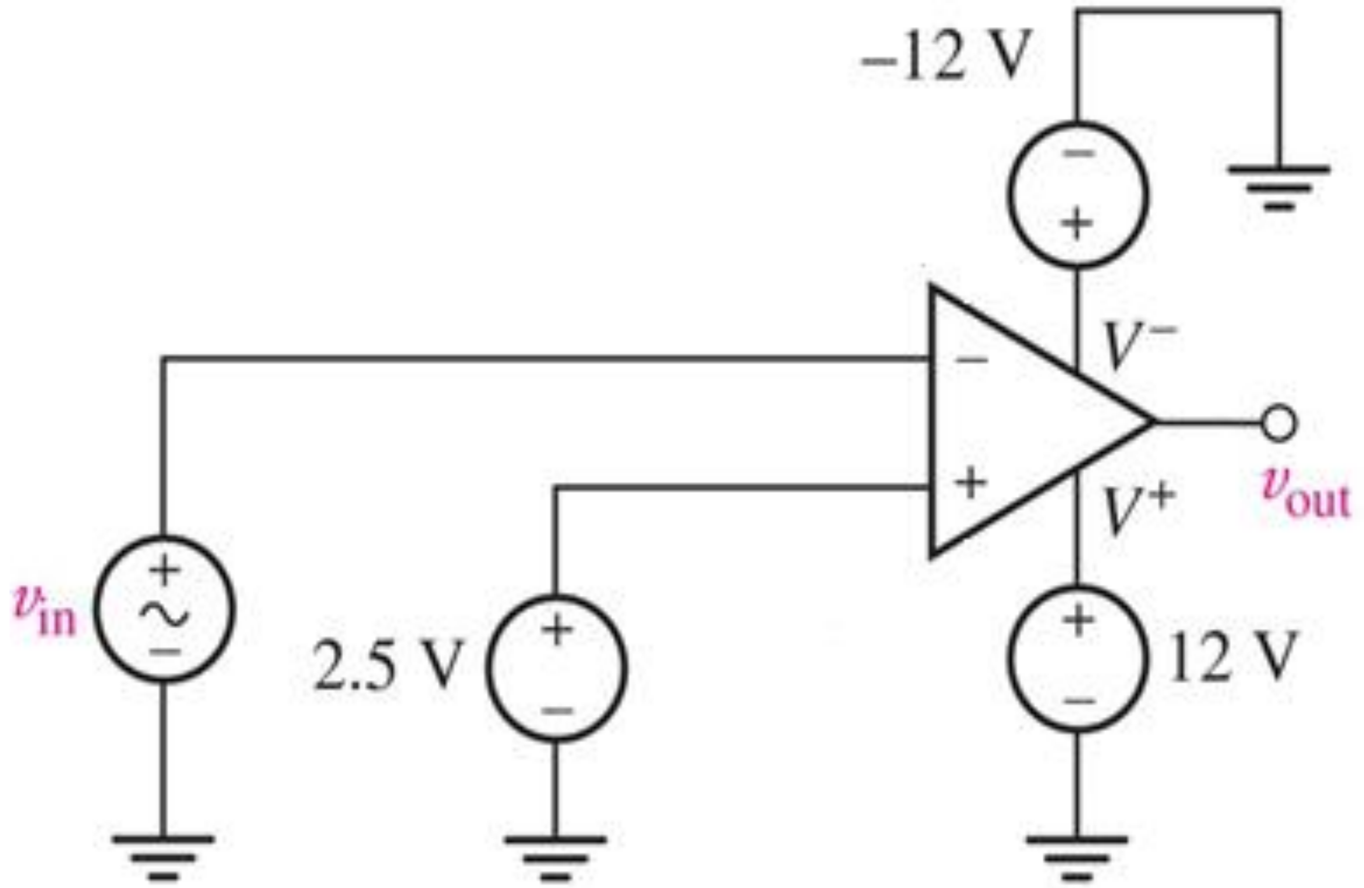
- Thus, the negative feedback stabilizes the operation by keeping v_d close to zero

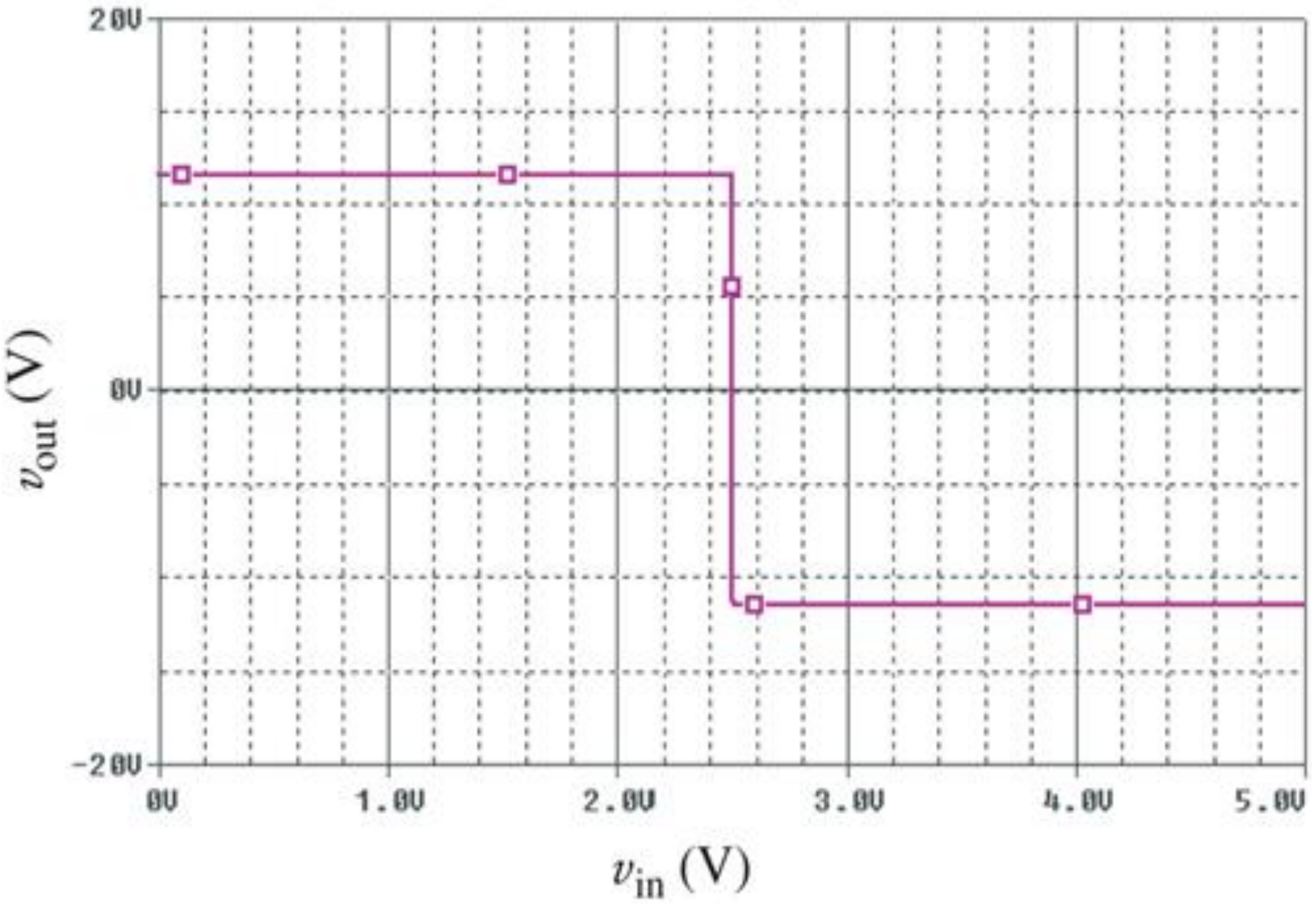


Voltage Comparator

- An operational amplifier can be used as a voltage comparator when operated in open-loop configuration (without feedback).

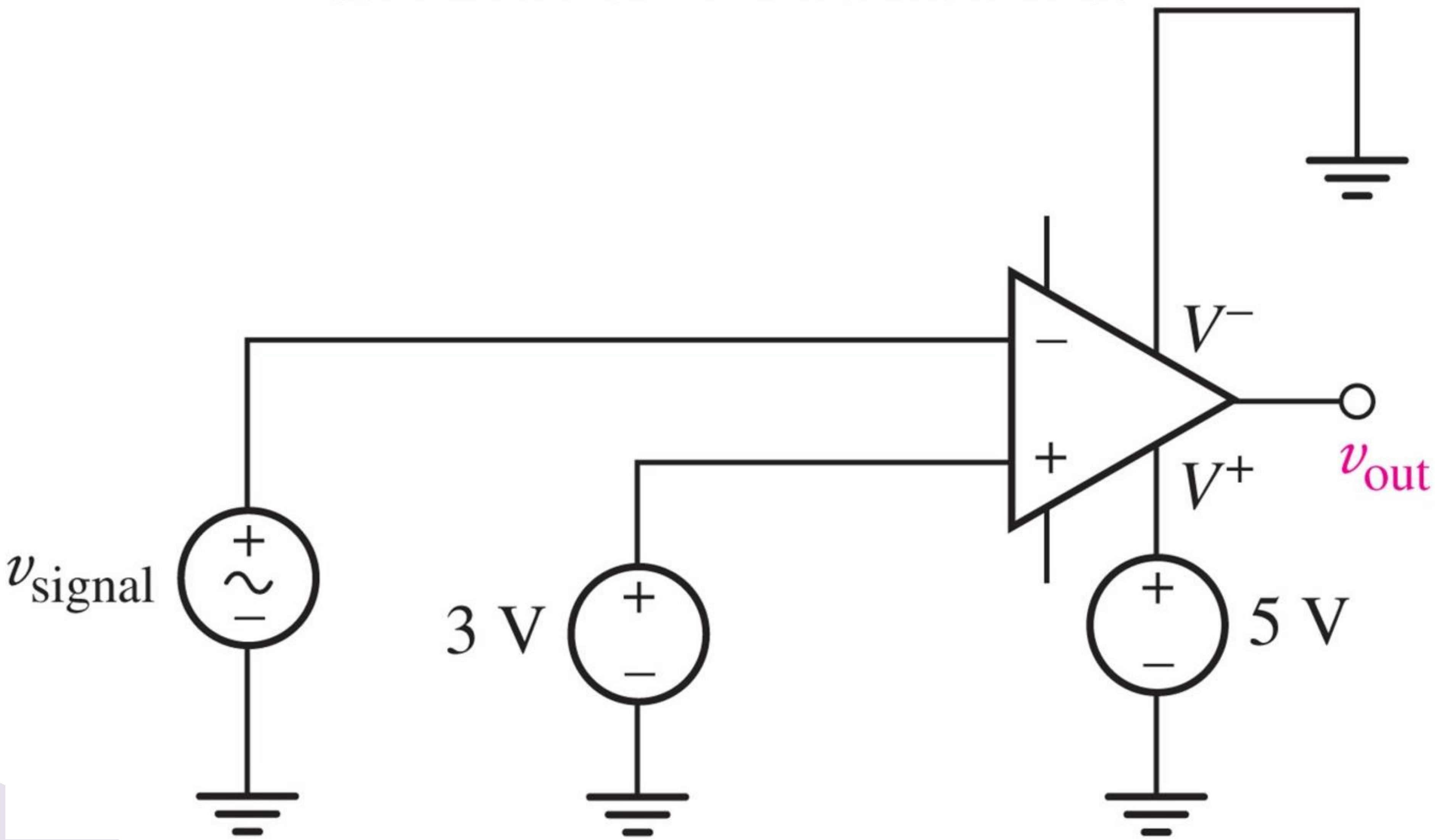
$$v_{out} = \begin{cases} 12 & v_{in} < 2.5 \\ -12 & v_{in} > 2.5 \end{cases}$$

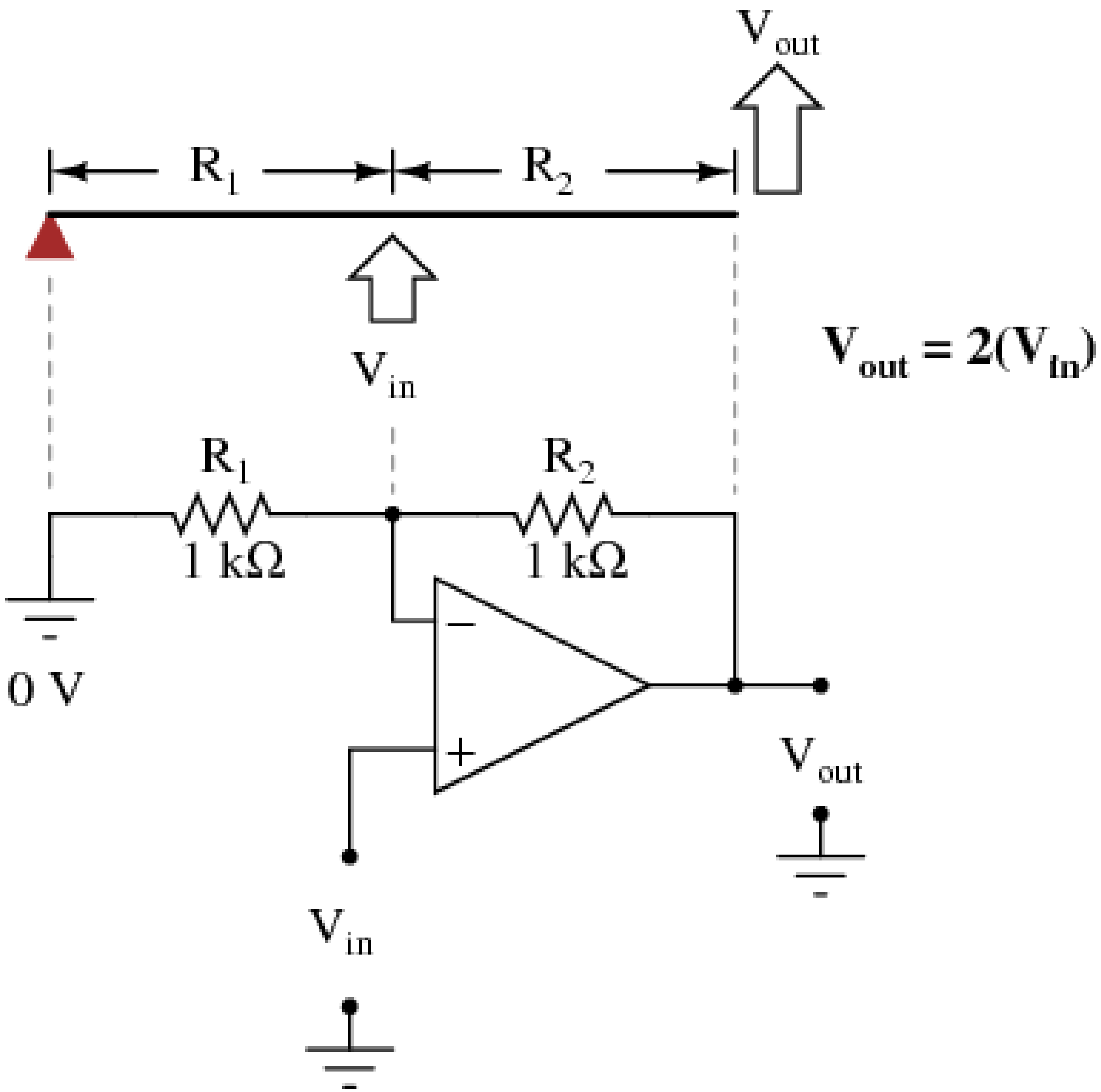


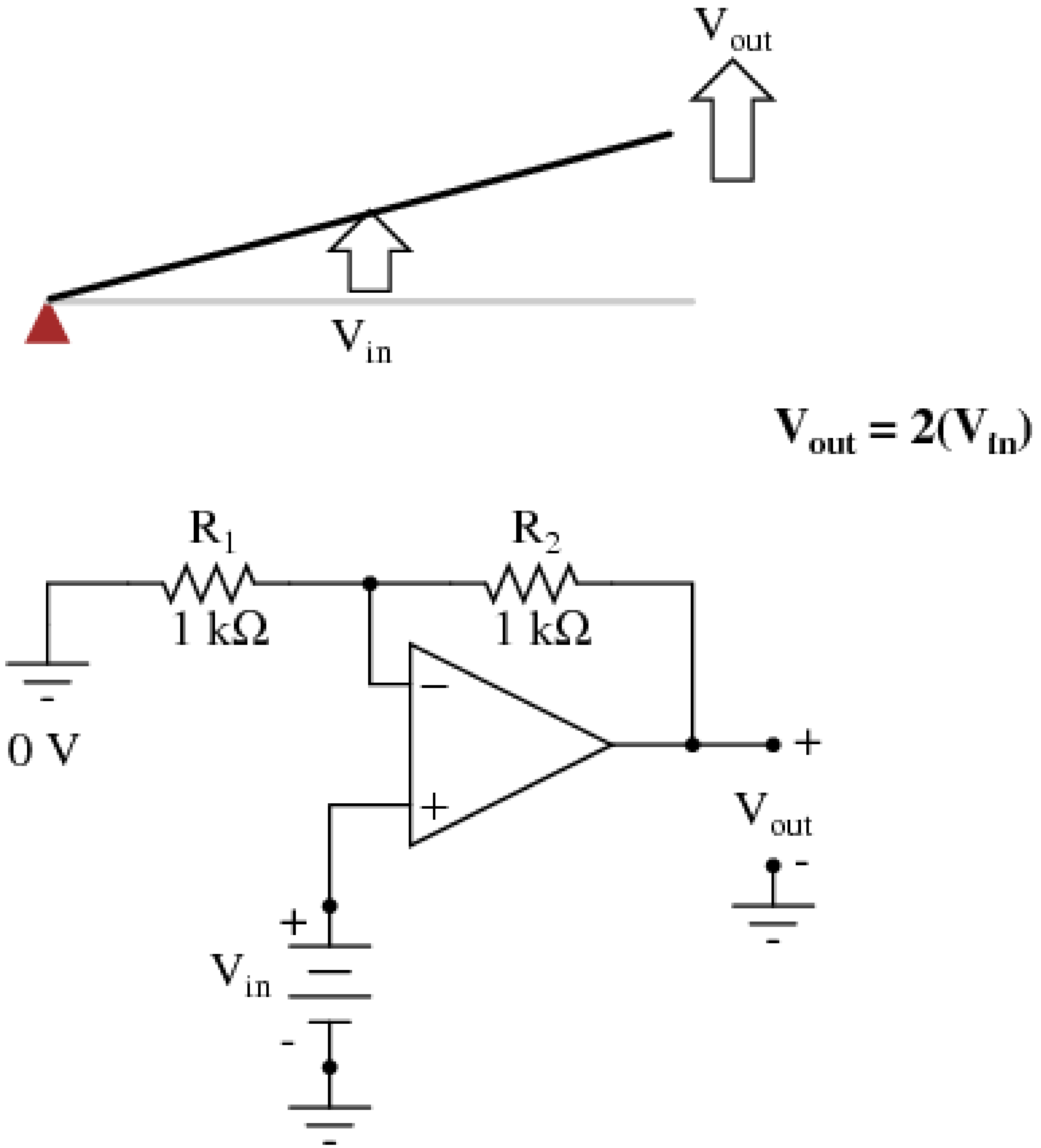


Example

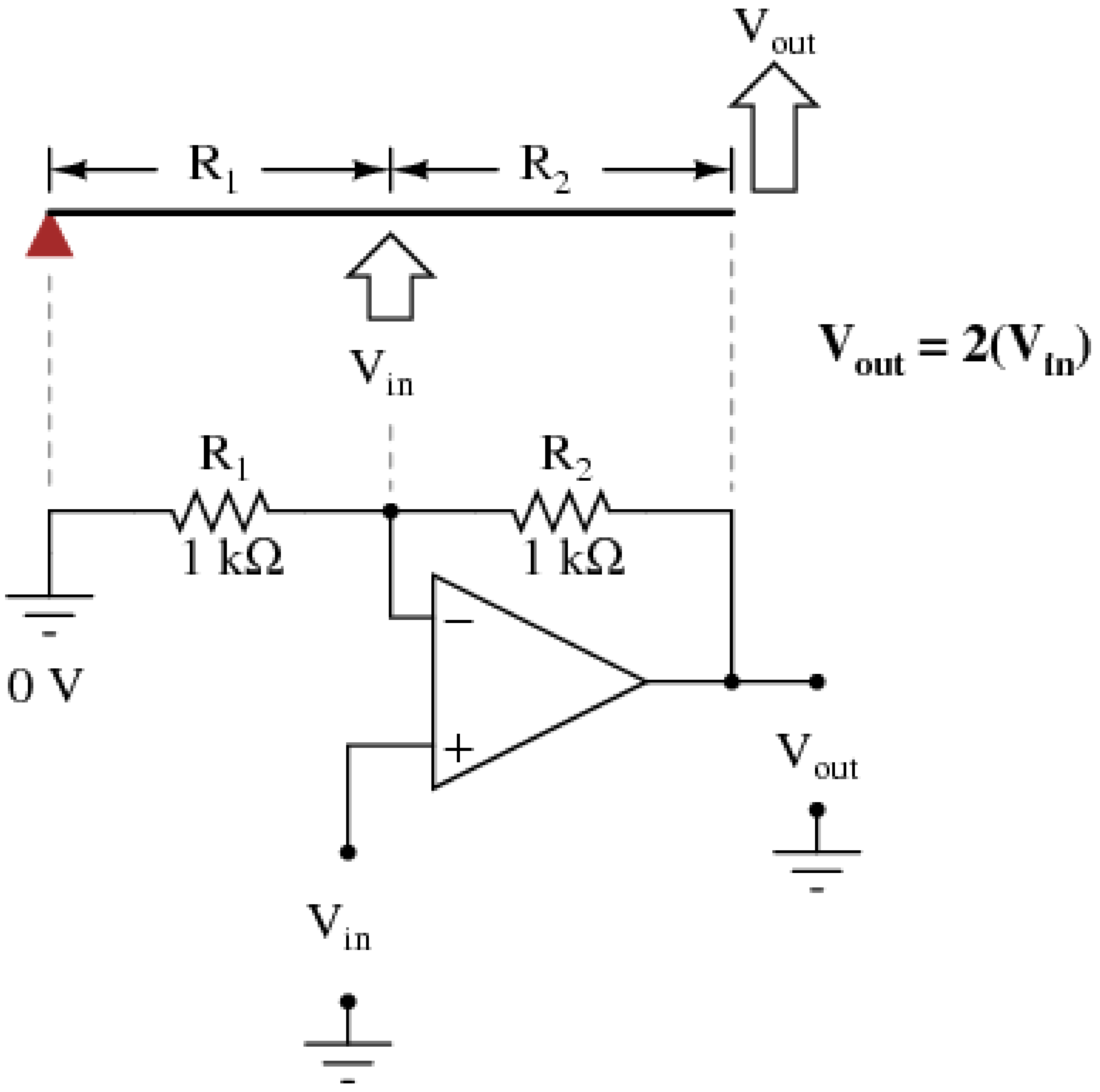
Assume we have a temperature sensor that generates a voltage ranging from 0 to 5 volts for temperatures between 0 and 100 degrees Celsius. Design a circuit that provides a logical output of 1 when the temperature is below 60 degrees.

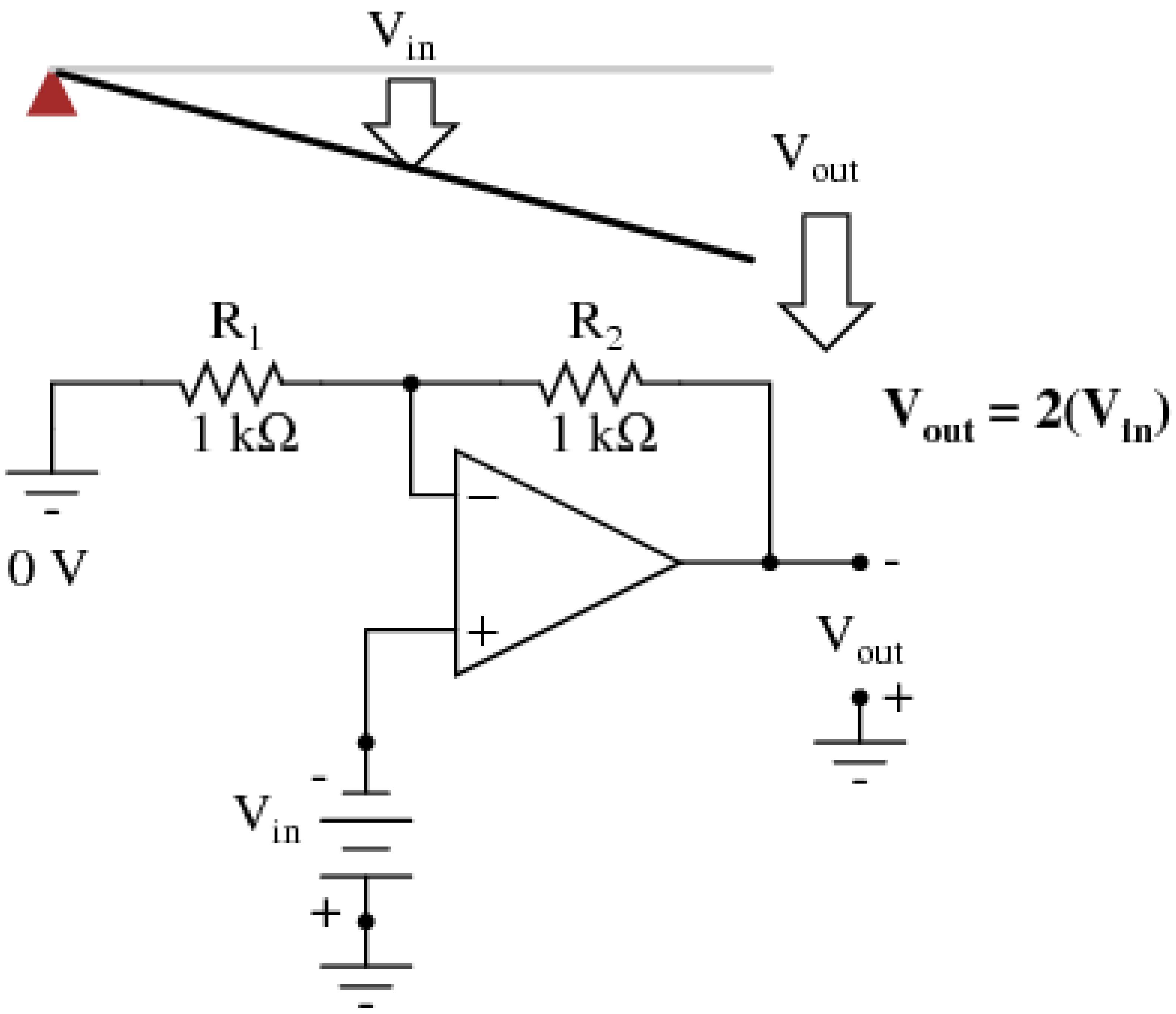



V_{out}
R_1
R_2
V_{in}
$V_{out} = 2(V_{in})$
R_1
R_2
1 kΩ
1 kΩ
0 V
V_{out}
V_{in}

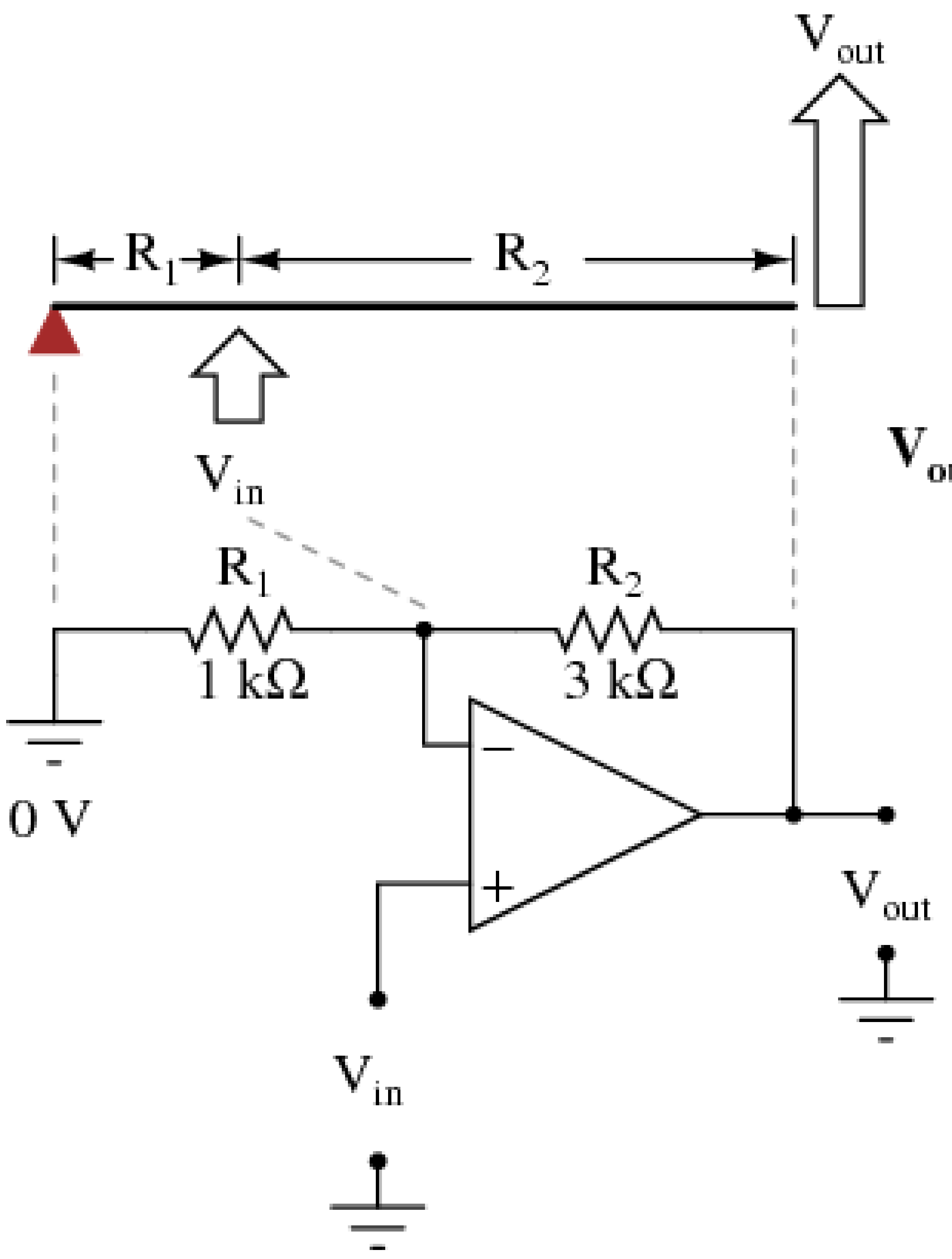

V_{out}
V_{in}
$V_{out} = 2(V_{in})$
R_1
R_2
1 kΩ
1 kΩ
0 V
V_{out}
V_{in}

Similarity of Non-Inverting Amplifier to a Lever

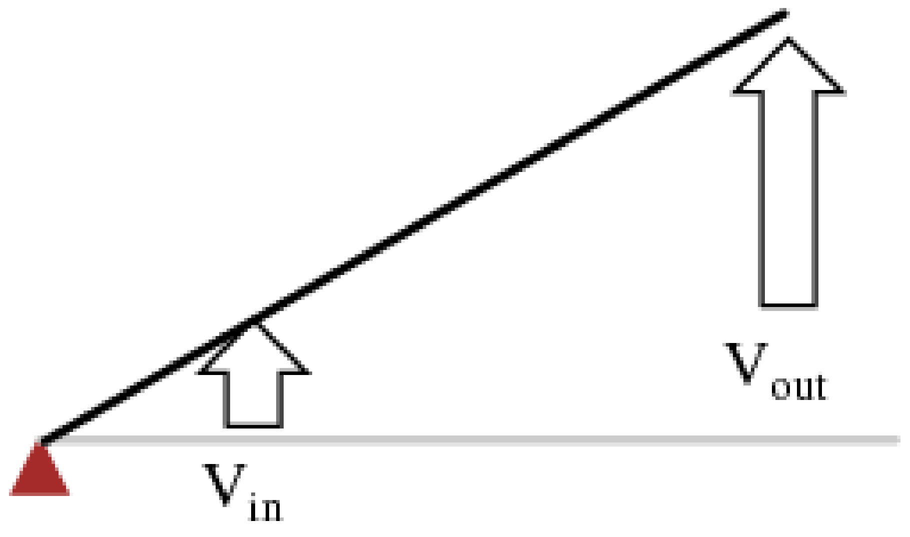




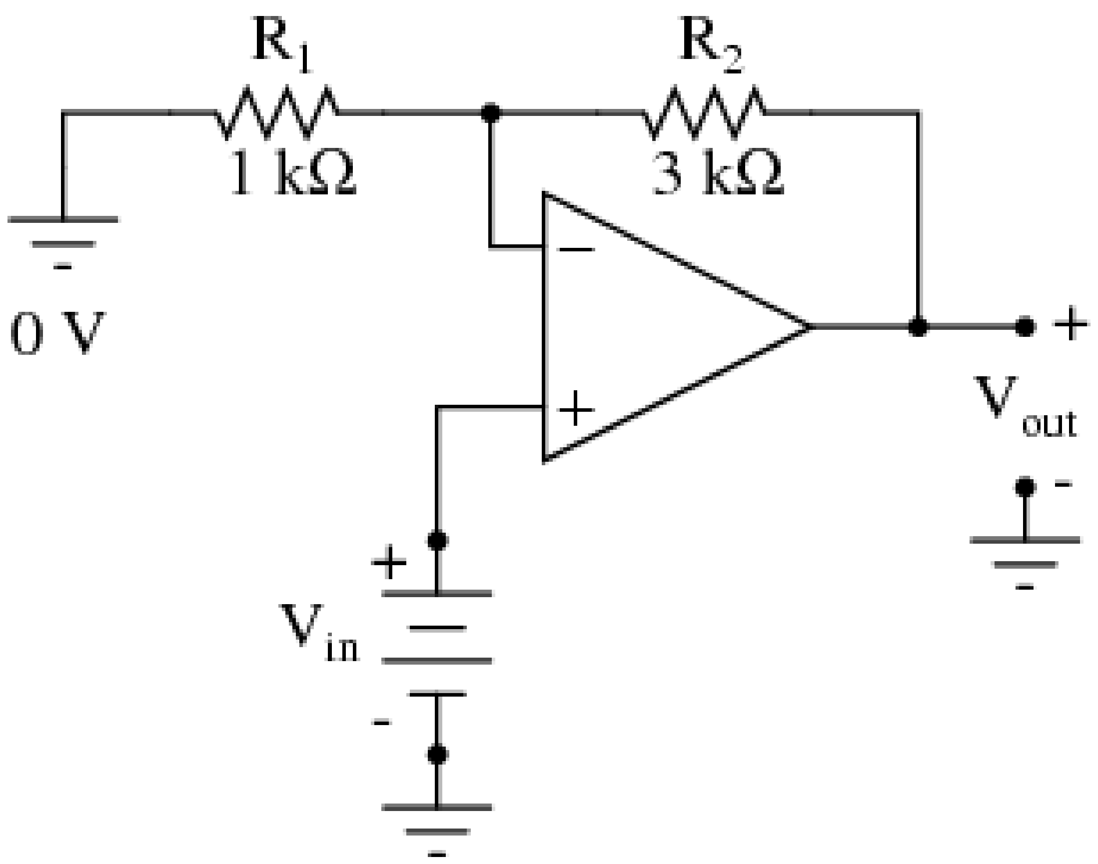
Similarity of Non-Inverting Amplifier to a Lever



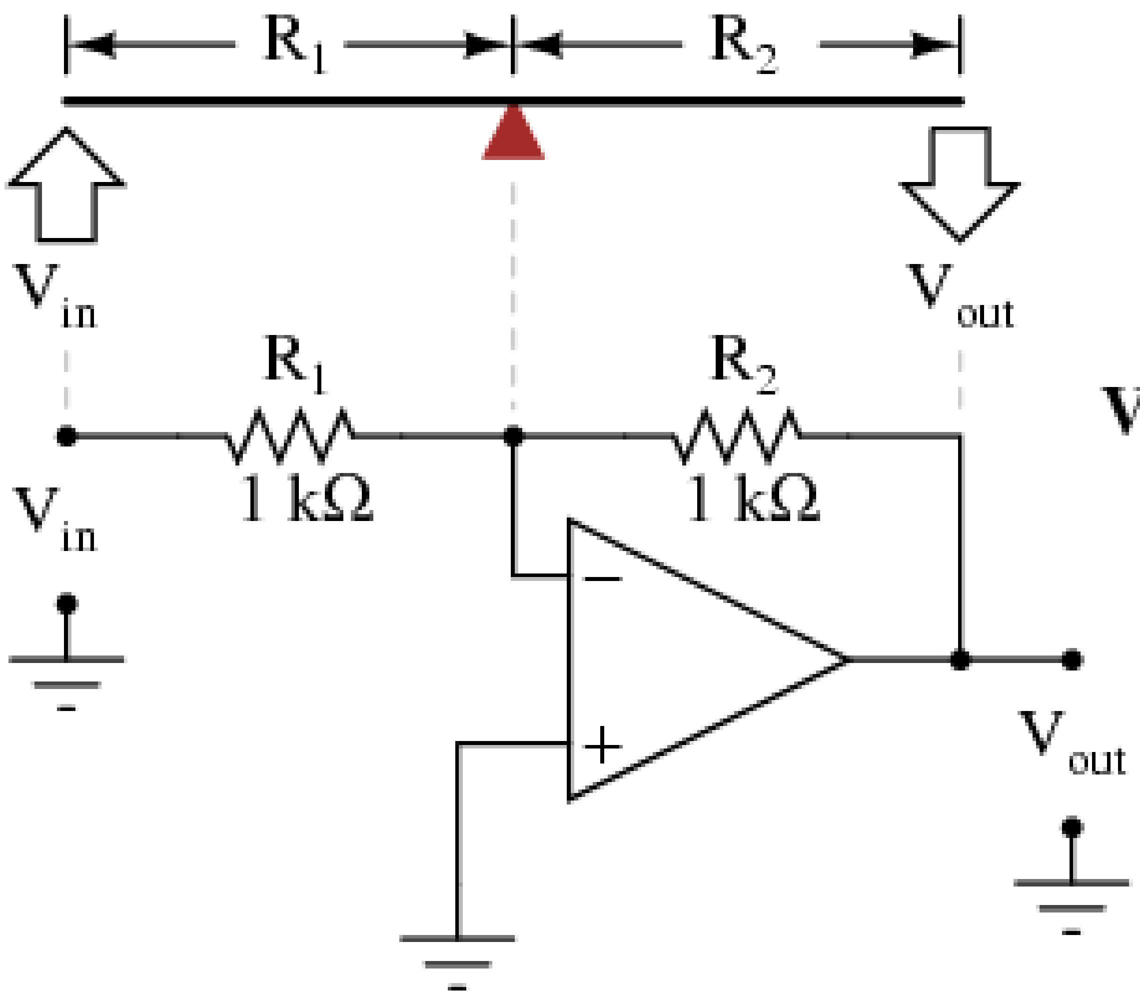
$V_{out} = 4(V_{in})$



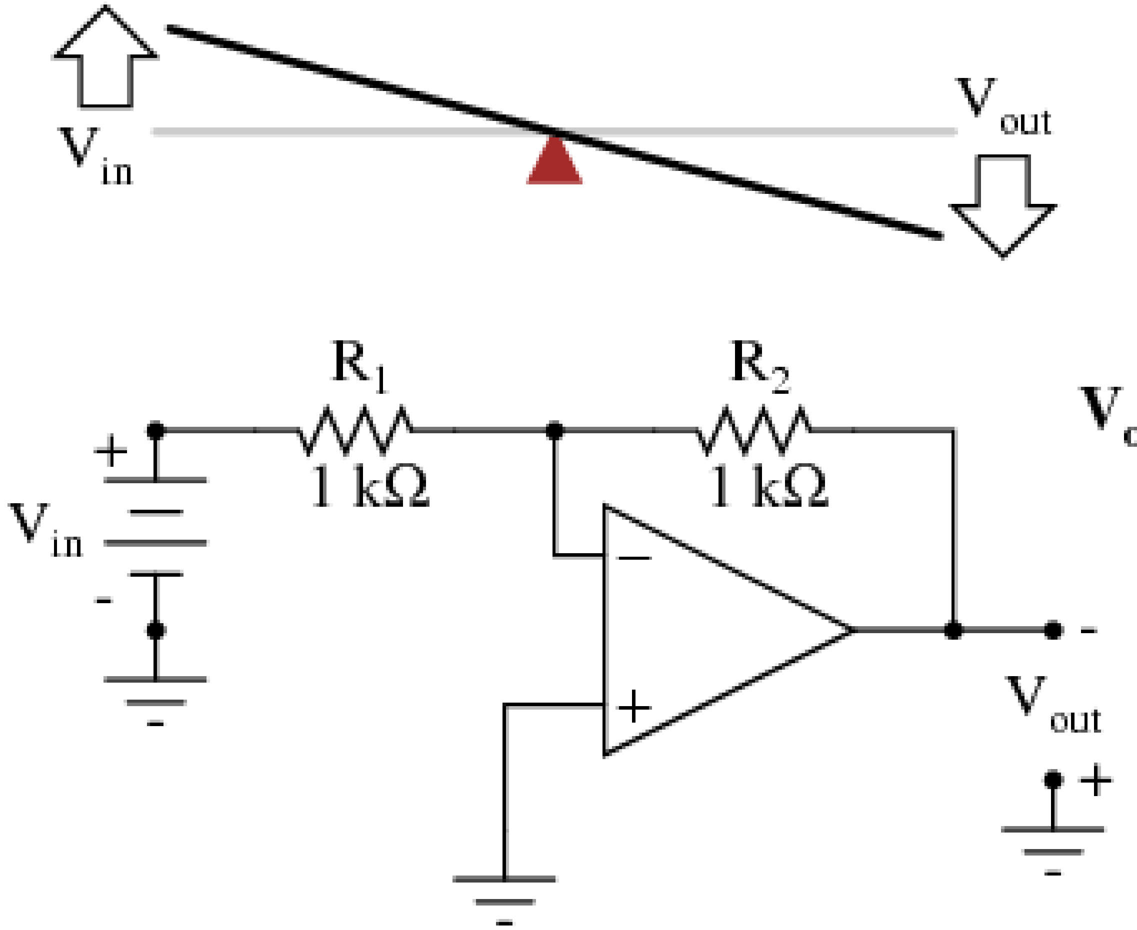
$V_{out} = 4(V_{in})$



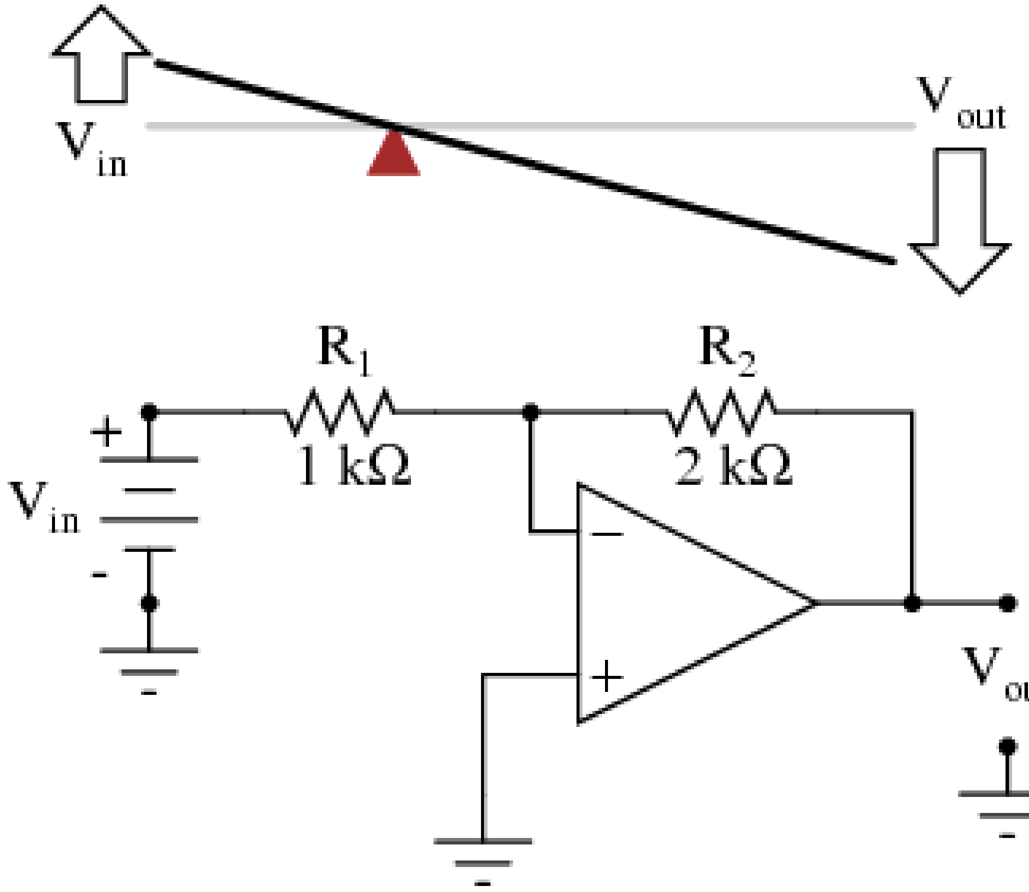
Similarity of Inverting Amplifier to a Lever



$V_{out} = -(V_{in})$



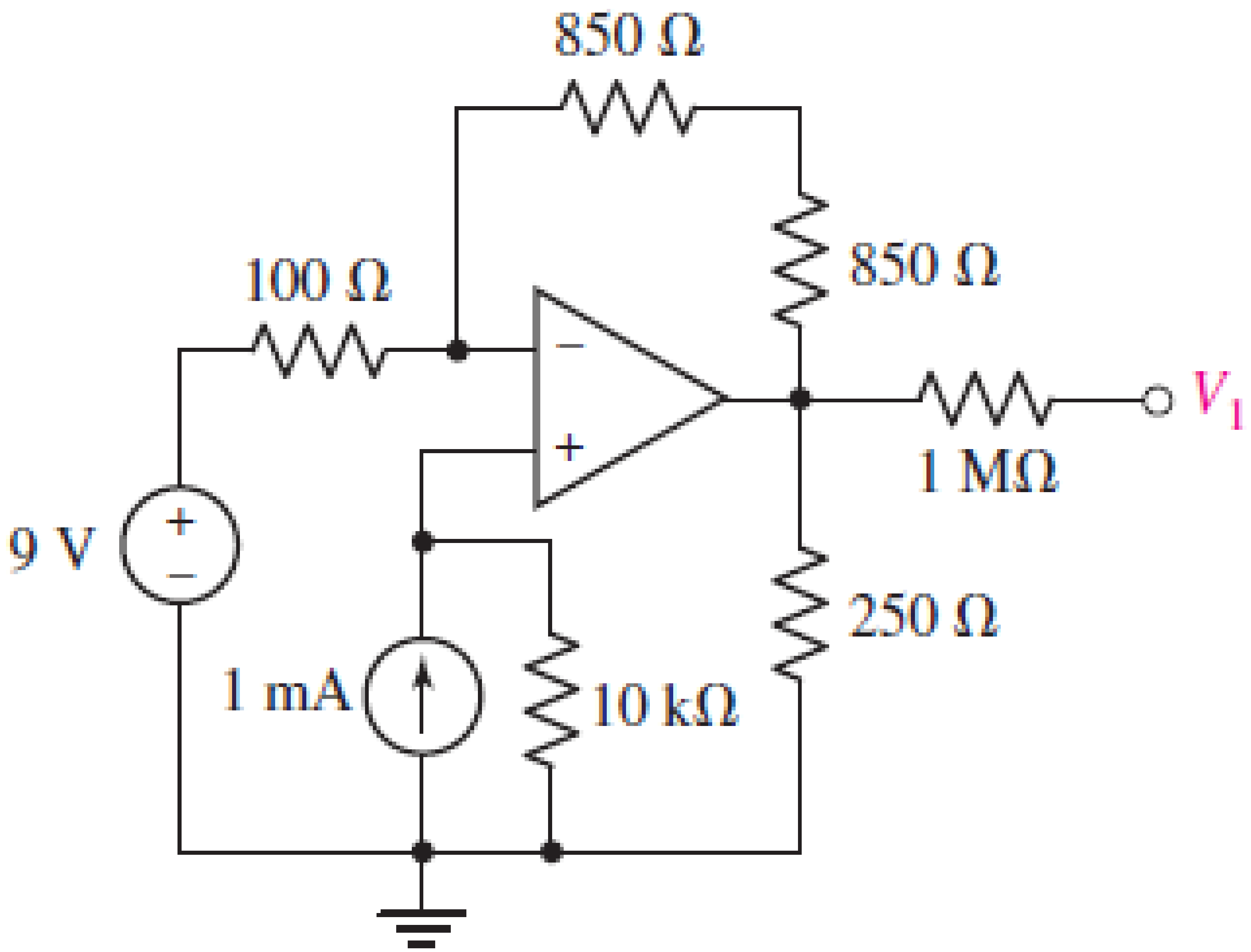
$V_{out} = -(V_{in})$



$V_{out} = -2(V_{in})$

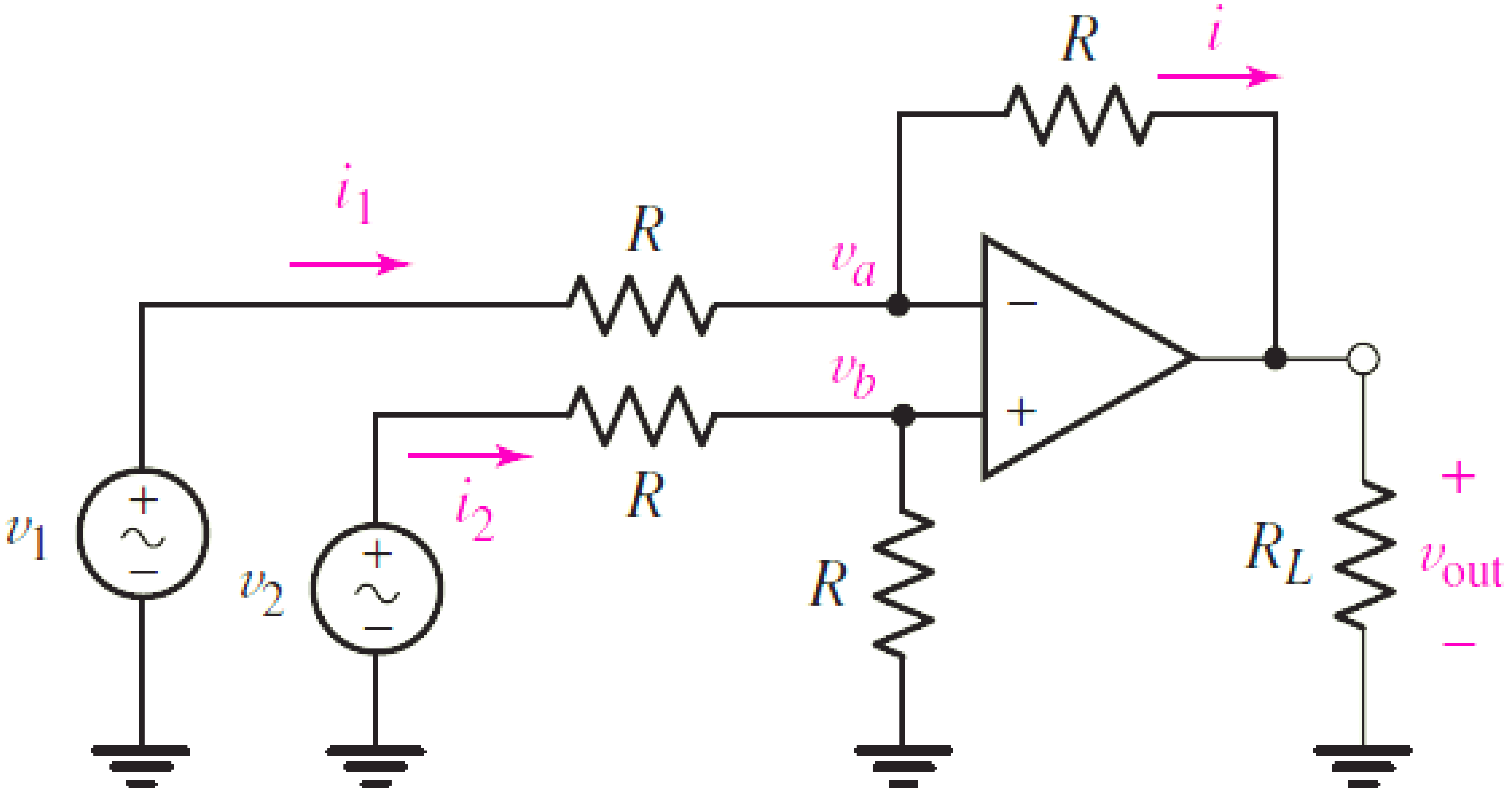
Class Exercise 1

V_1=?

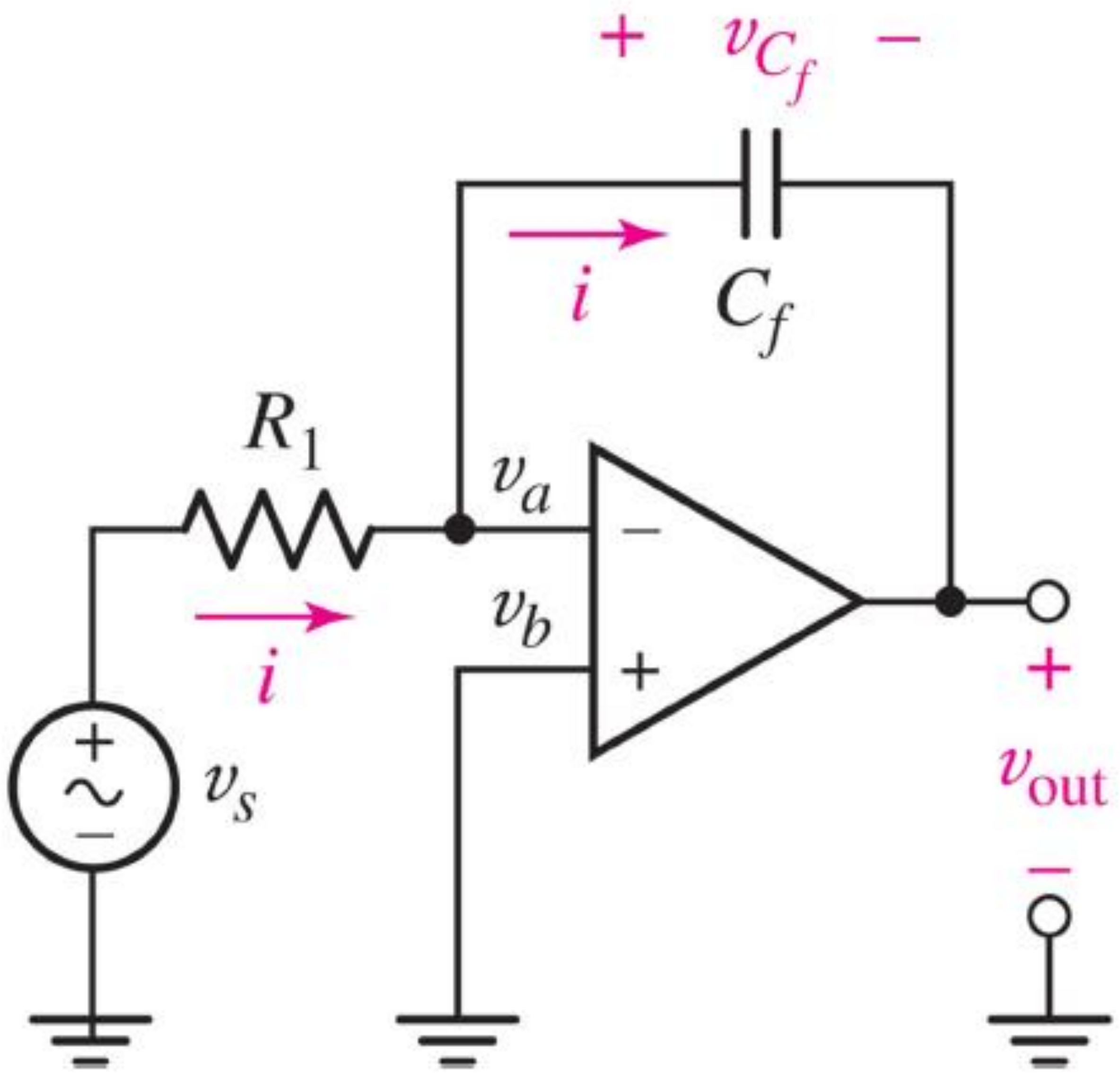


Class Exercise 2

V_{out} =?



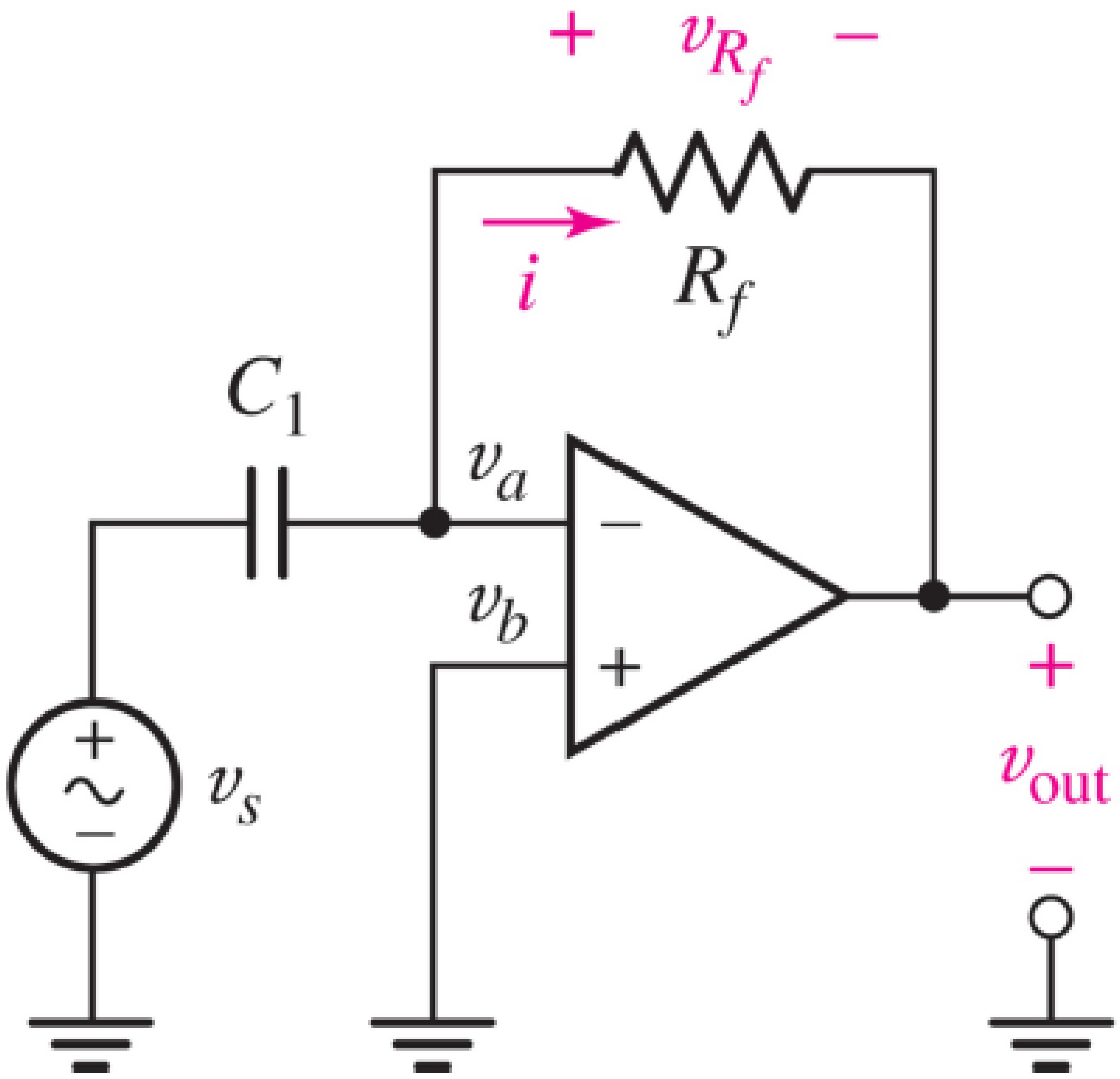
V_{out} =?



$$v_{out} = -\frac{1}{R_1 C_f}\int_0^t v_s\,dt' - v_{C_f}(0)$$

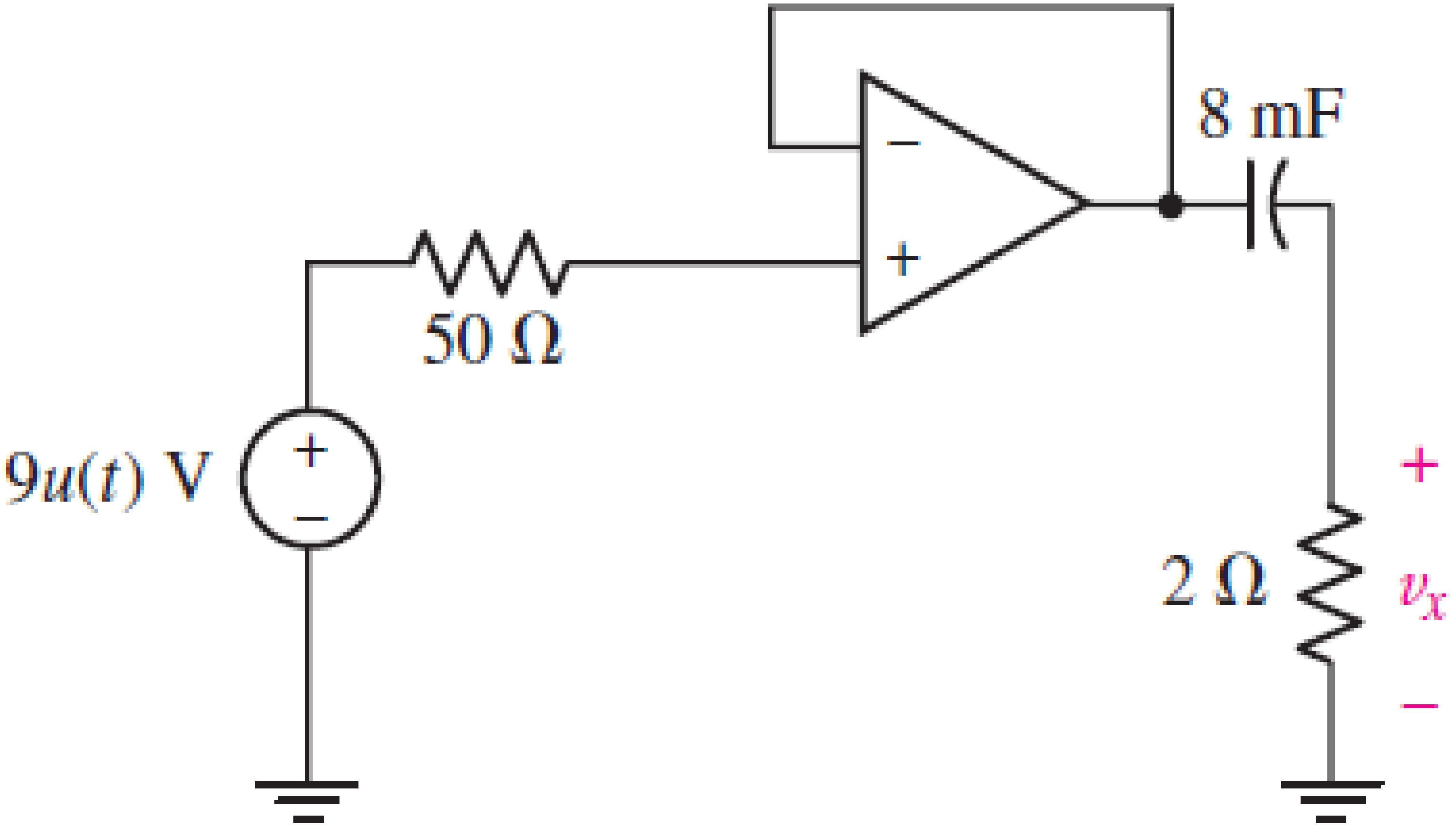
Class Exercise 4

$V_{out} = ?$



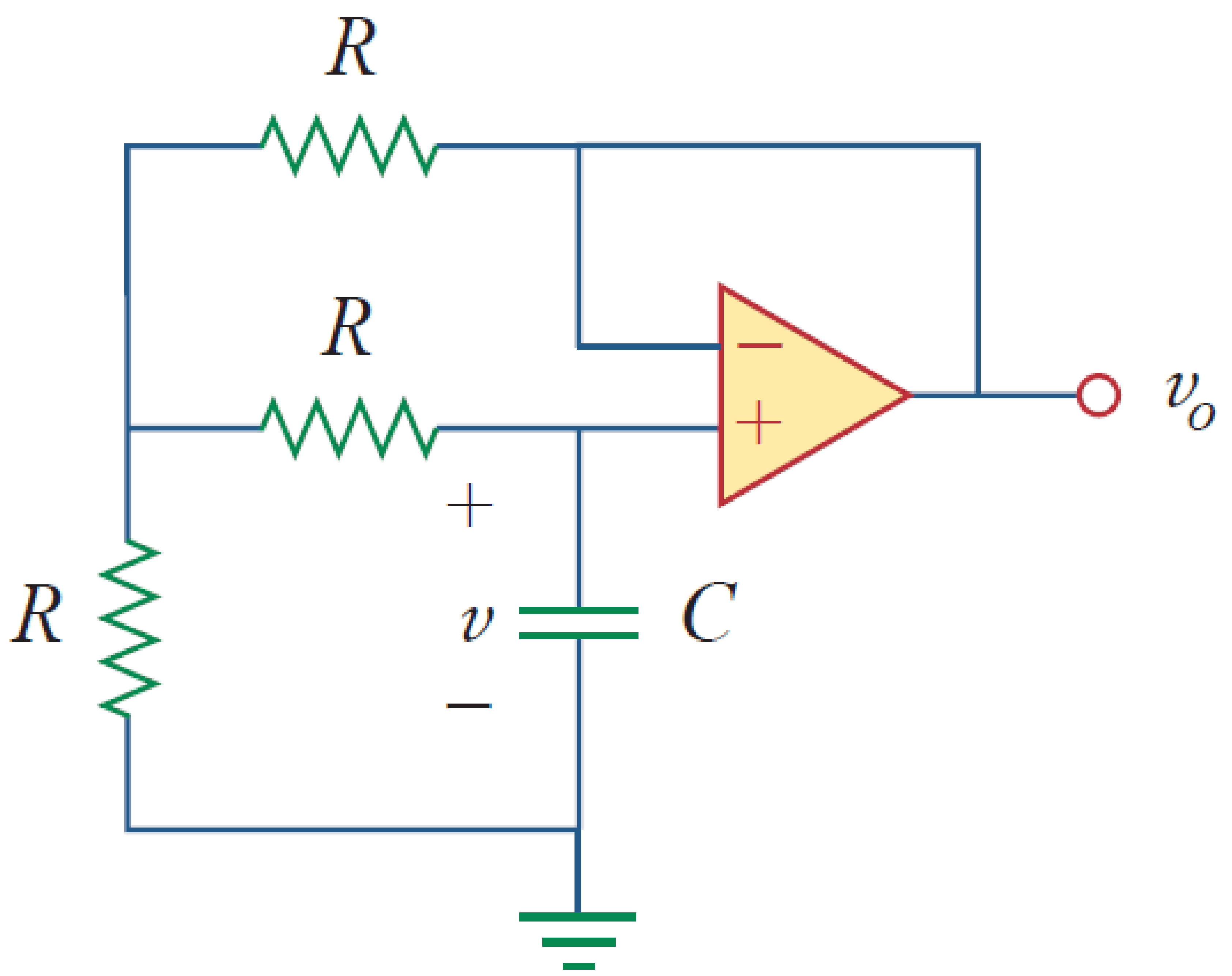
$$v_{out} = -C_1 R_f \frac{dv_s}{dt}$$

v_x=?

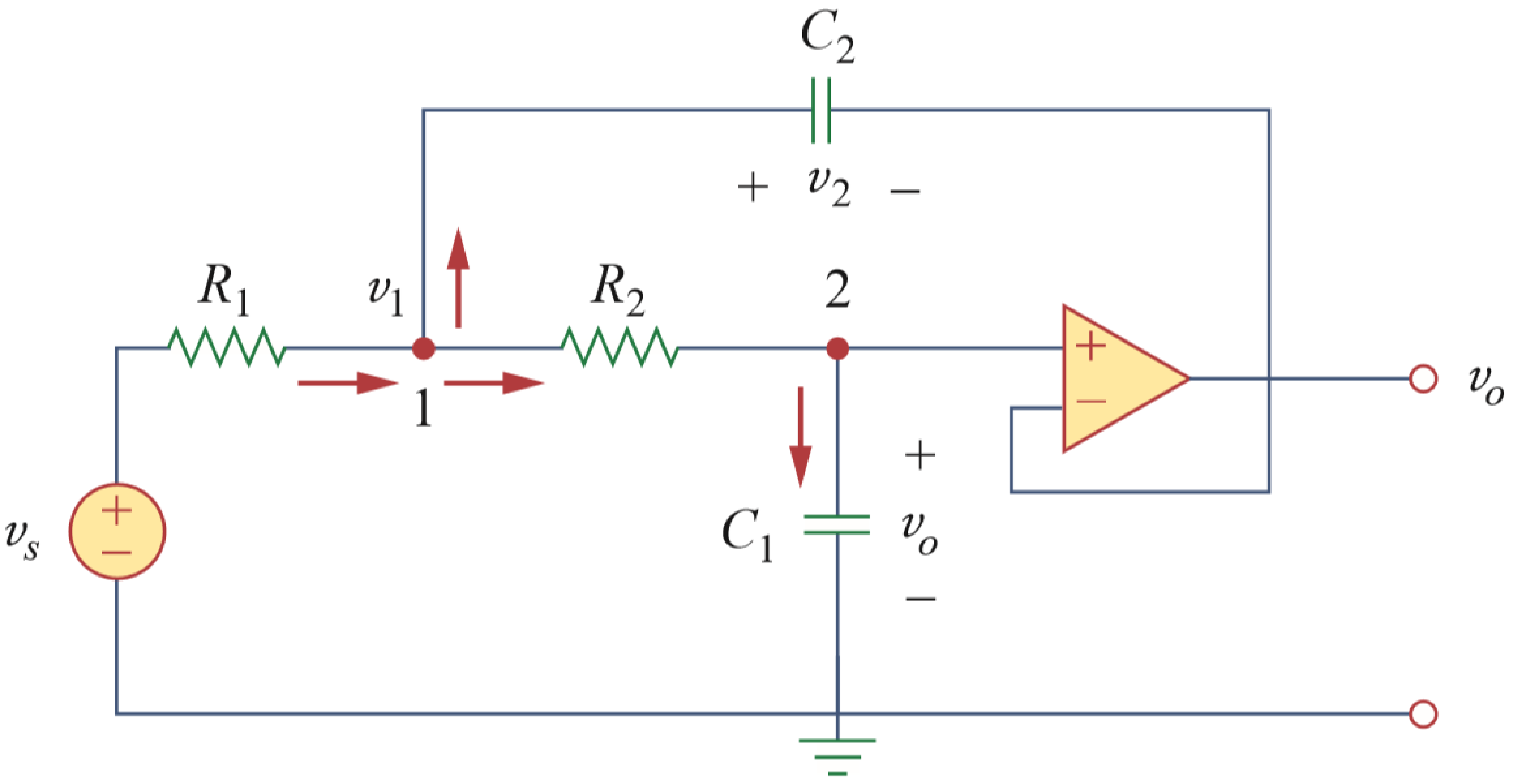


Class Exercise 6

If the initial voltage across the capacitor at time $t = 0$ is 10 volts, determine the output voltage v_o.

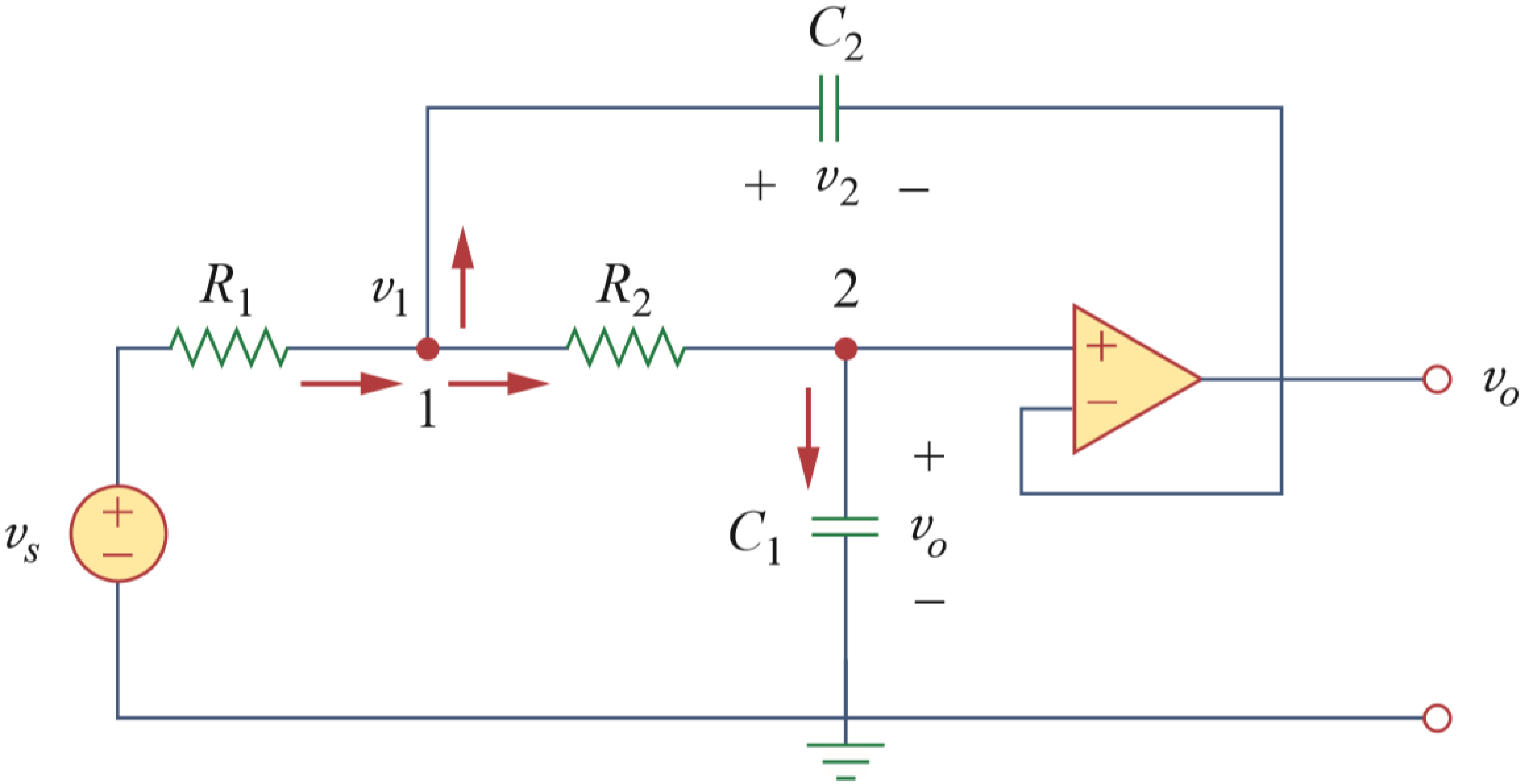


- $v_s = 10u(t)mV, \;\; R_1 = R_2 = 10K\Omega, C_1 = 20\mu F, \; C_2 = 100\mu F$
- $v_o = ?$



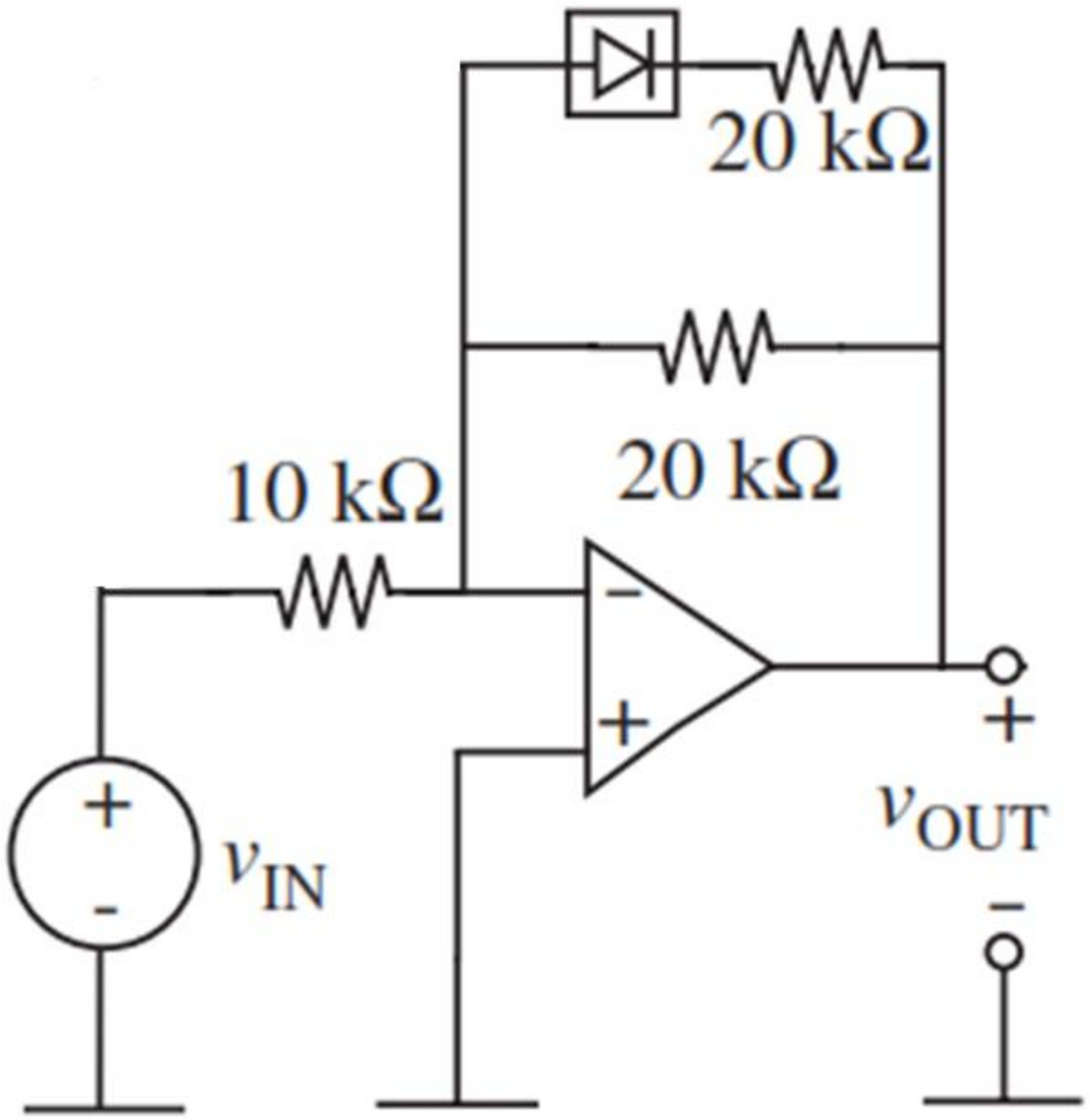
Class Exercise 8

- $v_s = \cos(t)mV,\ R_1 = R_2 = 10K\Omega,\ C_1 = 20\mu F,\ C_2 = 100\mu F$
- $v_o = ?$ (Use phasors).



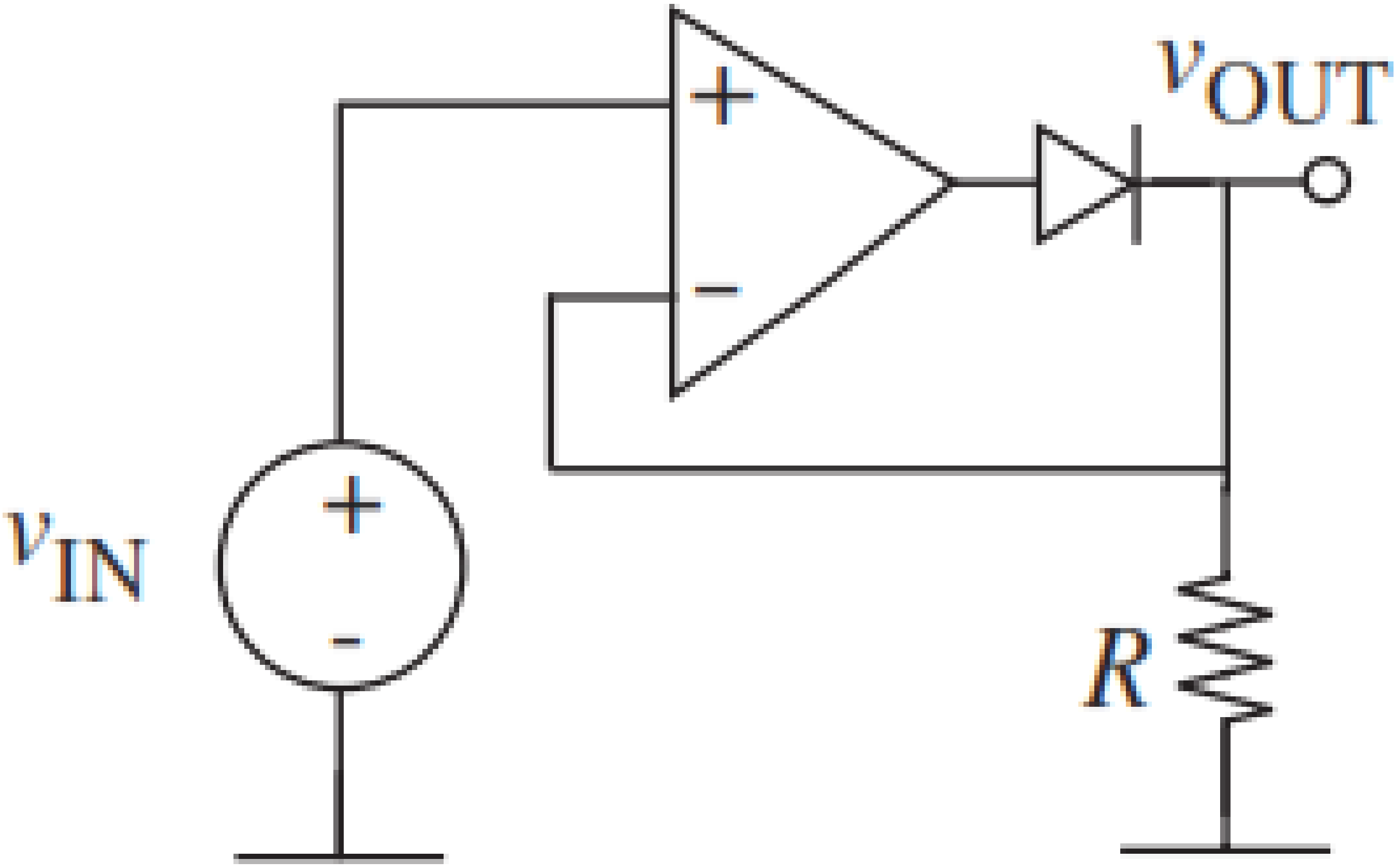
Class Exercise 9

If $v_{in} = \sin(200\pi t)$, plot the output voltage v_{out}.



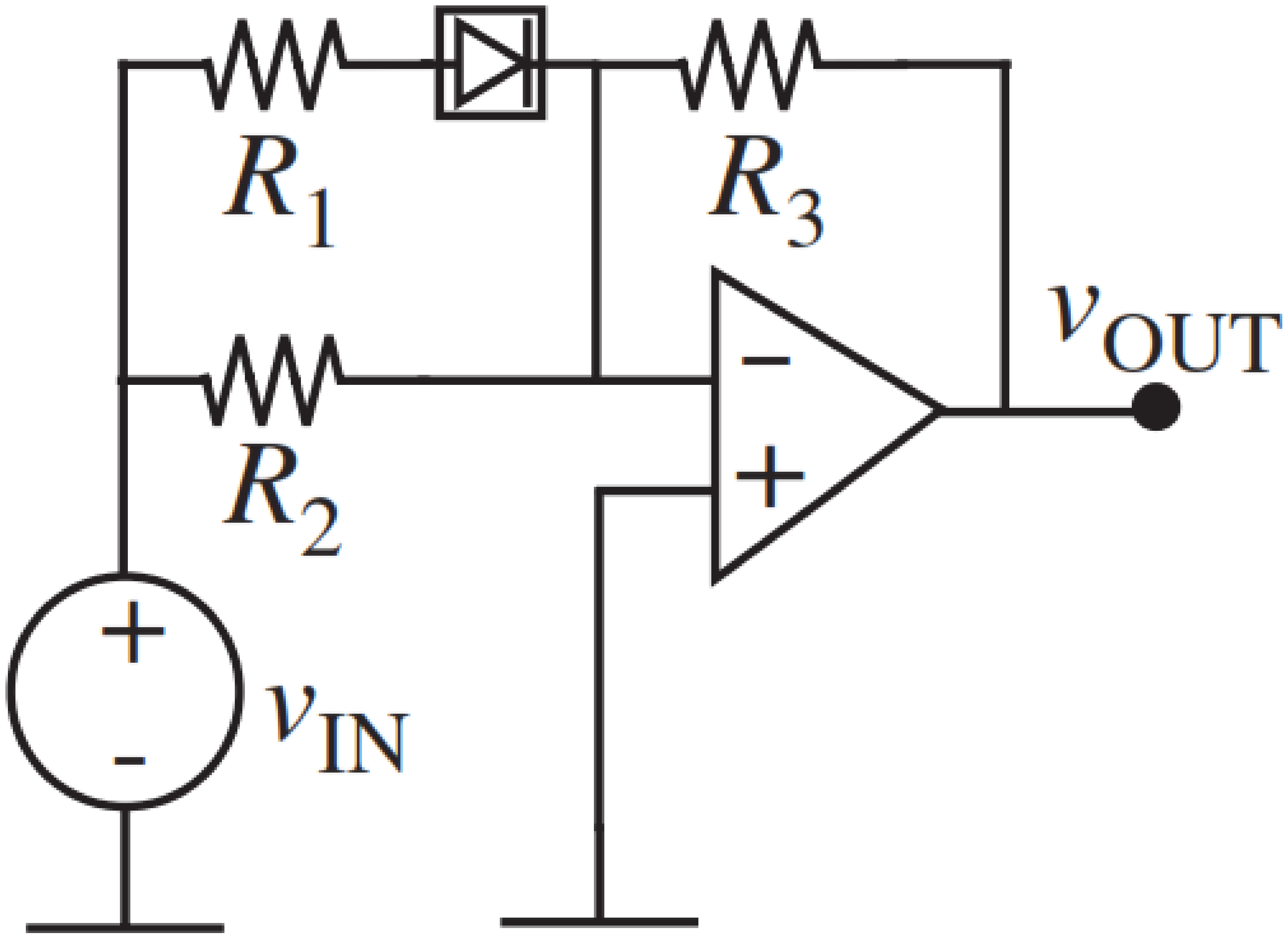
Class Exercise 10

- Plot the diagram of v_{out} as a function of v_{in}.



Class Exercise 11

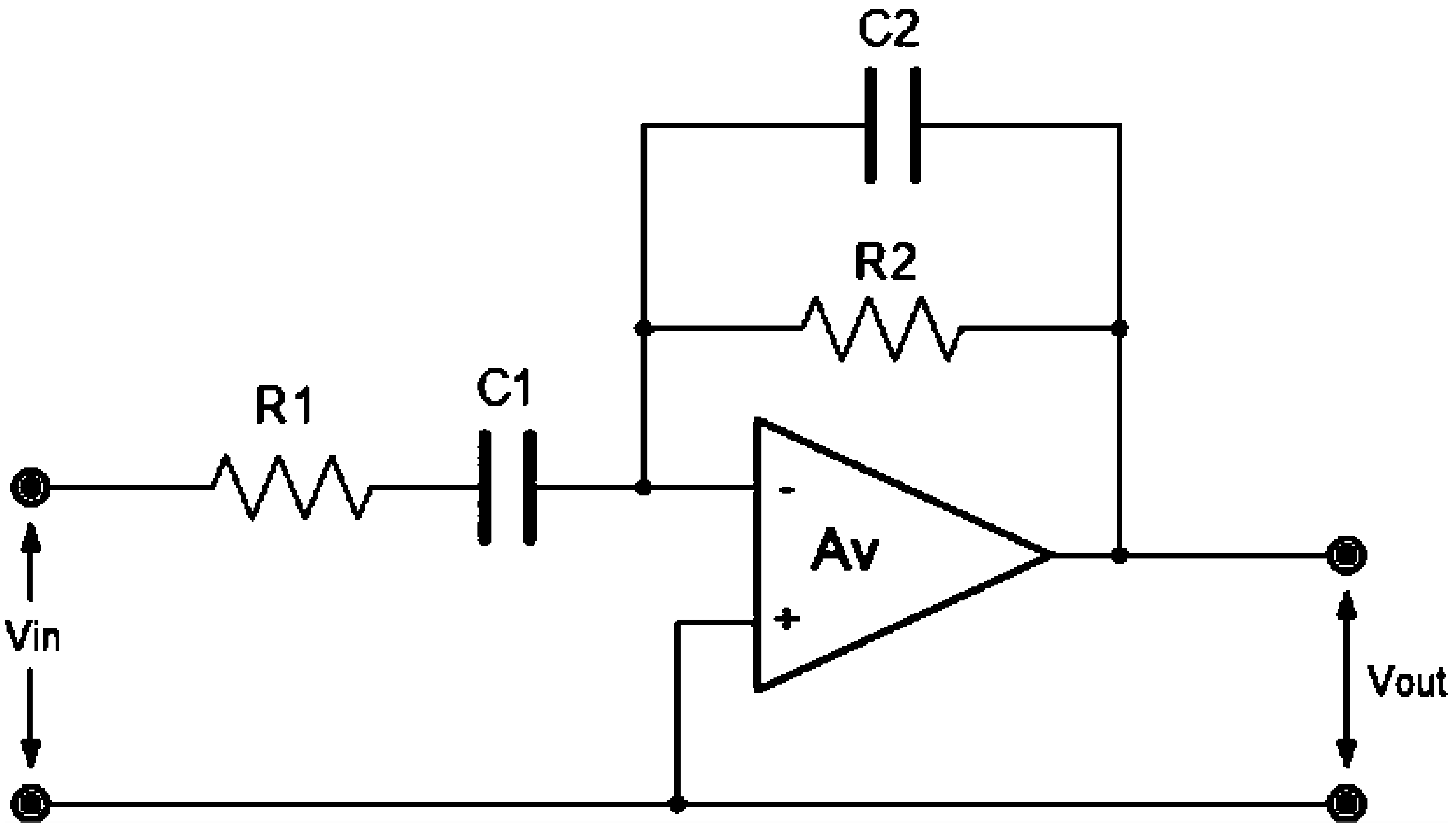
➢ Plot the diagram of v_{out} as a function of v_{in}.



Class Exercise

This is an active filter (using an op-amp in a frequency filter). Calculate:

- ✓ The type of filter
- ✓ The filter gain
- ✓ The cut-off frequency



Thanks